Time domain restrictions on and the (Casimir) cavity expectation values of the quantum noise

Piotr Marecki
Leipzig University

Laserinterferometrie Seminar AEI Hannover, 25 April 2008

Outline:

1. Quantum Inequalities as restrictions on the fluctuations of the quantum electric field, $\langle : E^2(t, \vec{x}) : \rangle_S$
2. Remarks about balanced homodyne detection
3. BHD response to ground state quantum noise in Casimir cavities

Motivation: verified manifestations of quantum fluctuations

- Spontaneous emission of light from atoms
- Balanced homodyne detection of light
- Casimir forces between electrically neutral objects

(1) Quantum Inequalities
(L.Ford, T.Roman, C.Fewster)

Formal problems with quantum fluctuations:

$\langle E(t,x)E(t,x)\rangle_S$ is UV-divergent; "correct" operators are normal-ordered

$$:E^2(t,x): = E(t,x)E(t+\epsilon,x) - \langle E(t,x)E(t+\epsilon,x)\rangle_{vac}$$

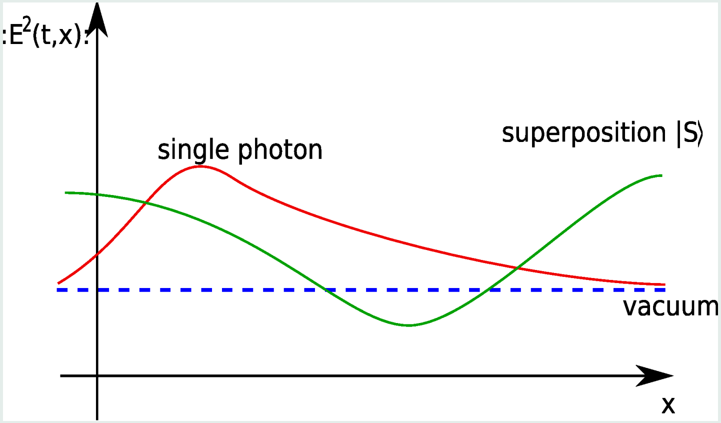
Limit $\epsilon \longrightarrow 0$ corresponds to the rule "all creation operators to the left"

$$:E^2(t,x):\, |vac\rangle = 0.$$

Is this an "absolute" zero? No! For $|S\rangle = N(|vac\rangle + \alpha|2\rangle)$ we find

$$\langle :E^2(t,x):\rangle_S < 0 \quad \text{for some } (t,x)$$

$\langle{:}\,E^2(t,x){:}\rangle$ for various states of light



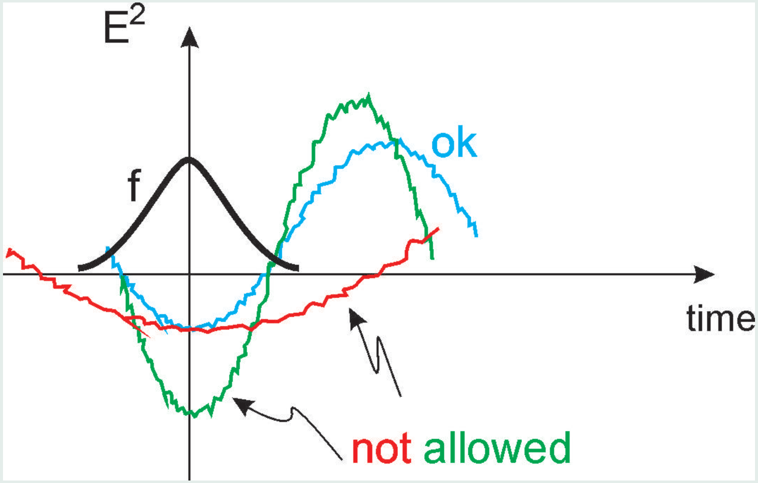
Can sub-vacuum levels persist for long times?

Quantum Inequalities:
Sub vacuum levels of quantum fluctuations cannot persist for long times.

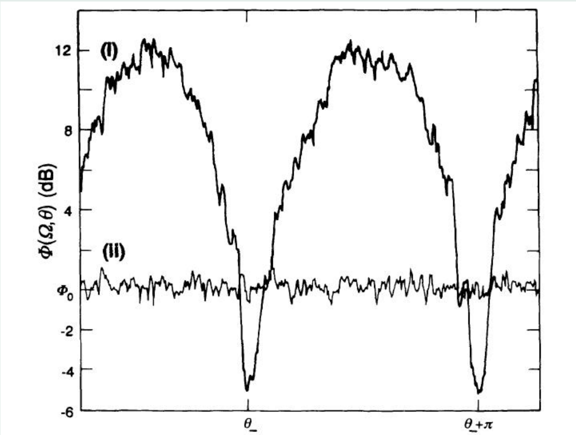
- The quantum electric field must obey

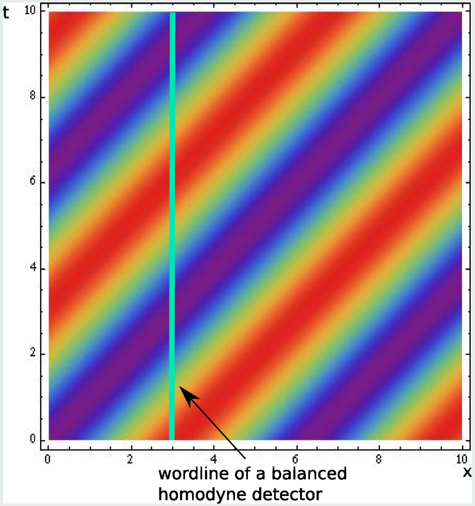
$$\int f(t)\langle :E^2(t,x): \rangle_S \;\geq -\frac{const}{\text{duration}^4}$$

- f(t): is a probability density (time of measurement) - e.g. a Gaussian
- *const* is a functional of f and does not depend on S



Example: Squeezed states of light





The fluctuations of the electric field, $\langle E^2(t)\rangle$ (restricted in frequencies to ω) for a squeezed and vacuum states.
Up to frequency-dependent and polarization-factors the electric field

$$E(t) = \frac{i}{\sqrt{2}}(e^{i\omega t}a - e^{-i\omega t}a^*)$$

with single-mode sqeezing: $\langle aa\rangle = -\cosh(r)\sinh(r), \quad \langle a^\dagger a\rangle = \sinh^2(r),$

has

$$\langle E^2(t)\rangle_S = \langle E^2(t)\rangle_{vac} + \sinh(r)\left\{\sinh(r) - \cosh(r)\cos[2(\omega t - px)]\right\}.$$

- It can be verified that single-mode squeezed light fulfills the QI.
- Full expression for QI

$$\int_{-\infty}^{\infty} f(t)\langle :E^2(t,x):\rangle_S \, dt \geqslant -\frac{1}{2\pi^2}\int_0^{\infty} d\omega \int d^3p \, |p| \left| \widehat{(\sqrt{f})}(\omega + |p|) \right|$$

- The functional on the RHS has the $-1/\text{duration}^4$ property.
- RHS is independent of the state S
- The QI has a clear interpretation and must be fulfilled by all states (multimode squeezed, superpositions of coherent or squeezed states etc.) Although for in the single-mode regime the QI is related to the Heisenberg inequality, the merit of QI is the clear picture it offers for multi-mode excited states of the quantum fields.
- QI have been generalized to general wordlines of detectors and curved spacetimes; in these cases the form of the RHS gets modified.
- In original form QIs have been developed for the energy density of quantum fields (applications to GR)

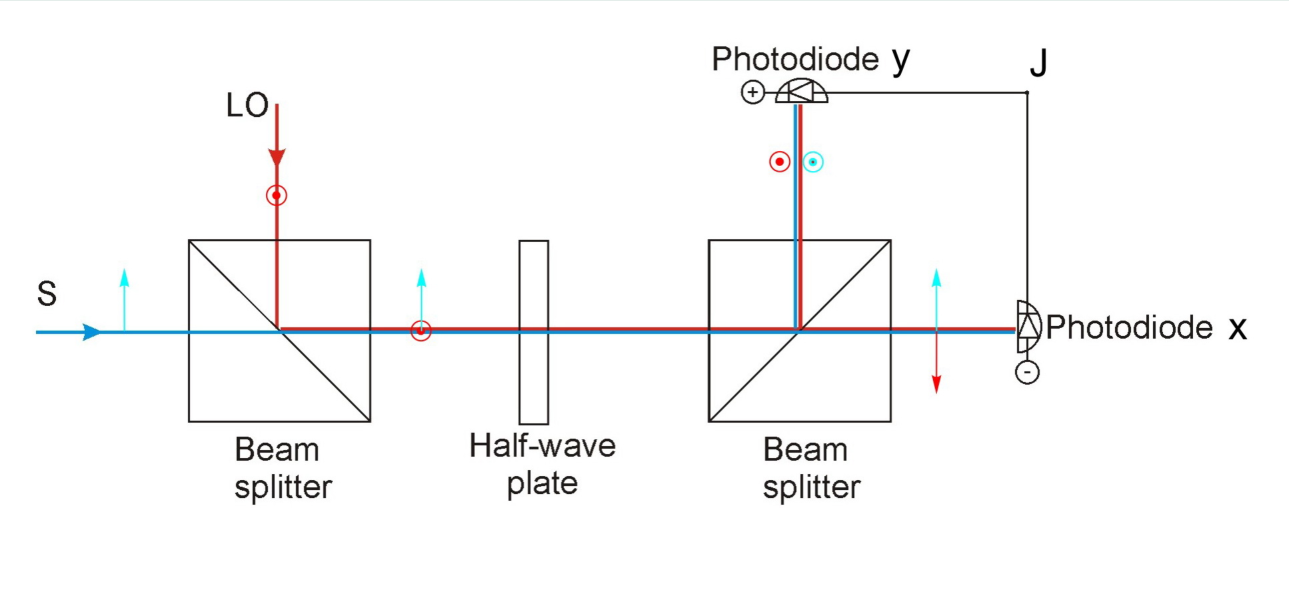
Message of Quantum Inequalities:

long waves are flat, deep waves are short

(for sub-vacuum *fluctuations of the free quantum electric field*).

(2) Balanced homodyne detector with a local oscillator

- Two photodiodes with their output subtracted
- Additional coherent light of frequency ω ("Local Oscillator")
- The quantum field (S) de-balances the detector (stochastic process of measurement)



BHD with LO: output

$$\langle J\rangle_S = \alpha_{el}\cdot E^i_{LO}\cdot\Big\langle E_i(t,\vec{x}) + E_i(t,\vec{y})\Big\rangle_S$$

$$\langle J^2\rangle_S = \alpha_{el}^2\cdot \underbrace{E^i_{LO}E^j_{LO}}_{LO\ power}\cdot\underbrace{\Big\langle [E_i(t,\vec{x})\ + E_i(t,\vec{y})][E_j(t,\vec{x}) + E_i(t,\vec{y})]\Big\rangle_S}_{Related\ to\ quantum\ field\ 2pt\ funct.}$$

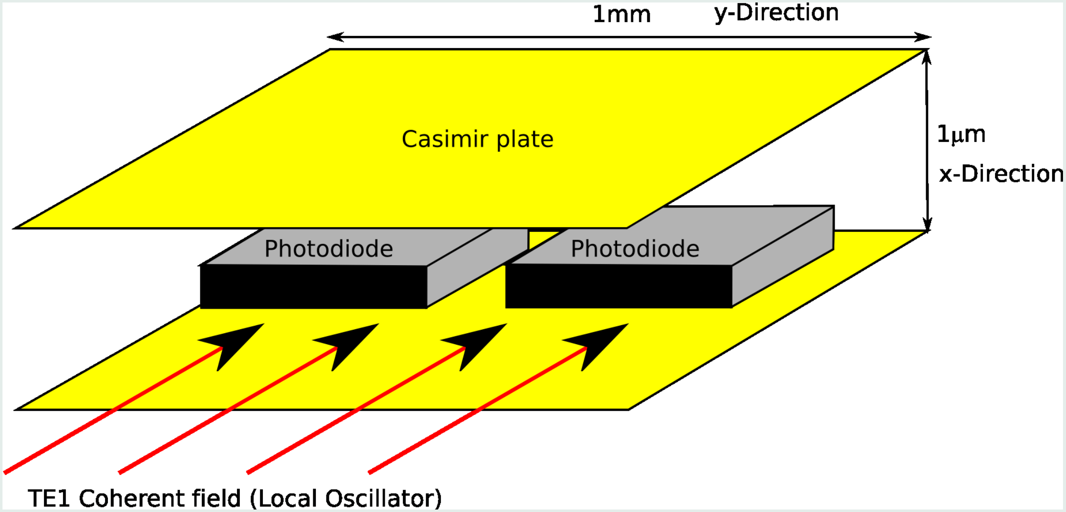
- the RHS contains expectation values of field operators; all of these operators are restricted to the frequency of the LO
- For stationary states (thermal or ground) the $\langle J^2\rangle_S$ is related to the spectral density

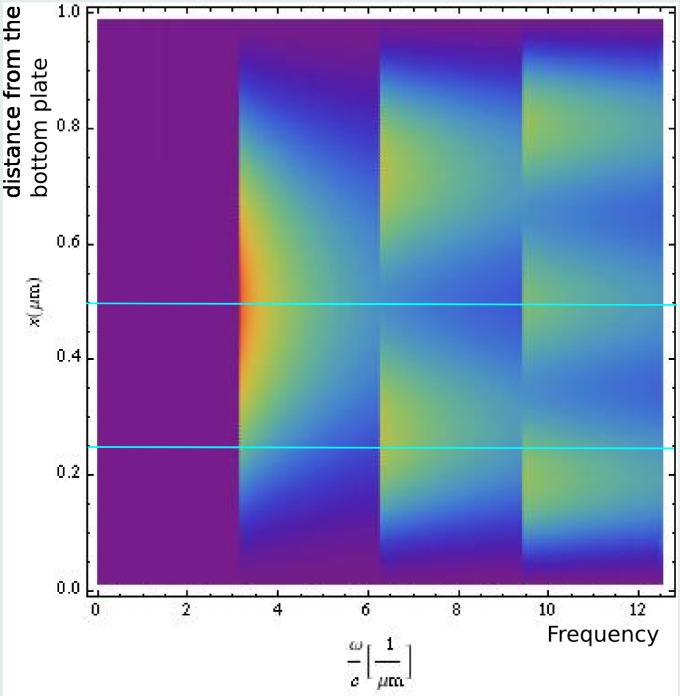
$$\sigma_{ij}(\omega,\vec{x},\vec{y}) = \int dt\, e^{i\omega t}\langle E_i(t,\vec{x})E_j(0,\vec{y})\rangle_S.$$

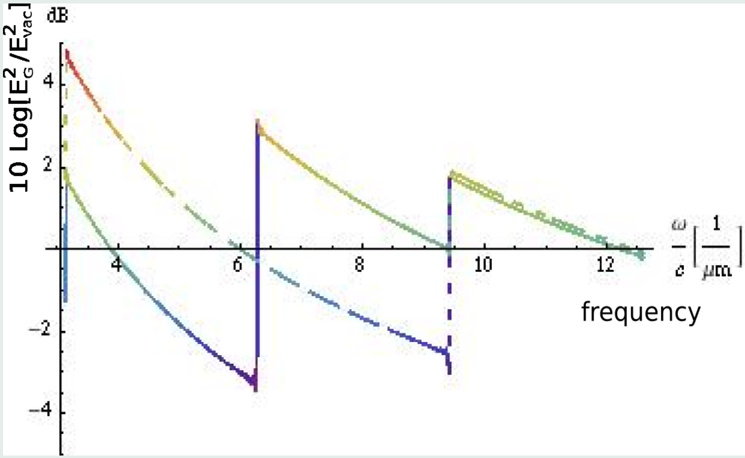
- Almost all results of QFT under influence of external conditions are derivable from the spectral density σ_{ij}. This quantity is usually known analytically. It is of great interest to measure this quantity for interesting states!

(3) BHD response in Casimir cavities

Hypothetical experimental setup:







Spectral density σ_{yy} related to the expected output of a BHD with the LO polarized along the y-direction (parallel to the plates) for the ground state in the Casimir cavity (S.Hacyan) [left]. Fluctuations $\langle E^2 \rangle_S$ in the ground state (for field operators restricted to the frequency ω) relative to vacuum fluctuations (in the absence of the plates) for a BHD at $x = 0.25\mu m$ and $x = 0.5\mu m$ within the cavity [right].

Remarks:

- The ground state is stationary $\Rightarrow$ quantum noise in the Casimir cavity time-independent (i.e. independent of the phase of the Local Oscillator)
- An application of the proposed type is would amount to a "tomography" of the ground state of the Casimir cavity
- Spectral densities reveal much finer details of the quantum ground state than the already measured Casimir forces.

Message from Casimir cavities:

Static external conditions may suppress vacuum fluctuations

(will BHD-type detectors be sensitive enough to verify this prediction?).

Summary

- Regions with sub-vacuum fluctuations must be followed by regions with greatly increased fluctuations no matter what the state of the quantum field is

- Casimir geometries provide environments with non-trivial, $\vec{x}$ and ω dependent, time-independent often sub-vacuum $\langle E^2 \rangle$. An experimental verification of this prediction would be highly desirable.

Details in J. Phys. A: Math. Theor. 41 164037 (2008) (also: arXiv:0711.1541)

(Simplest) derivation of Quantum Inequalities (C.Fewster)
Normal ordering: $\langle : E^2(t) : \rangle_S = \lim_{s \to t} \left[\langle E(t)E(s) \rangle_S - \langle E(t)E(s) \rangle_{vac} \right]$
Identity for symmetric function $h(t,s)$:

$$\frac{1}{\pi} \int_0^\infty d\omega \int_{-\infty}^\infty dt\, ds\, e^{-i\omega(t-s)} h(t,s) = \int_{-\infty}^\infty dt\, h(t,t)$$

$$\int_{-\infty}^\infty g^2(t) \langle : E^2(t) : \rangle_S =$$
$$\int_0^\infty \frac{d\omega}{\pi} \int_{-\infty}^\infty dt\, ds\, e^{-i\omega(t-s)} g(t) g(s) \underbrace{\left[\langle E(t)E(s) \rangle_S - \langle E(t)E(s) \rangle_{vac} \right]}_{\text{symmetric function}}$$

We get: $\displaystyle \int_{-\infty}^\infty g^2(t) \langle : E^2(t) : \rangle_S = \int_0^\infty d\omega \left[\langle A_\omega^\dagger A_\omega \rangle_S - \langle A_\omega^\dagger A_\omega \rangle_{vac} \right]$

with $A_\omega = \int dt\, \exp(-i\omega t)\, g(t)\, E(t)$; first term on RHS is positive $\Rightarrow$QI.

In details:

$$E_i(t,\vec{x}) = \int d\mu(p^a)\ \left[e^{-i\omega_p t}\mathcal{E}_i(p^a,\vec{x})b(p^a) + e^{i\omega_p t}\overline{\mathcal{E}_i(p^a,\vec{x})}b^\dagger(p^a)\right]$$

where p^a is a multiindex (polarizations,momenta), $\mathcal{E}_i$ classical solution, and

$$[b(p^a), b^\dagger(k^a)] = \delta(p^a - k^a)$$

The Quantum Inequality is:

$$\int_{-\infty}^{\infty} g^2(t)\langle : E^2(t) :\rangle_S \geqslant -\int_0^\infty \frac{d\omega}{\pi}\int d\mu(p^a)\,|\hat{g}(\omega+\omega_p)|^2\,|\mathcal{E}_i(p^a,\vec{x})|^2.$$

In Minkowski spacetime (without boundaries etc)

$$\mathcal{E}_i = \frac{|\vec{p}|}{\sqrt{2(2\pi)^3}}\exp(-i\vec{p}\vec{x})\mathrm{e}_i^\alpha(\vec{p}).$$

Spectral densities in Casimir cavities:

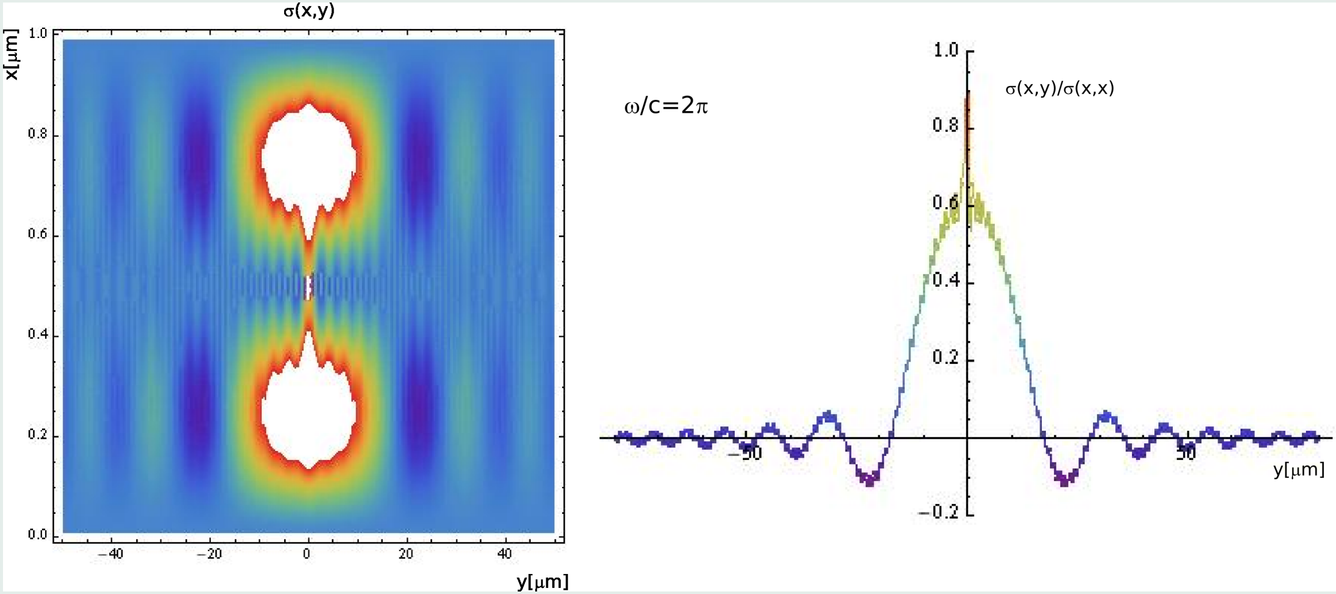
With (image-sums)

$$F^{\mp}(x,\tilde{x}) = -\frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty}\frac{1}{(x\mp\tilde{x}-nL)^2+(y-\tilde{y})^2+(z-\tilde{z})^2-(t-\tilde{t})^2}$$

the two-point function is found from

$$\langle E_y(x^a)E_y(\tilde{x}^a)\rangle_G = (\partial_x^2+\partial_z^2)\left[F^-(x^a,\tilde{x}^a)-F^+(x^a,\tilde{x}^a)\right],$$

Correlation between photodiodes?



Plot of $\sigma(\omega, \vec{x}, \vec{y})$ for $\omega = 2\pi c/1\mu m$, $\vec{x} = (x, 0, 0)$, $\vec{y} = (x, y, 0)$ [left] and $\sigma(\omega, \vec{x}, \vec{y})/\sigma(\omega, \vec{x}, \vec{x})$ for $x = 0.75\mu m$ [right].