Remarks on quantum noise, negative energy densities and Hadamard regularization

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QFT and Mathematical Physics Seminar, Hamburg University

28 April 2009

Introduction to the problems addressed in this talk

The problem: what is the contribution of quantum fields to the energy of matter?

A calculation of the energy density of quantum fields motivated by the success of quantum mechanics in condensed matter is quite wrong (Nernst 1916, Pauli[a] 1920s: *the radius of the static Einstein universe with this value of* ρ_Λ *"would not even reach to the moon"*)

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Ansätze

The mathematical solution employed in QFT on Minkowski space (normal ordering)... is specific to this space. It can be understood as a subtraction of a vacuum expectation value from the products of fields,

$$:\phi(x)^2:= \lim_{y \to x} \left[\phi(x)\phi(y) - \langle \phi(x)\phi(y) \rangle_{vac}\right]$$

In a curved spacetime we use the "Hadamard parametrix", $H(x, y)$, instead of the vacuum $\langle\rangle$, as the later is too ambiguous. A pattern of non-trivial $\langle :\phi^2(x): \rangle_S$ emerges for every state S. For "quiet states" these are, *luckily*, small (at least away from boundaries and horizons).

Outline of the talk

1. Quantum fields interacting with atoms
2. Experimental characterization of quantum fluctuations
3. Spectrum of the Casimir effect

Interactions of fields with simple quantum systems:

- Two-level atom interacting with quantum electromagnetic field
- Hilbert space: $\mathbb{C}^2 \otimes \mathcal{F}$, where $\mathcal{F}$: Fock space,
- ... $\mathcal{F}$ build upon Ω, not necessarily the vacuum (GNS); single excitations created by $E(f)|\Omega\rangle$
- standard dipole interaction, $V = e\,\vec{x}\cdot\vec{E}(t,\vec{x})$, when restricted to the two levels of the atom:

$$V = e\,\sigma_2 \otimes E_t(\chi),$$

$$E_t(\chi) = \int d^3x\, \vec{E}^i(t,\vec{x}) \cdot \underbrace{\overline{\psi}_e(\vec{x}) x_i \psi_g(\vec{x})}_{\chi:\ \text{wavefunctions}}$$

- The evolution is unitary for all times (χ is real).

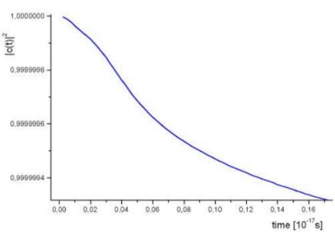
A particularly attractive description possible when states of the field restricted to $\Omega \oplus \{\text{single excitation subspace}\}$. Recalling $V = e\,\sigma_2 \otimes E_t(\chi)$, we find:

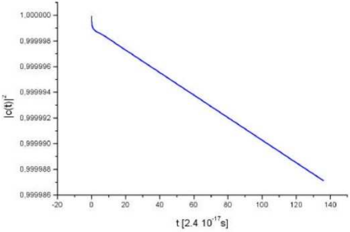
- Starting from $|1\rangle \otimes |\Omega\rangle$ the state never leaves the form

$$\Psi(t) = c(t)\,|1\rangle \otimes |\Omega\rangle + |0\rangle \otimes |E(f(t))\,\Omega\rangle$$

- After a short, exact, computation one arrives at the closed equation

$$\dot{c}(t) = -e^2 \int_0^t d\tau\, e^{i\Delta E\cdot(t-\tau)}\, (\Omega, E_t(\chi)E_\tau(\chi)\,\Omega)\, c(\tau)$$





- Multiscale problem: short structure is rich ($\sim 10^{-18}s$), the intermediate structure is extremely uniform $10^{-18}s - 10^{-10}s$...

- There are no free parameters in the model; numerical approach is complicated by the extreme span of scales.
- There emerges a "revival" if the momenta, $\vec{p}$, of the quantized fields are discretized ($\sim$ reflecting mirrors).
- The whole structure is encoded in (a) the 2-point function of the initial state of the field (b) the atomic wavefunctions.
- Extrapolating arguments (larger e^2) indicate the decay time by 1 order to large (perhaps need to include atomic recoil?)

Interactions of fields and atoms:

The system consisting of a quantum field and a few-level atom is simple enough to allow for an approximate solution of the spontaneous emission problem. In the evolution (the strength of which is controlled by the fine-structure constant e^2) the state of the atom initially entangles with the state of the field. Depending on the 2-point function of the initial state of the quantum field the amplitude of the excited state either becomes very small or “revives”. (quant-ph/0407186)

Photodetector (e.g. photodiode)

- initial state: $|0 \otimes S\rangle$, with a bound-state $|0\rangle$ well-localized around certain x_0, and the state of interest, S, of the quantum field
- final states of the electron: scattering states (P_{sc})
- Perturbative calculation of the response. First order result:

$$W_{exc}(g) = \langle 0 \otimes S|\, U_g^*(P_{sc} \otimes \mathbf{1})U_g\, |S \otimes |0\rangle$$

explicitly

$$W_{exc}(g) = \int g(\tau)g(s)\, d\tau\, ds\, \underbrace{\langle 0|x^i(\tau)P_{sc}\, x^j(s)|0\rangle}_{\text{electronic correlation funct.}} \cdot \underbrace{\langle E_i(\tau, x_0)E_j(s, x_0)\rangle_S}_{\text{field correlation funct.}}$$

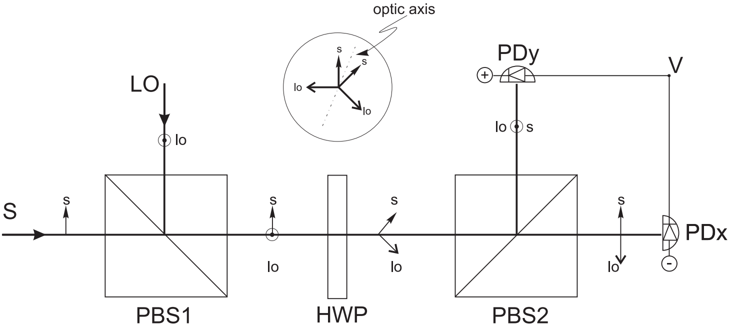
- for many interesting states $W_{exc}(g)$ is unmeasurably small

Balanced homodyne detector, frequency, phase

- Two photodiodes with their output subtracted
- External, coherent, monochromatic light (LO) "blended" with S

$$\langle pol[E(t,x)]\rangle_{S\ and\ LO} = \langle pol[E_{LO}(t,x) + E(t,x)]\rangle_{S}$$

- Balancing: $|E_{LO}(t,x_0)| = |E_{LO}(t,y_0)|$
- Statistic properties of the state S de-balance the detector (stochastic process of measurement)



Charge J accumulated between the diodes

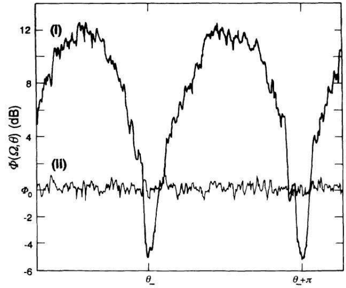
- expectation value

$$\langle J \rangle = \alpha_{el} \cdot E^i_{LO} \cdot \big\langle E_i(t_0, x_0) \big\rangle_S$$

- standard deviation (if exp. value vanishes)

$$\langle J^2 \rangle = \alpha^2_{el} \cdot \underbrace{E^i_{LO} E^j_{LO}}_{LO\ power} \cdot \underbrace{\big\langle E_i(t_0, x_0)\, E_j(t_0, x_0) \big\rangle_S}_{Quantum\ field\ 2pt\ funct.}$$

- purpose: measure properties of the state S for a well-characterized LO
- all field operators are restricted to the frequency of the LO
- α_{el} depends on the electronic structure of the semiconductor
- t_0 is the LO phase and can be varied easily in experiments
- $\langle J^2 \rangle$ is proportional to LO power



Summary (detectors):

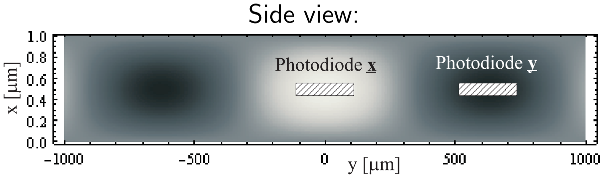
By exploiting *a trick* with subtraction of the output balanced photodiodes it is apparently possible to *quantify* the fluctuations of the quantum field (even in the vacuum!). Quantity of interest: $\left\langle E_i(t_0, x_0)\, E_j(t_0, x_0)\right\rangle_S$ (fields restricted to the frequency of the local oscillator). Relative character of the zero-level set by the vacuum is uncovered by the squeezed states of light (above). In some regions they are "darker than vacuum". (quant-ph/0703076)

Electromagnetic fields in waveguides; quantization; ground-state

- Electromagnetic fields in stationary, $z-$invariant cavities expressed thru two scalar potentials $\mathcal{E}, \mathcal{M}$, each of which fulfills the d'Alembert equation with Dirichlet, Neumann boundary conditions on the surface.
- Electromagnetic fields expressed by the second-order partial derivatives of the potentials. In the TE case, e.g.

$$B^x = \partial_z \partial_x \mathcal{M},\ E^x = \partial_t \partial_y \mathcal{M},\ B^z = -(\partial_x^2 + \partial_y^2)\mathcal{M}.$$

- The potentials are quantized as independent scalar fields. The two-point functions have a form of "sums of images" ($\sim$ electrostatics).



- Central idea: consider the Fourier transform of the two-point function

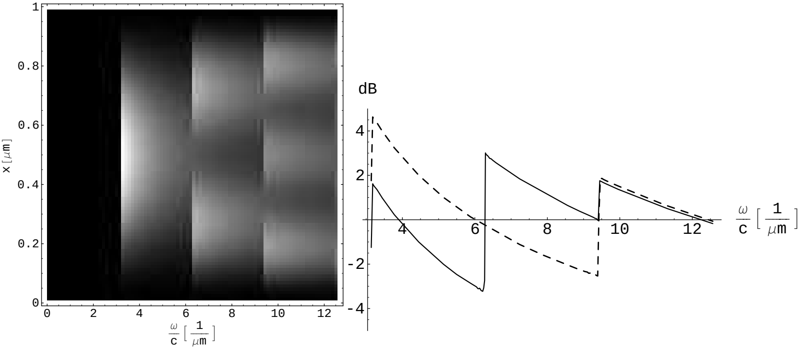
$$\sigma_{ij}(\omega, \vec{x}, \vec{y}) = \mathcal{F}_t \langle E_i(t, \vec{x}) E_i(0, \vec{y}) \rangle_G$$

- This quantity (spectral density) is simply related to the output of an balanced detector with LO of frequency ω

$$E^i_{LO} E^j_{LO} \cdot \left\langle E_i(t_0, \vec{x})\, E_j(t_0, \vec{x}) \right\rangle_S \approx \int d\omega\, k^i(\omega) k^j(\omega)\, \sigma_{ij}(\omega, \vec{x}, \vec{x})$$

- Shortly: $\sigma_{ij}(\omega, \vec{x}, \vec{x})$ is the fluctuation of the field of frequency ω at $\vec{x}$.

Spectral density, sub-vacuum fluctuations (Hadamard)



[Left:] Casimir ground-state spectral density normalized by Hadamard density, $[\sigma_S(\omega,\vec{x},\vec{x}) - \sigma_H(\omega,\vec{x},\vec{x})]/\sigma_H(\omega,\vec{x},\vec{x})$, (with $\vec{x} = (x,0,0)$), as a function of the position $x \in [0,a]$ between the plates (plate separation, $a = 1\mu$ m is assumed). Negative values (suppression of fluctuations) are black. [Right:] Suppression of fluctuations in dB, that is $10 \cdot \log_{10}[\sigma_S(\omega,\vec{x},\vec{x})/\sigma_H(\omega,\vec{x},\vec{x})]$, for $x = 0.25a$ (solid) and $x = 0.5a$ (dashed). Frequency range is $\omega \in [0, 4\pi c/a]$

Summary (Casimir):

Casimir setups are the simplest nontrivial modifications of the homogeneous (Minkowski) situation. Fluctuations in the ground state are lower than the Minkowski-vacuum ones in some regions, for some frequencies. Minkowski-vacuum two-point function provides the simplest case of the Hadamard parametrix. (arXiv:0711.1541)

Summary of the talk

- There are various situations where subtle QFT effects can be seen.
- In the case of an atom the spontaneous emission is directly influenced by the 2pt function (measure of fluctuations) of the initial state of the quantum field.
- Balanced detectors provide a tool quantifying the diagonal values of the (frequency-restricted) two point functions.
- In the Casimir situation there is a rich (frequency-, position-, polarization-) dependent pattern to look for in, hopefully, future experiments.

Spectral densities and measured quantities

- electric field in ground-state representation restricted in frequencies

$$E(t,x_0)|_{k(\omega)} = \int d\nu(p^a) k^i(\omega_p) \left[e^{-i\omega_p t}\, \psi_i(p^a,x_0) b(p^a) + e^{i\omega_p t}\, \overline{\psi_i(p^a,x_0)} b^*(p^a) \right],$$

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- spectral density $\sigma_{ij}(\omega,x_0,y_0)$: two-point function Fourier-transformed w.r.t t
- relation between $\langle J^2\rangle_G$ (LHS) and smeared spectral density (RHS)

$$\int d\nu(p^a)\,k^i(\omega_p)k^j(\omega_p)\,\psi_i(p^a,x_0)\overline{\psi_j(p^a,y_0)} = \int d\omega\,k^i(\omega)k^j(\omega)\,\sigma_{ij}(\omega,x_0,y_0)$$