Detector operators and sub-vacuum fluctuations in Quantum Optics

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Erwin Schrödinger Institute, March 30th 2007

Outline:

1. Quantum field-theoretical models of photodiodes and balanced homodyne detectors
2. Expectation values of $\langle E^2(t) \rangle$ and bounds from QEI's
3. States 'darker than vacuum' (squeezed states)

(1) Detectors; part I: photodiode

- Goal: a state S (vacuum, thermal, coherent, ...) of the quantum radiation field needs to be characterized (n-point functions).
- Typical solution in QO: use well-characterized quantum systems interacting in a simple way with the quantum radiation field
- model of an PIN junction: single electron interacting with a quantum radiation field
- simple interaction means: the state space of the electron can be severly restricted, interaction can be assumed linear in the quantum field, Born-approximation will be employed

Photodiode...

- initial state: $|0\rangle \otimes S$, with a bound-state $|0\rangle$ well-localized around certain x_0, and the state of interest, S, of the quantum field
- the final states, $|\vec{q}\rangle \otimes |\tilde{S}\rangle$ will be integrated over
- Simplification: the interacton is linear (dipole approximation)

$$V_{int} = ex^i \cdot E_i(t, \vec{x})$$

- Perturbative calculation of the response. First order result:

$$P_{exc}(g) = \underbrace{\int d\tau\, ds\, g(\tau)g(s)}_{\text{temporal sensitivity}} \underbrace{\int dq\, \langle 0|x^i(\tau)|\vec{q}\rangle\langle\vec{q}|x^j(s)|0\rangle}_{\text{electronic correlation funct.}} \underbrace{\langle E_i(\tau, x_0)E_j(s, x_0)\rangle_S}_{\text{field correlation funct.}}$$

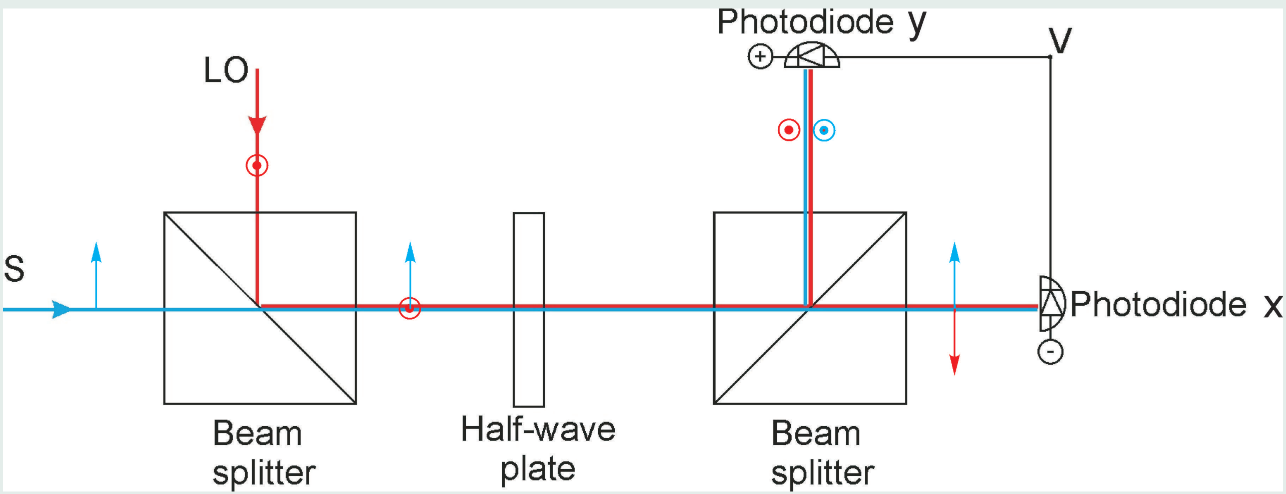
- $g(t)$ is smooth and equal to 1 during the measurement
- for many interesting states $P_{exc}(g)$ is unmeasurably small
- corresponding field-theoretical observable can be well-approximated by a local, positive observable

Balanced homodyne detector with a local oscillator

- Two photodiodes with their output subtracted
- External, coherent, monochromatic light (LO) “blended” with S

$$\langle pol[E(t,x)]\rangle_{S\ and\ LO} = \langle pol[E_{LO}(t,x) + E(t,x)]\rangle_S$$

- The quantum field (S) de-balances the detector (stochastic process of measurement)



Charge J accumulated at the point V for $g(t) \to 1$

- expectation value

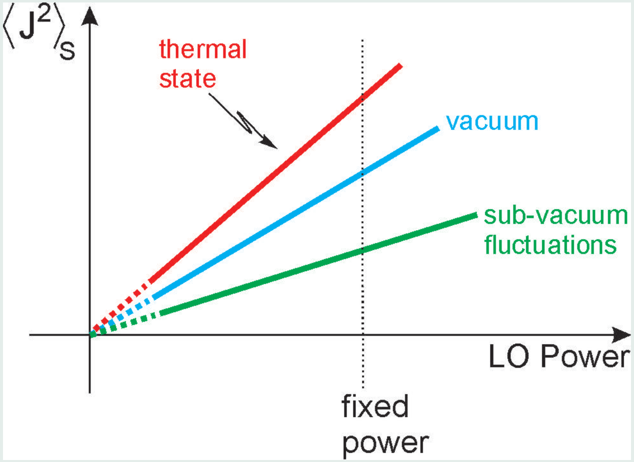
$$\langle J \rangle = \alpha_{el} \cdot E^i_{LO} \cdot \big\langle E_i(t_0, x_0) \big\rangle_S$$

- standard deviation (if exp. value vanishes)

$$\langle J^2 \rangle = \alpha_{el}^2 \cdot \underbrace{E^i_{LO} E^j_{LO}}_{LO\ \ power} \cdot \underbrace{\big\langle E_i(t_0, x_0)\, E_j(t_0, x_0) \big\rangle_S}_{Quantum\ \ field\ \ 2pt\ \ funct.}$$

- all field operators are restricted to the frequency of the LO
- α_{el} depends on the electronic structure of the semiconductor
- t_0 is the LO phase and can be varied easily in experiments
- $\langle J^2 \rangle$ is proportional to LO power

Various states of Quantum Fields on BHD with LO



Balanced detectors with local oscillators are amplifiers capable of measuring the one- and two-point functions of arbitrary states of quantum fields.

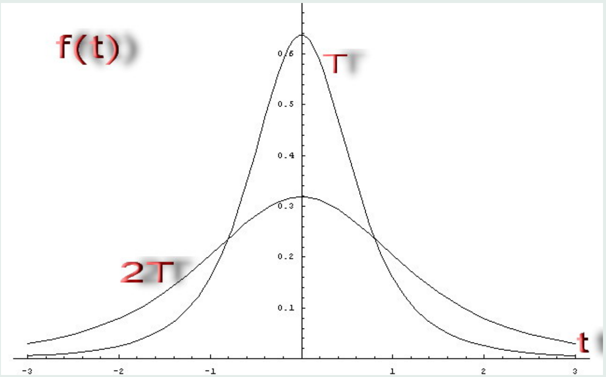
(2) Quantum Inequalities for $\langle E^2(t) \rangle$

- The electric field must obey

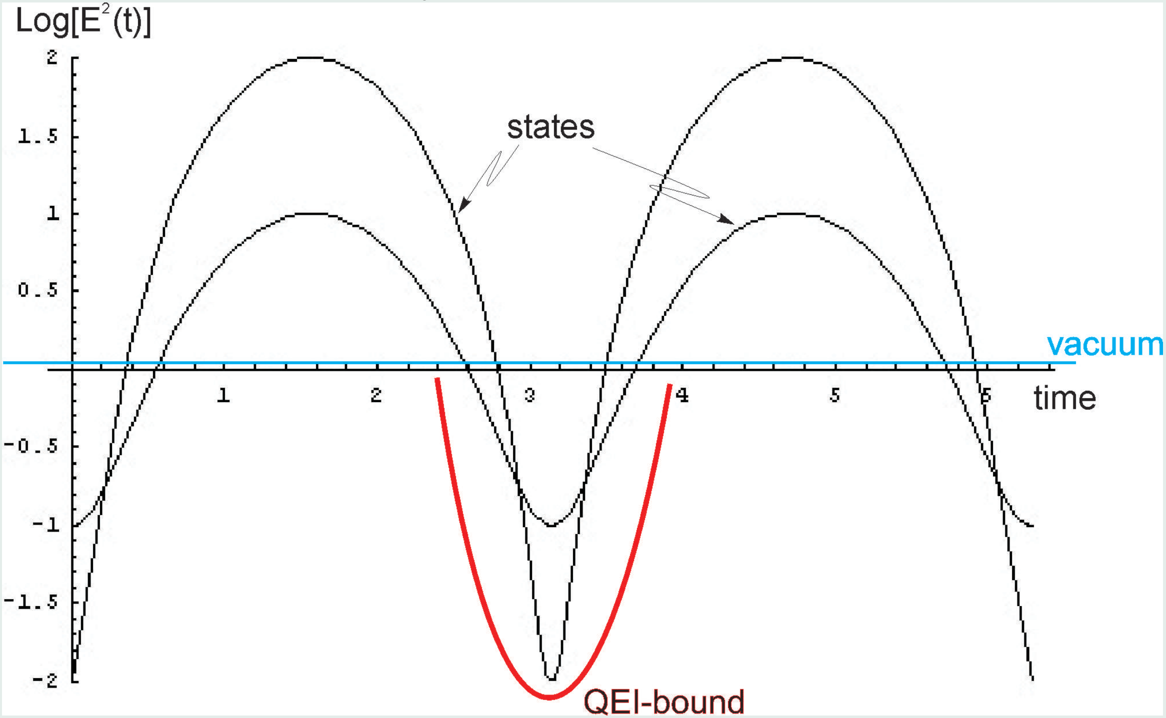
$$\int dt\, f(t) \left\{ \langle : E^2(t,x) : \rangle_S \right\} \geq -\frac{1}{2\pi^2} \int_0^\infty d\omega \int_{R^3} p\, d^3p \left| \widehat{\sqrt{f}}(\omega + p) \right|^2$$
$$\gtrsim -\frac{const}{T^4}$$

- f(t): real, positive, probability density; *const* does not depend on S; T is the 'duration' of measurement
- if restricted in frequencies (fields and the d^3p integral), then

$$\langle : E^2(t,x) : \rangle_S = \langle E^2(t,x) \rangle_S - \langle E^2(t,x) \rangle_\Omega$$

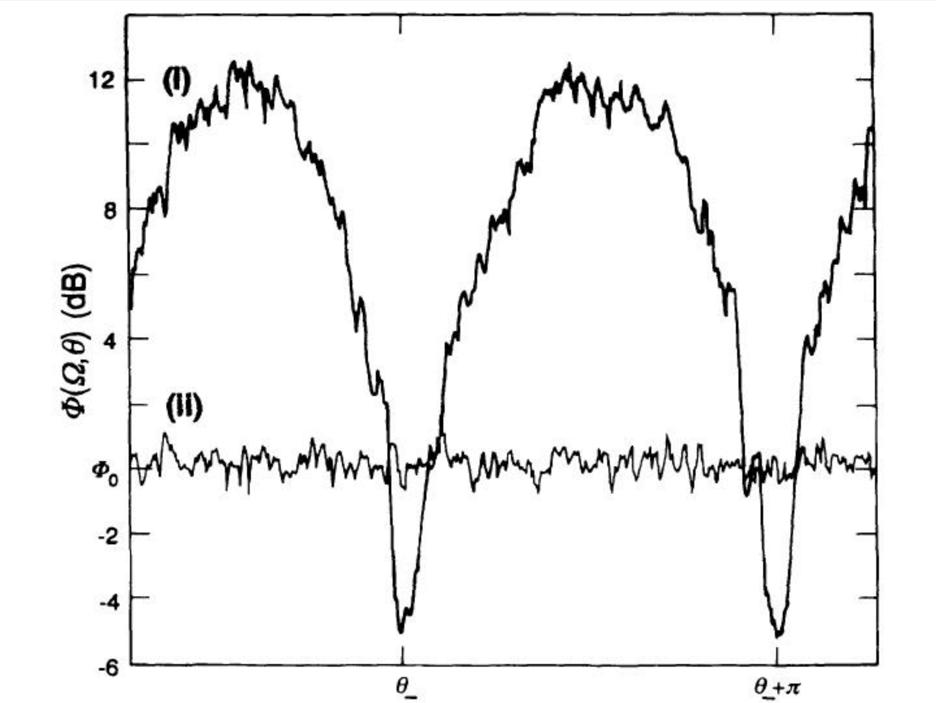


QEI's: qualitatively...



The magnitude times duration of sub-vacuum expectation value of $E^2(t,x)$ is bounded by QEI-like inequality.

(3) Squeezed states of light



Expectation value of $E^2(t)$ (output of a BHD) for a squeezed state [Polzik, Carri, Kimble, PRL 1992].

Description of squeezed states

- States of the type

$$|S\rangle = N \exp[r a^* a^*] \, |\Omega\rangle$$

(superpositions of pairs of photons)

- Expectation value of the squared electric field (restricted to ω)

$$\langle E^2(t) \rangle_S = \cosh(r) - \sinh(r) \cos(2\omega t)$$

(with $\langle E^2(t) \rangle_\Omega = 1$)

- Calculation shows, that $\langle T_{00}(t) \rangle_S$ also has sub-vacuum periods

Summary

- BHDs are capable of measuring one- and two-point functions of QFT-states (even for the vacuum)
- Sub-vacuum $\langle E^2(t,x)\rangle_S$ are restricted by QEI's
- Squeezed states have regions with sub-vacuum $\langle E^2(t,x)\rangle_S$ and $\langle T_{00}(t,x)\rangle_S$
- Details to be found in
  - quant-ph/0703076 (detectors in QFT)
  - Phys. Rev. A 66, 053801 (squeezed states and QEI's)