Spin-Wave Theory for Magnetic Insulators

From textbook knowledge towards BEC of magnons

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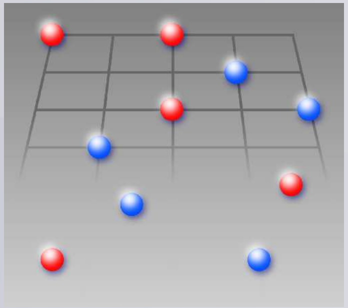
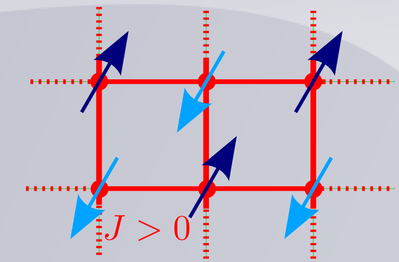


Outline

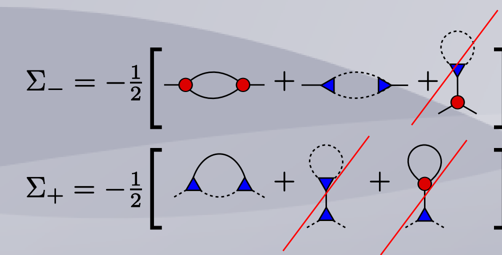
1. Introduction to spin-wave theory

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

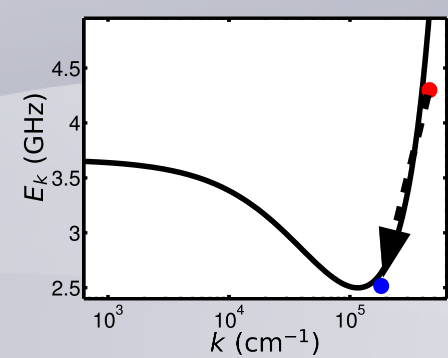


2. Applications

   1. Antiferromagnet in a magnetic field: nonanalytic properties of the magnon spectrum

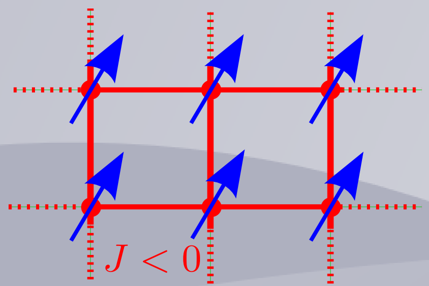


   2. Spin waves in thin film ferromagnets with dipole-dipole interactions

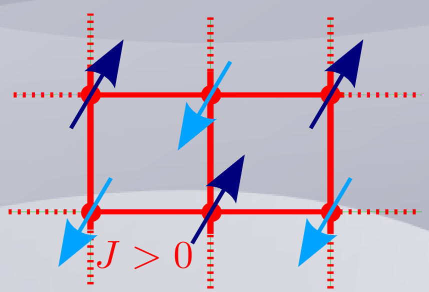


1. Introduction: Spin-wave theory

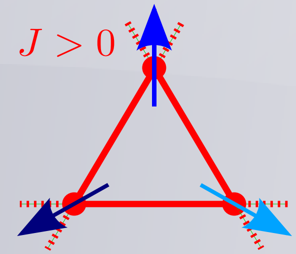
- Heisenberg model $$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$
- determine ordered classical groundstate
  - ferromagnet
    classical groundstate=quantum groundstate



  - anti-ferromagnet
    (2 sublattices, Néel groundstate)



  - triangular anti-ferromagnet
    (3 sublattices, frustration)



Chernychev, Zhitomirsky ('09)
Veillette *et al.* ('05)
AK, Kopietz (in preparation)

1. Spin wave theory

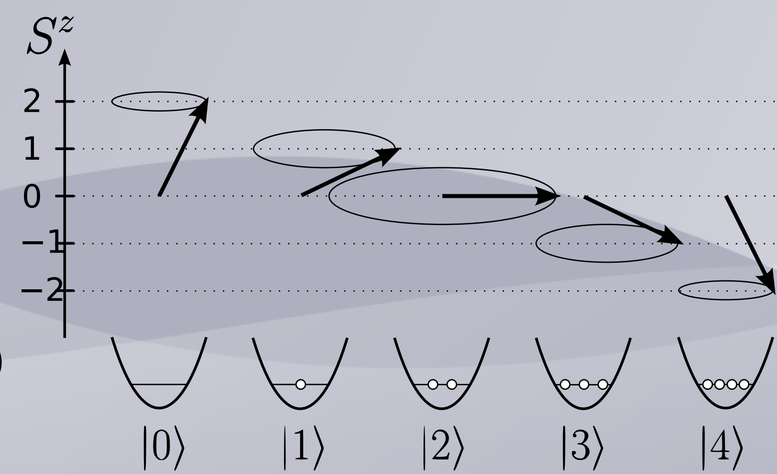
- expand in terms of bosons (1/S expansion), Holstein-Primakoff transformation

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\hat{S}^z = S - \hat{n} \qquad \hat{n} = \hat{b}^\dagger \hat{b} \qquad \left[\hat{b}, \hat{b}^\dagger\right] = 1$$

$$\hat{S}^+ = \sqrt{2S}\sqrt{1 - \frac{\hat{n}}{2S}}\,\hat{b}$$

$$\hat{S}^- = \sqrt{2S}\,\hat{b}^\dagger\sqrt{1 - \frac{\hat{n}}{2S}} \qquad \sqrt{1 - \frac{\hat{n}}{2S}} = 1 - \frac{\hat{n}}{4S} + \mathcal{O}(\frac{1}{S^2})$$



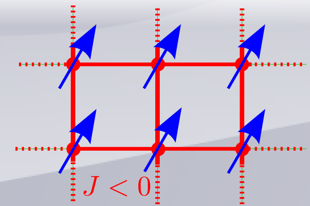
Holstein, Primakoff, Phys. Rev. **58**, 1098 (1940)

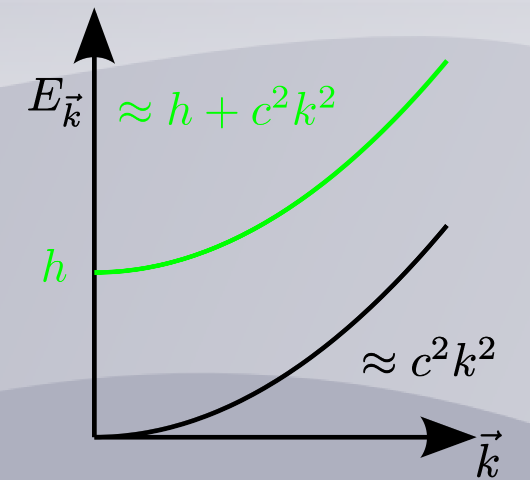
- determine properties of resulting interacting theory of bosons

$$H = \sum_{\vec{k}} E_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}} + \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3} \Gamma^3(\vec{k}_1, \vec{k}_2, \vec{k}_3) b^\dagger_{\vec{k}_1} b_{\vec{k}_2} b_{\vec{k}_3} + \sum_{1,2,3,4} \Gamma^4(1,2;3,4) b^\dagger_1 b^\dagger_2 b_3 b_4 + \ldots$$

1. Spin wave theory: General results

- ferromagnet

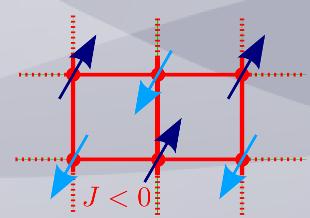


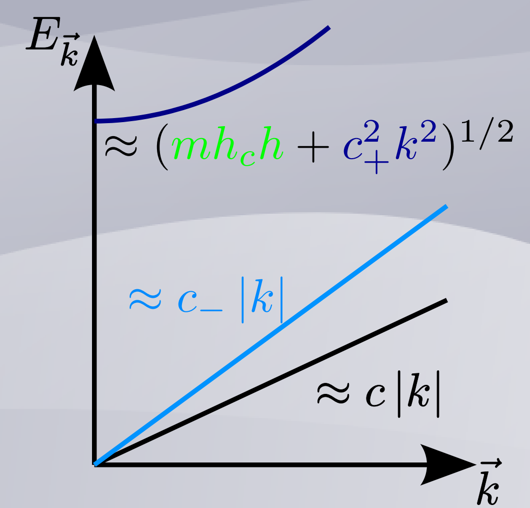


  - quadratic excitation spectrum
  - vanishing interaction vertices

$$\Gamma^4 \sim -(\vec{k}_1 \cdot \vec{k}_2 + \vec{k}_3 \cdot \vec{k}_4)$$

- antiferromagnet





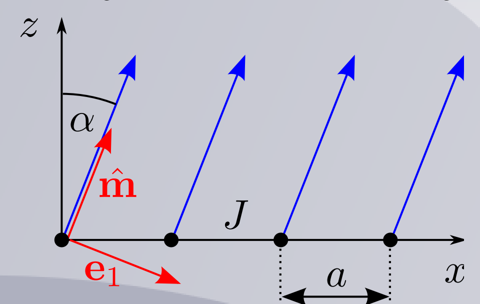
  - linear spectrum (Goldstone mode)
  - two modes in magnetic field (2 sublattices)
  - divergent interaction vertices

$$\Gamma^4 \sim \sqrt{\frac{|\vec{k}_1||\vec{k}_2|}{|\vec{k}_3||\vec{k}_4|}} \left(1 \pm \frac{\vec{k}_1 \cdot \vec{k}_2}{|\vec{k}_1||\vec{k}_2|} \right)$$

Hasselmann, Kopietz ('06)

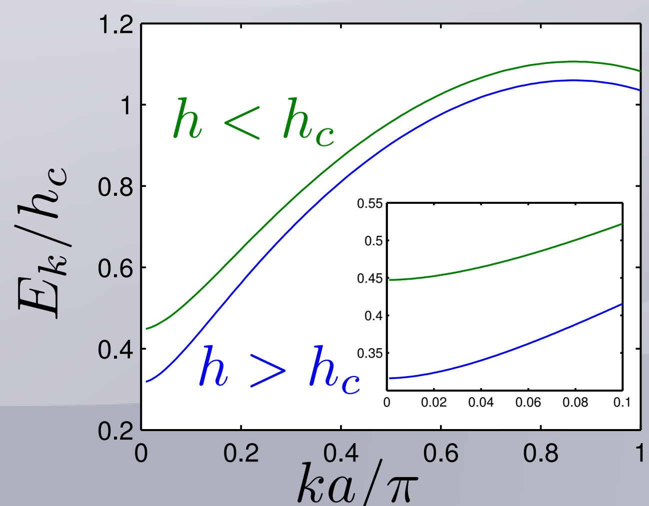
1. Ferromagnet with in D=1 with dipole-dipole interactions

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\vec{S}_i\cdot\vec{S}_j - \vec{h}\cdot\sum_i \vec{S}_i - \frac{1}{2}{\sum_{ij}}' \frac{\mu^2}{|x_{ij}|^3}[3(\vec{S}_i\cdot\vec{e}_x)(\vec{S}_j\cdot\vec{e}_x) - \vec{S}_i\cdot\vec{S}_j)$$



- critical point at $h_c = 3\zeta(3)\dfrac{S\mu^2}{a^3}$
- dispersion has gap:

$$E_k = \begin{cases} h_c\sqrt{r_0 + r_1(ka)^2\ln(r_2/|ka|)} & h > h_c \\ h_c\sqrt{2|r_0| + r_1(ka)^2\ln(r_2/|ka|)} & h > h_c \end{cases}$$



$$r_0 = \frac{h - h_c}{h_c} \qquad r_1 = 1/3\zeta(3) \qquad r_2 = e^{3/2}$$

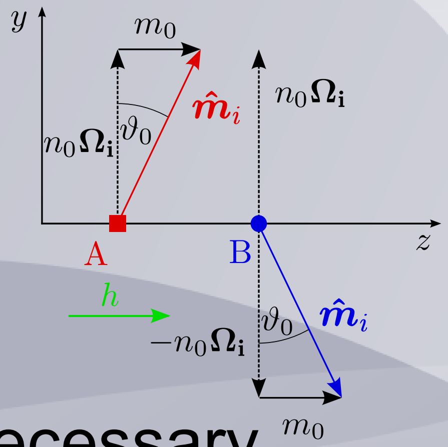
$$E_0 = \Delta = \begin{cases} \sqrt{\frac{h-h_c}{h_c}} & h > h_c \\ \sqrt{2\frac{h_c-h}{h_c}} & h > h_c \end{cases}$$

A. Ruppel, P. Kopietz (in preparation)

2.1 Quantum Antiferromagnet in a weak magnetic field

- quantum Antiferromagnet in a magnetic field

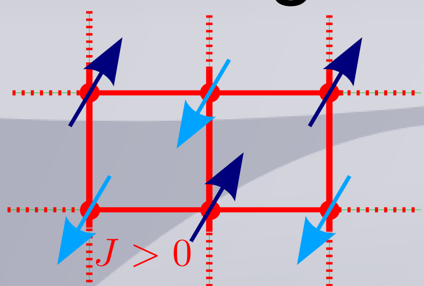
$$\hat{H} = \frac{1}{2}\sum_{ij} J_{ij}\mathsf{S}_i \cdot \mathsf{S}_j - h\sum_i S_i^z$$



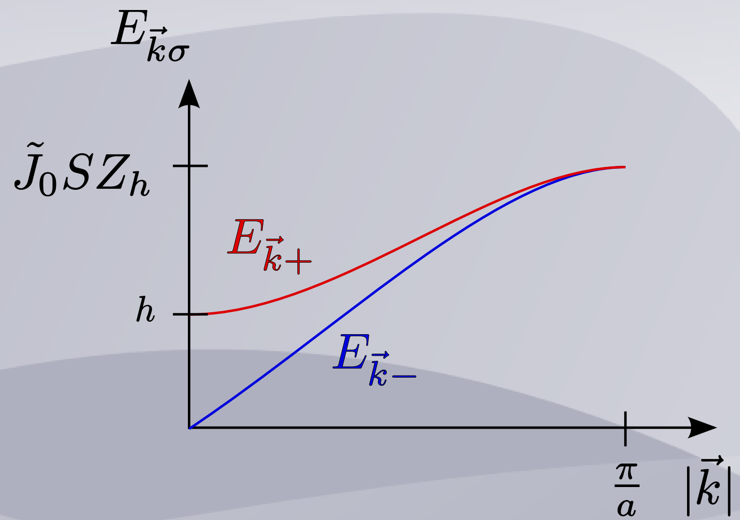
- **Non Linear Sigma Model**
  [Chakravarty, Halperin, Nelson, PRB **39**, 2344 (1989)]
  - effective continuum theory yields long wavelength results but ultraviolett cutoff necessary
  - simple interaction vertices without singularities
- **1/S-expansion**
  [Oguchi, Phys. Rev. **117**, 117 (1960), Harris *et al.* PRB **3**, 961 (1971)]
  - perturbative expansion in powers of 1/S
  - lattice theory to describe short wavelength results
  - tedious interaction vertices
  - obtain all physical quantities

2.1 Energy dispersion in leading order spin-wave theory

- Long-wavelength limit



$$E^2_{\vec{k}+} = h^2 + c_+^2 \vec{k}^2,$$
$$E^2_{\vec{k}-} = c_-^2 \vec{k}^2,$$



with the spin-wave velocities

$$c_+^2 = c_0^2 (1 - \frac{3}{\Delta_0^2} h^2),$$
$$c_-^2 = c_0^2 (1 - \frac{1}{\Delta_0^2} h^2),$$

AK, Sauli, Hasselmann, Kopietz ('08)

where $\Delta_0 = 4DJS$ and c_0 is the leading large-S result for spin-wave velocity for $h = 0$.

2.1 Spin-wave theory with Hermitian field operators

$$\hat{\Psi}_{\vec{k}\sigma} = p_\sigma \left[\sqrt{\frac{\nu_{\vec{k}\sigma}}{2}} \hat{X}_{\vec{k}\sigma} + \frac{i}{\sqrt{2\nu_{\vec{k}\sigma}}} \hat{P}_{\vec{k}\sigma} \right]$$

magnon operator

- commutation relations: $[\hat{X}_{\vec{k}\sigma}, \hat{P}_{\vec{k}'\sigma'}] = i\delta_{\vec{k},-\vec{k}'}\delta_{\sigma,\sigma'}$
- physical meaning for $k \ll 1$

$\hat{P}_{\vec{k}\sigma}$: uniform spin fluctuations

$\hat{X}_{\vec{k}\sigma}$: staggered spin fluctuations

- those operators yield regular vertices!
- construct effective action for staggered fluctuations

P. W. Anderson, Phys. Rev. 86, 694 (1952), AK, Hasselmann, Kopietz ('07)
N. Hasselmann and P. Kopietz, Europhys. Lett. 74, 1067 (2006)

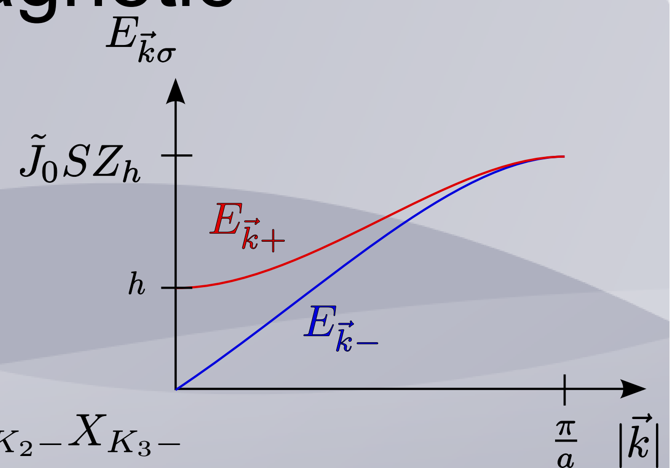
2.1 Effective Model

- eliminate degrees of freedom associated with the operator of the ferromagnetic fluctuations

$$e^{-S_{\mathrm{eff}}[X_\sigma]} = \int \mathcal{D}[P_\sigma] e^{-S[P_\sigma, X_\sigma]} \qquad K = (\mathsf{k}, i\omega_n)$$

$$S_{\mathrm{eff}}[X_\sigma] = S_0 + \frac{\beta}{2} \sum_{K\sigma} \frac{E^2_{\mathsf{k}\sigma} + \omega^2}{\Delta_{\mathsf{k}\sigma}} X_{-K\sigma} X_{K\sigma}$$

$$+\beta\sqrt{\frac{2}{N}} \sum_{K_1 K_2 K_3} \delta_{K_1+K_2+K_3,0} \left[\frac{1}{3!} \Gamma^{(3)}_{---}(K_1, K_2, K_3) X_{K_1 -} X_{K_2 -} X_{K_3 -} + \frac{1}{2!} \Gamma^{(3)}_{-++}(K_1; K_2, K_3) X_{K_1 -} X_{K_2 +} X_{K_3 +} \right]$$



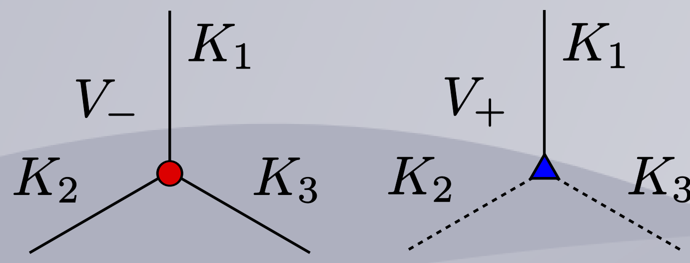
- generalization of Non-Linear-Sigma-Model for QAF subject to magnetic field

2.1 1/S Corrections

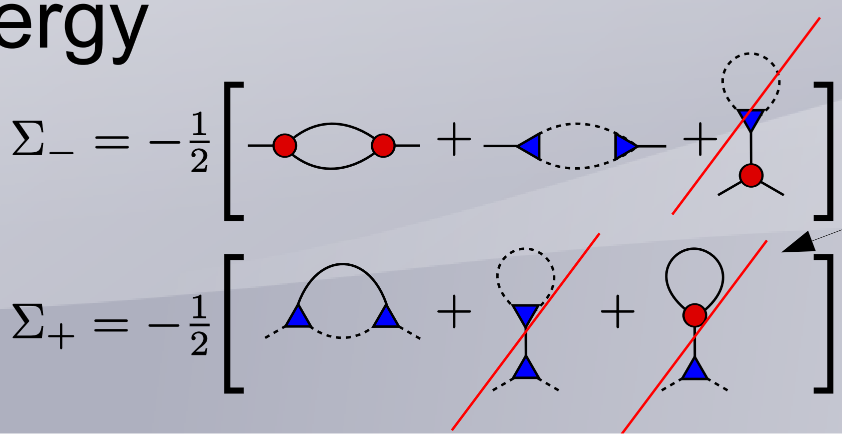
- diagrammatic perturbation theory

$$S_{\text{eff}}^{\text{int}}[X_\sigma] \quad = \beta\sqrt{\tfrac{2}{N}}\sum\Big[\tfrac{1}{3!}V_-^{(3)}X_-X_-X_- + \tfrac{1}{2!}V_+^{(3)}X_-X_+X_+\Big]$$

$$G_\sigma(K) = \frac{\Delta_{\mathsf{k}\sigma}}{E_{\mathsf{k}\sigma}^2+\omega^2}$$



- perturbation theory: 1/S corrections to self energy



no frequency dependence, negligible

2.1 Results

- leading order expansion of self-energy

$$\Sigma_-(K) = C^\omega \omega^2 + C^k c_0^2 \mathsf{k}^2 + \mathcal{O}(\omega^4, k^4)$$

Zhitomirsky, Chernychev, '98

- full propagator

$$G_-(K) = \frac{\Delta_{\mathsf{k}-}}{\omega^2 + E_{\mathsf{k}-}^2 + \Delta_{\mathsf{k}-}\Sigma_-(K)} \approx \frac{Z_-\Delta_0 n^2}{\omega^2 + c_-(h)^2 \mathsf{k}^2}$$

non analytic in h^2

- spin-wave velocity of gapless mode

$$\frac{c_-^2}{c_0^2} \approx 1 - \Delta_0 n^2 C^\omega \approx 1 - \frac{6\sqrt{3}\,\tilde{h}^2}{\pi^2 S} \ln\left(\frac{2}{\tilde{h}}\right) \qquad D = 3$$

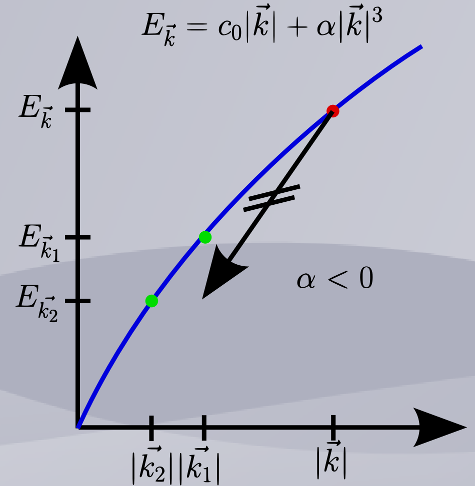
$$\frac{c_-^2}{c_0^2} \approx 1 - \Delta_0 n^2 C^\omega \approx 1 - \frac{2\tilde{h}}{\pi S} \qquad D = 2$$

$$\tilde{h} = \frac{h}{\Delta_0}$$

2.1 Spin-wave damping (Quasiparticle decay)

- spontaneous decay (3-point vertex)
  - energy-momentum conservation

$$E_{\vec{k}} = E_{\vec{k}_1} + E_{\vec{k}_2} \qquad \vec{k} = \vec{k}_1 + \vec{k}_2$$



$$\Gamma_{\vec{k}} \approx 0$$

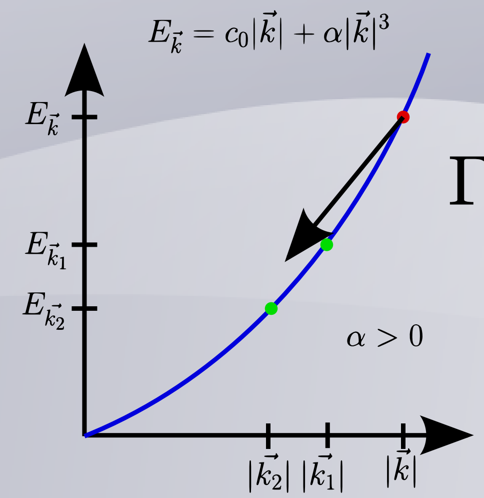
- energy dispersion

$$E_{\vec{k}-} \approx c_-|\vec{k}|(1 + \bar{A}_- \vec{k}^2)\ ,$$

$$c_- \approx 2\sqrt{D} JSa$$

$$A_-(\hat{\vec{k}}) = -\frac{a^2}{4}\left[\frac{1-2m^2}{D(1-m^2)} + \frac{1}{3}\sum_\mu \hat{k}_\mu^4\right]$$

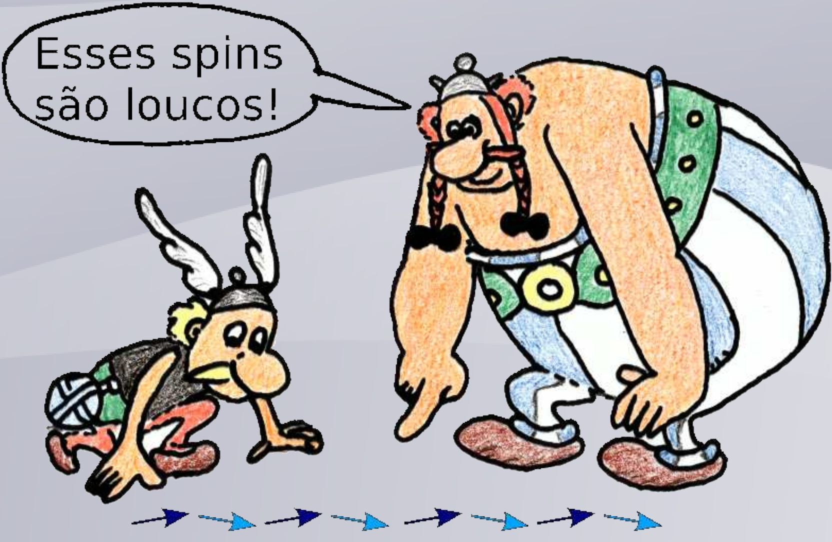
$$\Gamma_{\vec{k}-} \propto \frac{1}{S}\left(\frac{h}{h_c}\right)^2 \left(\sqrt{6\bar{A}_-}\right)^{D-3} a^{D+1}|\vec{k}|^{2D-1}$$



$$\Gamma_{\vec{k}} \propto k^{2D-1}$$

2.1 Recapitulation

- new formulation for the quantum antiferromagnet in a magnetic field: Combine NLSM with 1/S expansion (spin-wave theory)
- advantage: physical interpretation of field operators
- results: non analytic dependences of the spin-wave velocity
- damping via LSWT



2.2 Spin-wave theory for thin film ferromagnets

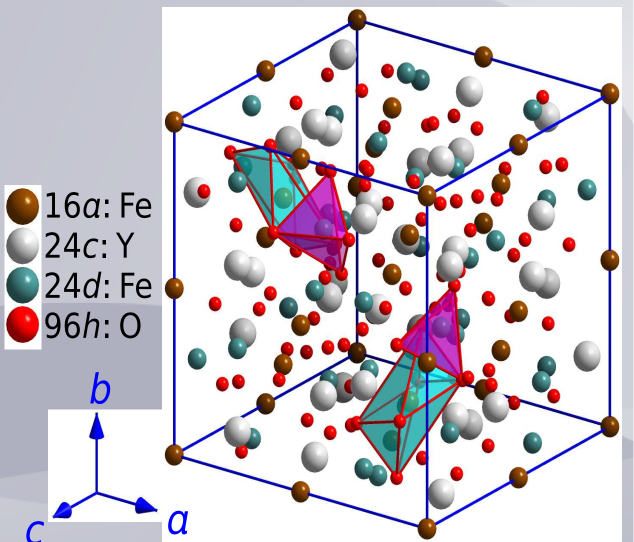
- Motivation: Experiments on YIG
  - Crystal structure:

space group: **Ia3d**
Y: 24(c) white
Fe: 24(d) green
Fe: 16(a) brown
O: 96(h) red

Gilleo *et al.* ‘58

Magnetic system:
40 magnetic ions in elementary cell
40 magnetic bands

Elastic system:
160 atoms in elementary cell
3x160 phonon bands

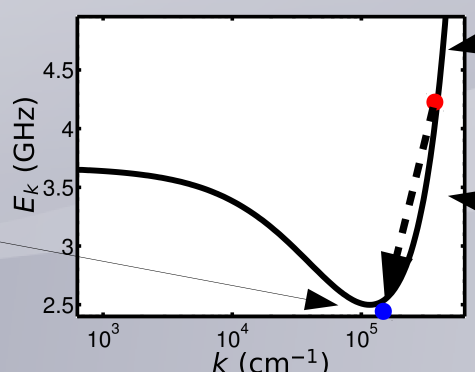


  - low spin wave damping
  - good experimental control

Observation of the occupation number using microwave antennas or Brillouin Light Scattering (BLS)

Bose-Einstein Condensation of magnons at room temperature!

Demokritov *et al.* Nature **443**, 430 (2006)

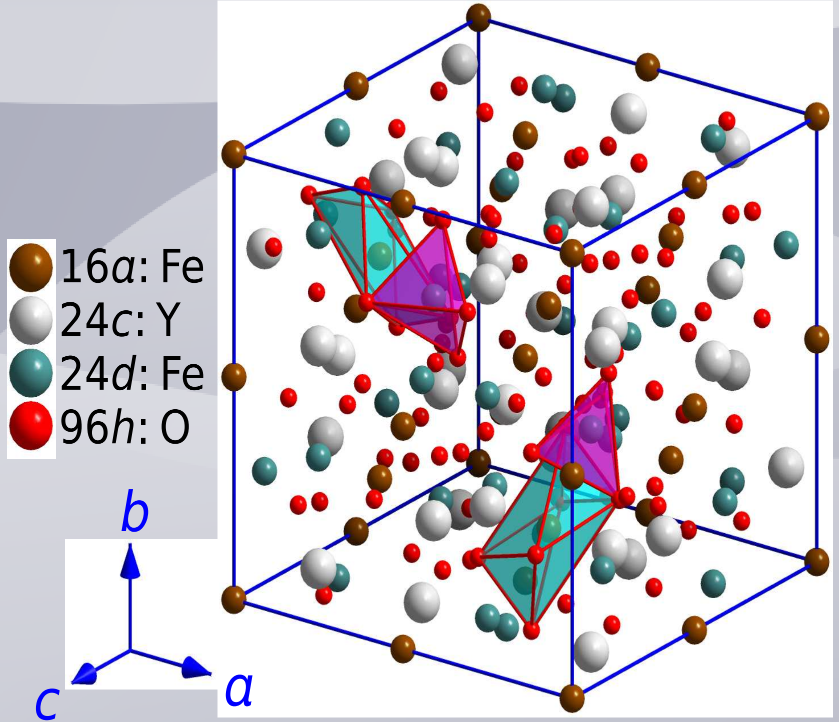


Parametric pumping of magnons at high k-vectors creates magnetic excitations

Question:
Time evolution of magnons:
Non-equilibrium physics of interacting quasiparticles

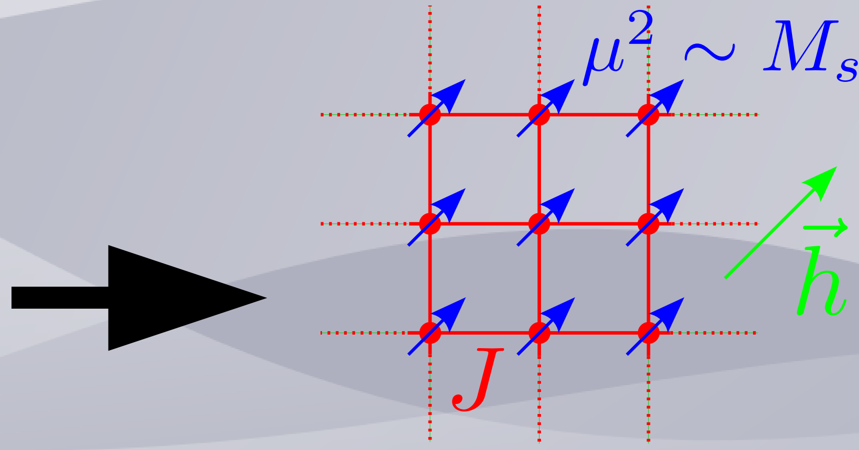
2.2 Simplifications to relevant physical properties

crystal structure of YIG



AK, Sauli, Bartosch, Kopietz ('09)

microscopic Hamiltonian



quantum spin S ferromagnet

Zeeman term

dipole-dipole interactions

$$\hat{H}_{\text{mag}} = -\frac{1}{2}\sum_{ij} J_{ij}\mathsf{S}_i \cdot \mathsf{S}_j - h\sum_i S_i^z$$

$$-\frac{1}{2}\sum_i \sum_{j\neq i} \frac{\mu^2}{|\mathsf{r}_{ij}|^3}\left[3(\mathsf{S}_i \cdot \hat{\mathsf{r}}_{ij})(\mathsf{S}_j \cdot \hat{\mathsf{r}}_{ij}) - \mathsf{S}_i \cdot \mathsf{S}_j\right]$$

2.2 Microscopic Hamiltonian: Heisenberg model (lowest band)

quantum spin S ferromagnet

Zeeman term

dipole-dipole interactions

$$\hat{H}_{\mathrm{mag}} = -\frac{1}{2}\sum_{ij} J_{ij}\,\mathbf{S}_i\cdot\mathbf{S}_j - h\sum_i S_i^z$$

$$-\ \frac{1}{2}\sum_i\sum_{j\neq i}\frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3(\mathbf{S}_i\cdot\hat{\mathbf{r}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{r}}_{ij}) - \mathbf{S}_i\cdot\mathbf{S}_j\right]$$

$$= \ -\frac{1}{2}\sum_{ij}\sum_{\alpha\beta}\left[J_{ij}\delta^{\alpha\beta} + D_{ij}^{\alpha\beta}\right] S_i^\alpha S_j^\beta - h\sum_i S_i^z$$

- Dipolar tensor

$$D_{ij}^{\alpha\beta} = (1-\delta_{ij})\frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3\hat{r}_{ij}^\alpha\hat{r}_{ij}^\beta - \delta^{\alpha\beta}\right]$$

material parameters
$a = 12.376$ Å Gilleo *et al.* ‘58
$4\pi M_s = 1750$ G Tittmann ‘73
$\frac{\rho_{\mathrm{ex}}}{\mu} = 5.17\cdot 10^{-13}\mathrm{Oe\ m}^2$ Cherepanov *et al.* ‘93
alternatively $J = 1.29$ K $S = 14.2 \qquad \mu = 2\mu_B$ Tupitsyn *et al.* ‘08

2.2 Linear Spin Wave Theory

- classical groundstate for stripe geometry
- Holstein Primakoff transformation (bosons)

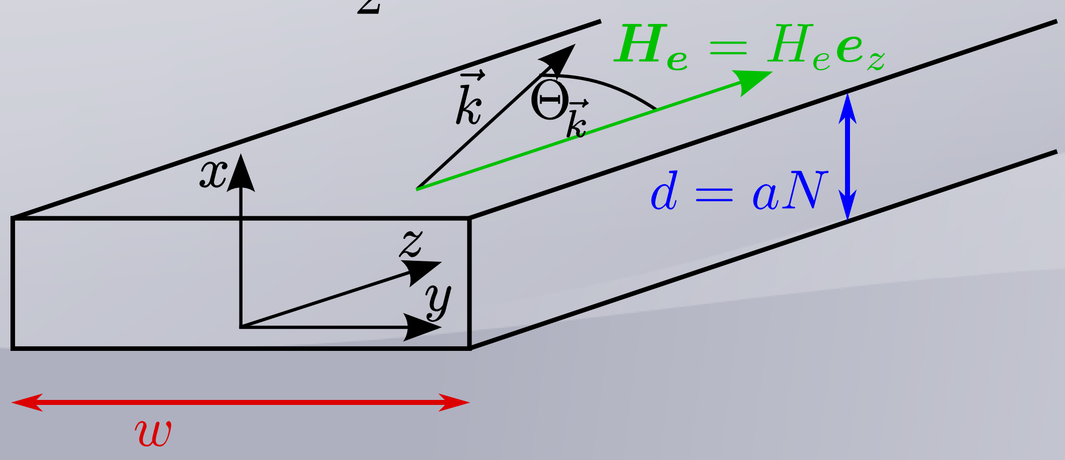
$$\hat{H}_2 = \sum_{ij} \left[A_{ij} b_i^\dagger b_j + \frac{B_{ij}}{2} \left(b_i b_j + b_i^\dagger b_j^\dagger \right) \right]$$

Filho Costa *et al.* Sol. State Comm. **108**, 439 (1998)

$$A_{ij} = \delta_{ij} h + S(\delta_{ij} \sum_n J_{in} - J_{ij}) + S \left[\delta_{ij} \sum_n D_{in}^{zz} - \frac{D_{ij}^{xx} + D_{ij}^{yy}}{2} \right],$$

$$B_{ij} = -\frac{S}{2} [D_{ij}^{xx} - 2i D_{ij}^{xy} - D_{ij}^{yy}]$$

dipolar tensor



2.2 Stripe geometry

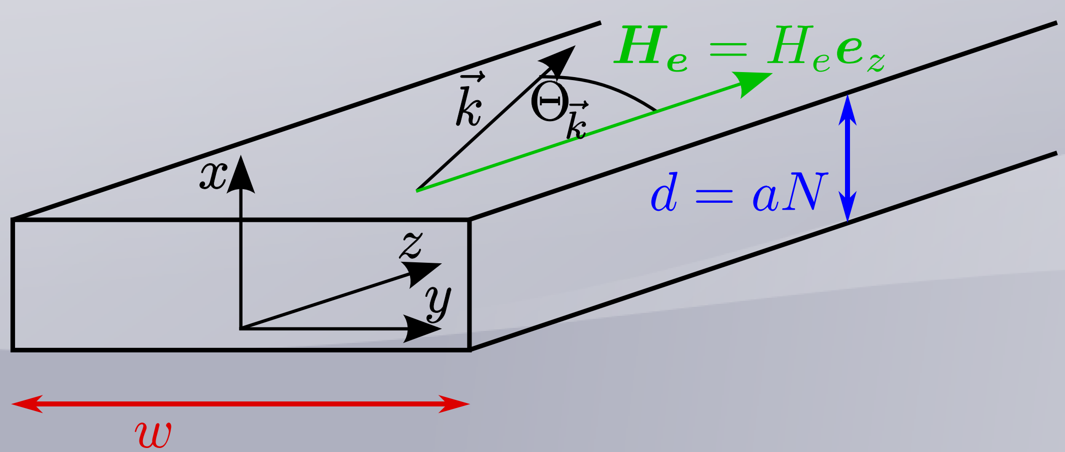
- partial Fourier transformation

$w \to \infty$

$$b_i = \frac{1}{\sqrt{N_y N_z}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}_i} b_{\vec{k}}(x_i)$$

- find all branches

$$\det \begin{pmatrix} E_{\vec{k}}\mathtt{I} - \mathtt{A}_{\vec{k}} & -\mathtt{B}_{\vec{k}} \\ -\mathtt{B}^*_{\vec{k}} & -E_{\vec{k}}\mathtt{I} - \mathtt{A}_{\vec{k}} \end{pmatrix} = 0$$



Problems:
1) dipolar sums
2) large matrices

2.2 Numerical approach

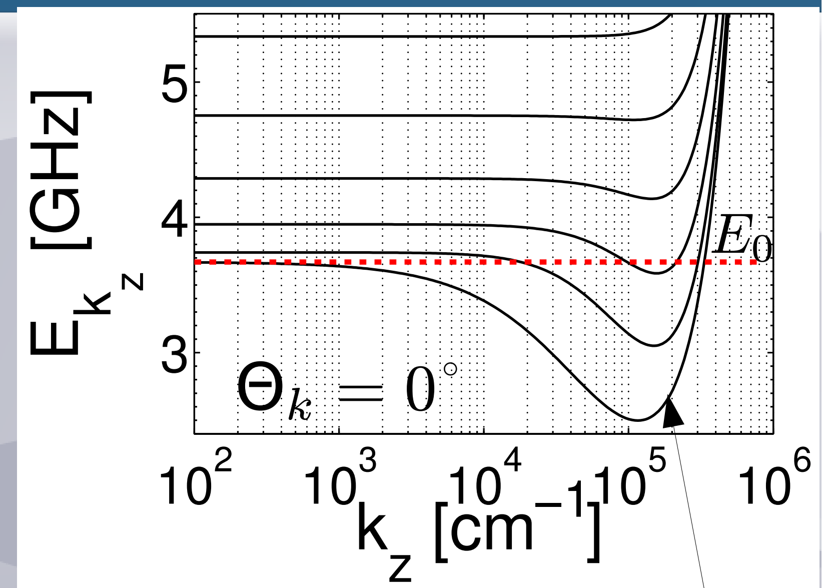
1) numerical diagonalization of 2Nx2N matrix

2) evaluation of dipol sums (Ewald summation technique)

Bartosch *et al.* '06

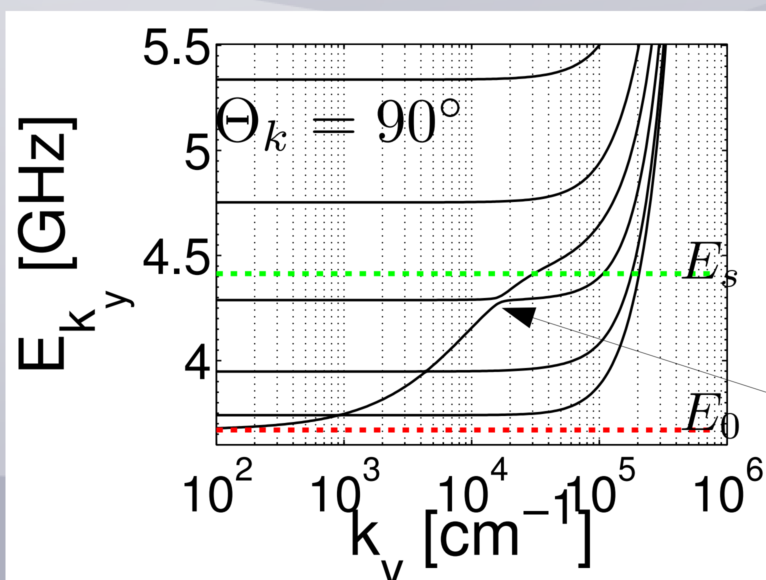
$$H_2 = \begin{pmatrix} \mathsf{A}_{\vec{k}} & \mathsf{B}_{\vec{k}} \\ -\mathsf{B}^{\mathsf{T}}_{\vec{k}} & -\mathsf{A}_{\vec{k}} \end{pmatrix}$$

$$d = 400a \approx 0.5\mu\text{m} \qquad N = 400 \quad H_e = 700\ \text{Oe}$$



minimum for BEC

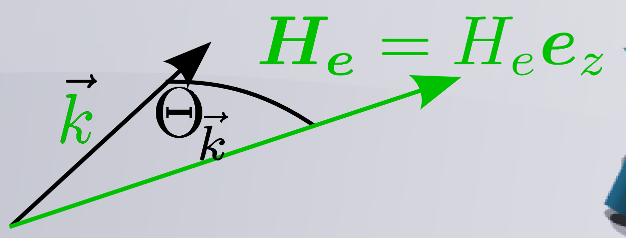
Demokritov *et al.* '06



$$E_0 = \sqrt{h(h + 4\pi\mu M_s)}$$

$$E_s = h + 2\pi\mu M_s$$

hybridization: surface mode



2.2 Analytical approach

$$\hat{H} = \sum_{\vec{k}} \sum_{x_i x_j} \Big[A_{\vec{k}}(x_{ij}) b^{\dagger}_{\vec{k}}(x_i) b_{\vec{k}}(x_j) + \frac{B_{\vec{k}}(x_{ij})}{2} b_{\vec{k}}(x_i) b_{\vec{k}}(x_j) + \frac{B^{*}_{\vec{k}}(x_{ij})}{2} b^{\dagger}_{\vec{k}}(x_i) b^{\dagger}_{\vec{k}}(x_j) \Big]$$

- uniform mode approximation

$$b_{\vec{k}}(x_i) = \frac{1}{\sqrt{N}} b_{\vec{k}}$$

- lowest eigenmode approximation

$$b_{\vec{k}}(x_i) = \sqrt{\frac{2}{N}} \cos(\frac{\pi x_i}{d}) b_{\vec{k}}$$

2.2 Analytical results with approximation

$$\hat{H} = \sum_{\vec{k}} \left[A_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}} + \frac{B_{\vec{k}}}{2} b_{\vec{k}} b_{\vec{k}} + \frac{B^*_{\vec{k}}}{2} b^\dagger_{\vec{k}} b^\dagger_{\vec{k}} \right]$$

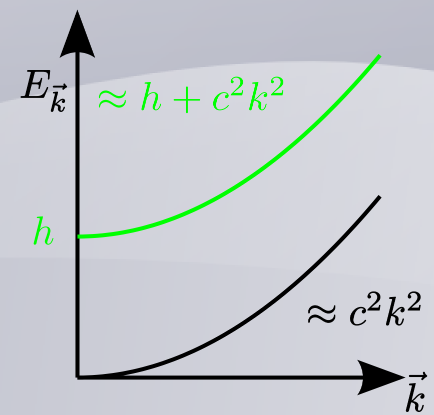
- dispersion via Bogoliubov transformation

$$E_{\vec{k}} = \sqrt{[h + \rho_{\mathrm{ex}} \vec{k}^2 + \Delta(1 - f_{\vec{k}}) \sin^2 \Theta_{\vec{k}}][h + \rho_{\mathrm{ex}} \vec{k}^2 + \Delta f_{\vec{k}}]}$$

$\Delta = 4\pi\mu M_S$

- no dipolar interaction: $\Delta = 0$

$$E_{\vec{k}} = h + \rho_{\mathrm{ex}} \vec{k}^2$$



2.2 Analytical results with approximation

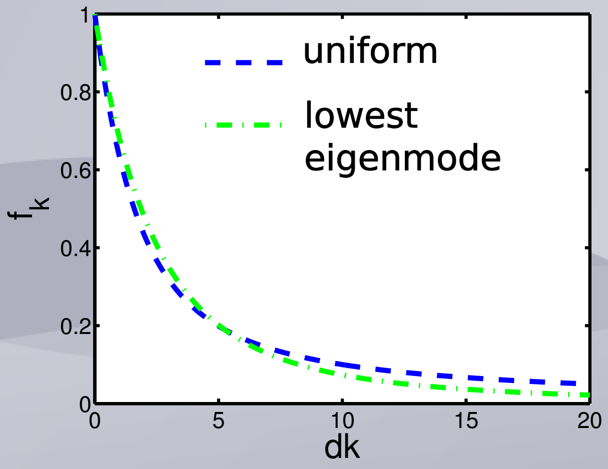
$$E_{\vec{k}} = \sqrt{[h + \rho_{\mathrm{ex}}\vec{k}^2 + \Delta(1 - f_{\vec{k}})\sin^2\Theta_{\vec{k}}][h + \rho_{\mathrm{ex}}\vec{k}^2 + \Delta f_{\vec{k}}]}$$

$$\Delta = 4\pi\mu M_S$$

- uniform mode approximation
  $\Rightarrow$ form factor

$$f_{\vec{k}} = \frac{1 - e^{-|\vec{k}|d}}{|\vec{k}|d}$$

compare: Kalinikos *et al.* '86
Tupitsyn *et al.* '08

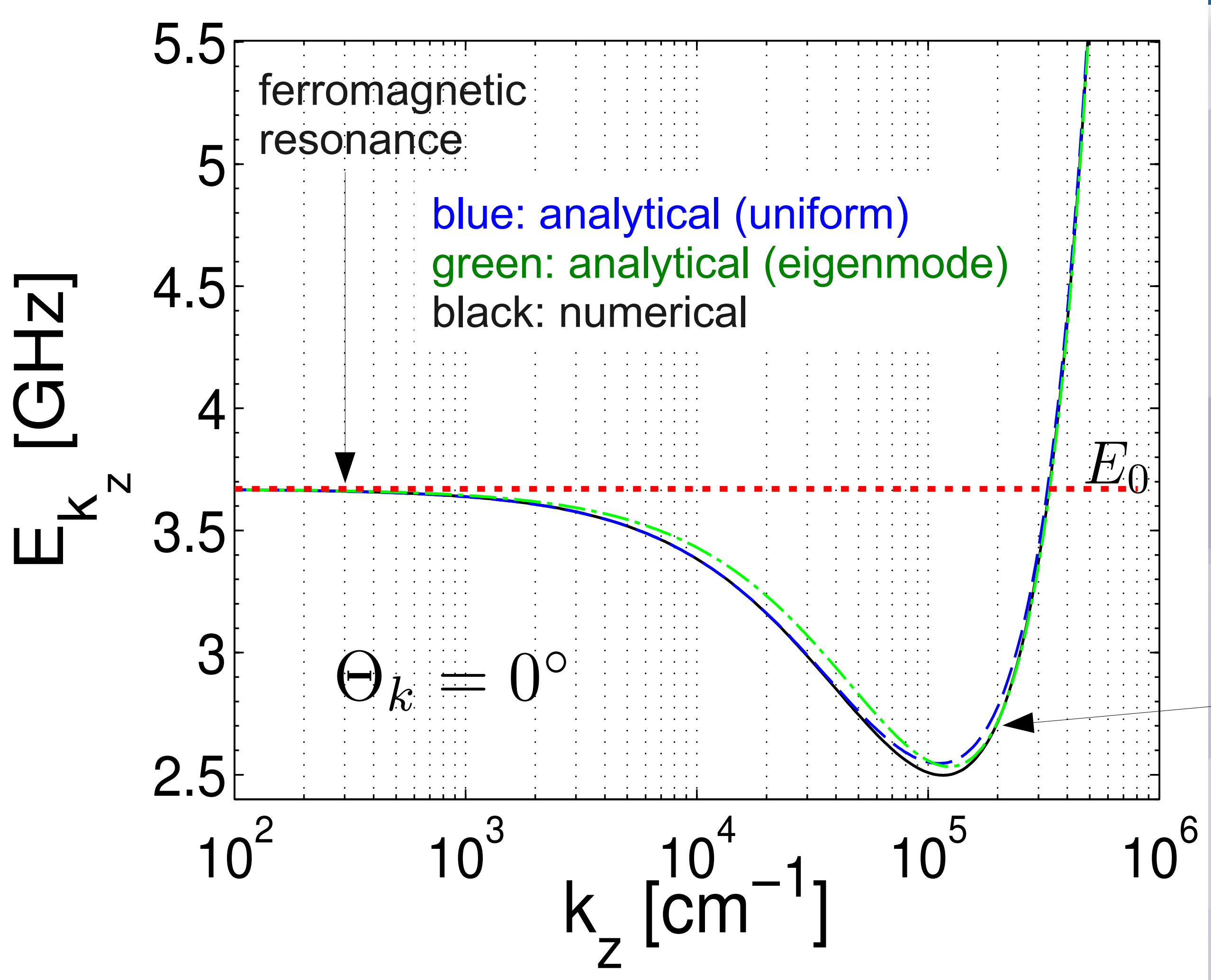


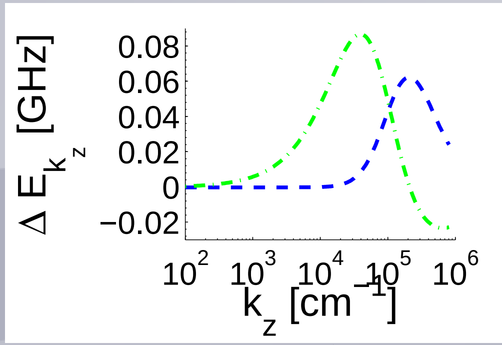
- eigenmode approximation
  $\Rightarrow$ different form factor:

$$f_{\vec{k}} = 1 - |\vec{k}d| \frac{|\vec{k}d|^3 + |\vec{k}d|\pi^2 + 2\pi^2(1 + e^{-|\vec{k}d|})}{(\vec{k}^2 d^2 + \pi^2)^2}$$

AK, Sauli, Bartosch, Kopietz ('09)

2.2 Comparison: lowest mode





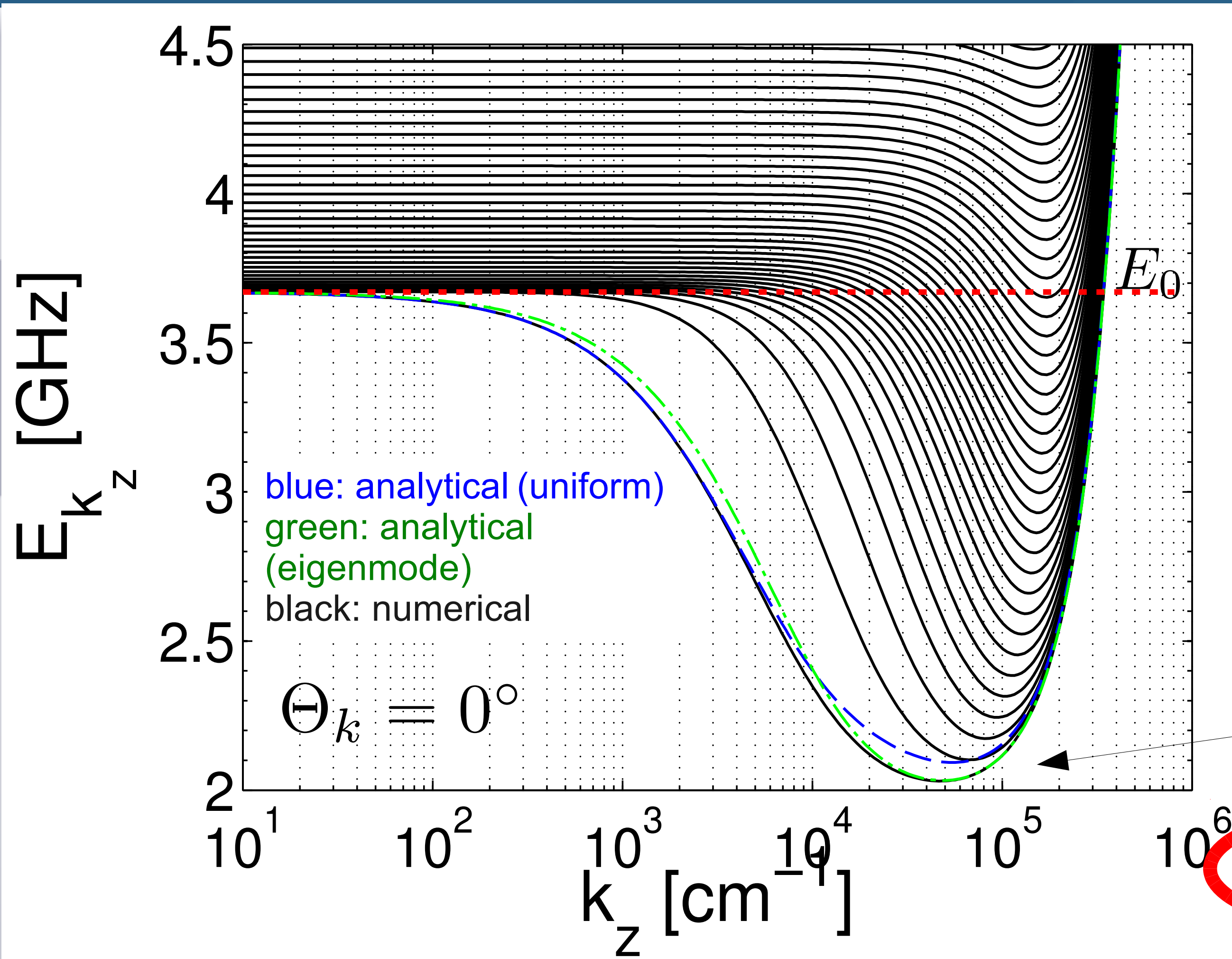
small deviations

$d = 400a \approx 0.5\mu\text{m}$

$H_e = 700\ \text{Oe}$

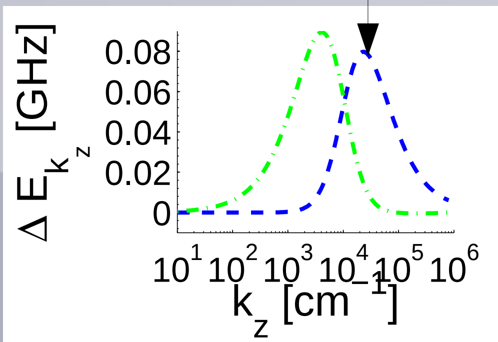
$$E_{\vec{k}} = \sqrt{[h + \rho_{\text{ex}}\vec{k}^2 + \Delta(1 - f_{\vec{k}})\sin^2\Theta_{\vec{k}}][h + \rho_{\text{ex}}\vec{k}^2 + \Delta f_{\vec{k}}]}$$

2.2 Real system: all modes



eigenmode approximation

$$f_{\vec{k}} = 1 - |\vec{k}d| \frac{|\vec{k}d|^3 + |\vec{k}d|\pi^2 + 2\pi^2(1 + e^{-|\vec{k}d|})}{(\vec{k}^2 d^2 + \pi^2)^2}$$



minimum for BEC

Demokritov *et al.* '06

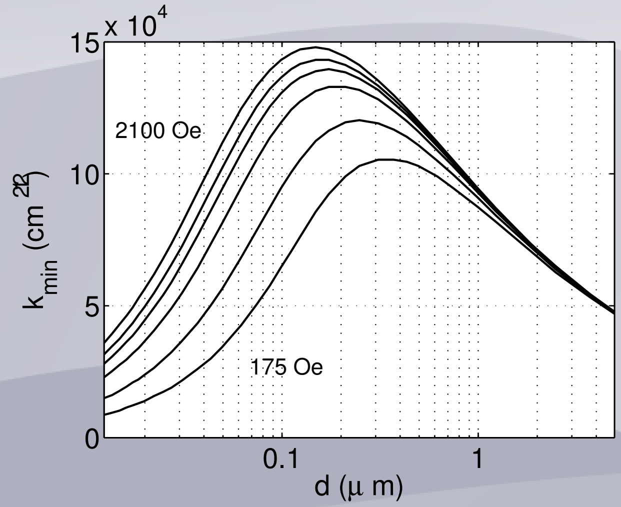
$$d = 4040a \approx 5\mu\mathrm{m}$$

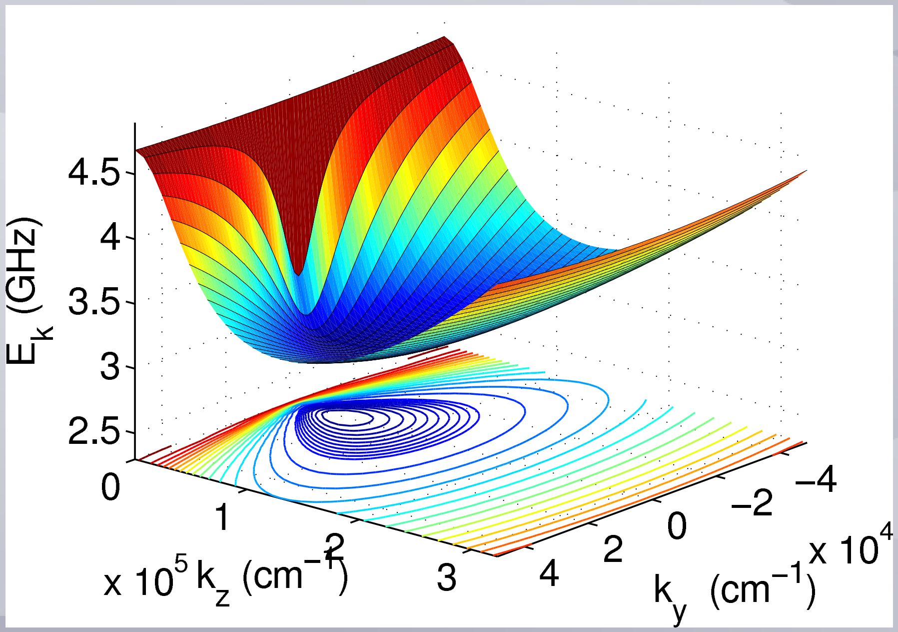
$$H_e = 700\ \mathrm{Oe}$$

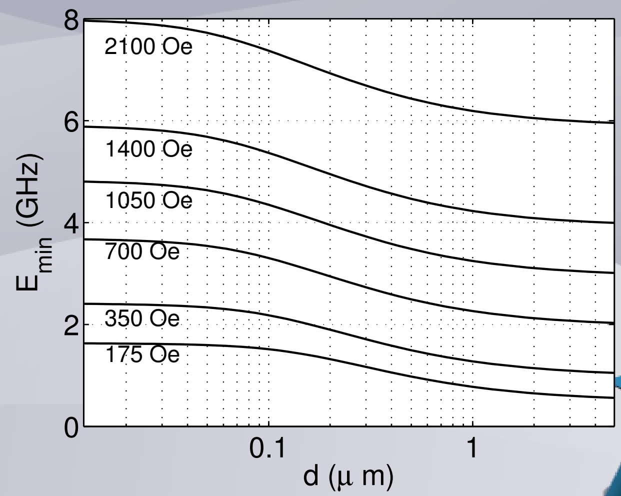
$$E_{\vec{k}} = \sqrt{[h + \rho_{\mathrm{ex}}\vec{k}^2 + \Delta(1 - f_{\vec{k}}) \sin^2 \Theta_{\vec{k}}][h + \rho_{\mathrm{ex}}\vec{k}^2 + \Delta f_{\vec{k}}]}$$

2.2 Spectrum: Summary

- Interplay
  - exchange interaction
  - dipolar interaction
  - finite thickness







2.2a Vertices in diagonal basis

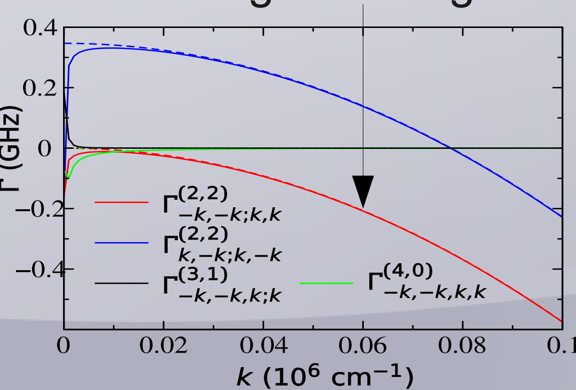
$$\hat{H}_2 = \sum_{\vec{k}} E_{\vec{k}} a^{\dagger}_{\vec{k}} a_{\vec{k}}$$

F. Sauli (in preparation)

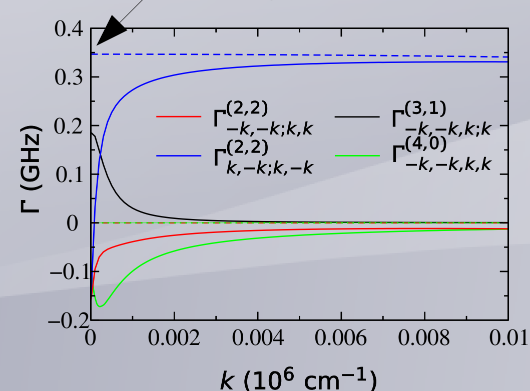
$$H_4 = \frac{1}{N} \sum_{\vec{k}_1 \cdots \vec{k}_4} \delta_{\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4,0} \Bigg(\frac{1}{(2!)^2} \Gamma^{(2,2)}_{\vec{k}_1,\vec{k}_2;\vec{k}_3,\vec{k}_4} b^{\dagger}_{\vec{k}_1} b^{\dagger}_{\vec{k}_2} b_{\vec{k}_3} b_{\vec{k}_4}$$
$$+ \frac{1}{3!} \left\{ \Gamma^{(3,1)}_{\vec{k}_1,\vec{k}_2,\vec{k}_3;\vec{k}_4} b^{\dagger}_{\vec{k}_1} b_{\vec{k}_2} b_{\vec{k}_3} b_{\vec{k}_4} + \text{h.c.} \right\} + \frac{1}{4!} \left\{ \Gamma^{(4,0)}_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} b_{\vec{k}_1} b_{\vec{k}_2} b_{\vec{k}_3} b_{\vec{k}_4} + \text{h.c.} \right\} \Bigg)$$

- properties of vertex functions

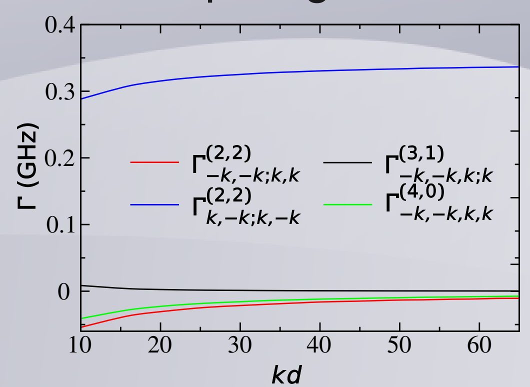
$\Gamma \sim -Jk^2$
ferromagnetic magnons



$\Gamma \sim M_s$
Sergio Rezende '09

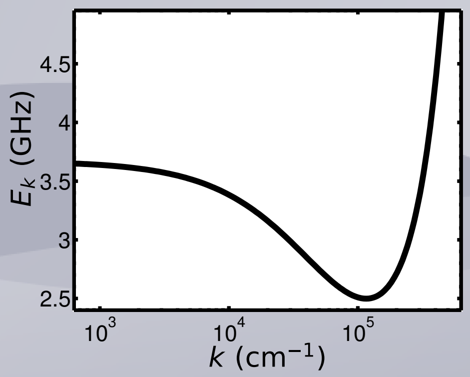


$k = k_{\min}$ finite size effects
vertices stay finite at minimum,
but competing interactions



2.2b BEC at finite momentum

- Bogoliubov shift $\Phi_{\vec{k}}(\tau) = \phi_{\vec{k}} + \delta\Phi_{\vec{k}}(\tau)$
- new features for YIG system
  - condensate at finite wave-vectors
    $\phi_k = \delta_{k,k_{\min}} \phi_0$
  - possible 2 condensates $\epsilon_{\vec{k}} = \epsilon_{-\vec{k}}$
    $\phi_k = \delta_{k,k_{\min}} \phi_0^+ + \delta_{k,-k_{\min}} \phi_0^-$
  - explicitly symmetry breaking term

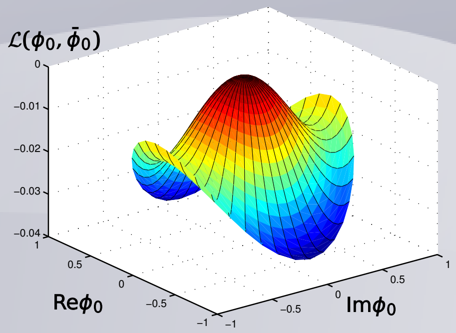


$$H_2 = \sum_{\vec{k}} \epsilon_{\vec{k}} b^\dagger_{\vec{k}} b_{\vec{k}} + \frac{1}{2} \sum \left(\gamma b^\dagger b^\dagger + \gamma^* bb\right)$$

parallel pumping

- Gross-Pitaevskii equation

$$0 = (\epsilon_{\vec{k}} - \mu)\phi_{-\vec{k}} + \gamma_{\vec{k}} \phi^*_{-\vec{k}} + (\text{Interactions})$$



Napoleons hat potential

2.2b BEC at finite momentum

- interactions provoke condensation at integer multiples of $\vec{k}_{\mathrm{min}}$

$$\phi^\sigma_{\mathsf{k}} = \sqrt{N} \sum_{n=-\infty}^{\infty} \delta_{\mathsf{k},n\mathsf{q}} \psi^\sigma_n$$

- discrete Gross-Pitaevskii equation

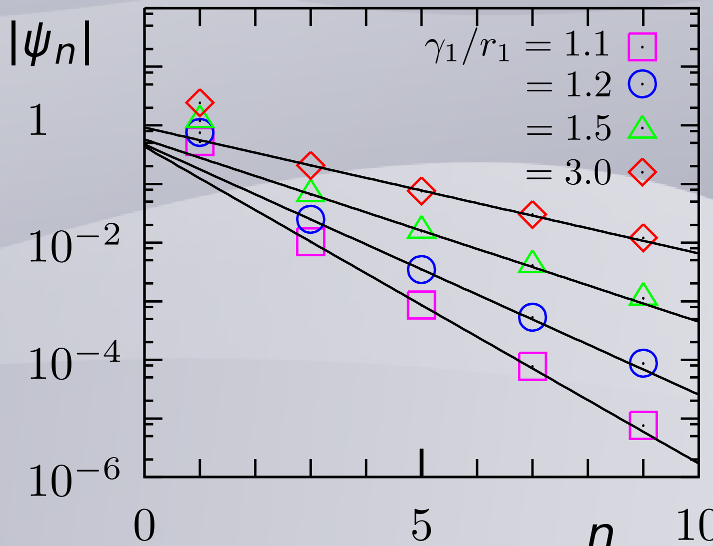
$$-(\epsilon_{n\mathsf{q}} - \mu)\psi^{\bar{\sigma}}_n - \gamma_n \psi^\sigma_n = \frac{1}{2} \sum_{n_1 n_2} \sum_{\sigma_1 \sigma_2} \delta_{n,n_1+n_2} V^{\sigma\sigma_1\sigma_2}_{n n_1 n_2} \psi^{\sigma_1}_{n_1} \psi^{\sigma_2}_{n_2}$$

$$+\frac{1}{3!} \sum_{n_1 n_2 n_3} \sum_{\sigma_1 \sigma_2 \sigma_3} \delta_{n,n_1+n_2+n_3} U^{\sigma\sigma_1\sigma_2\sigma_3}_{n n_1 n_2 n_3} \psi^{\sigma_1}_{n_1} \psi^{\sigma_2}_{n_2} \psi^{\sigma_3}_{n_3} .$$

- condensate density

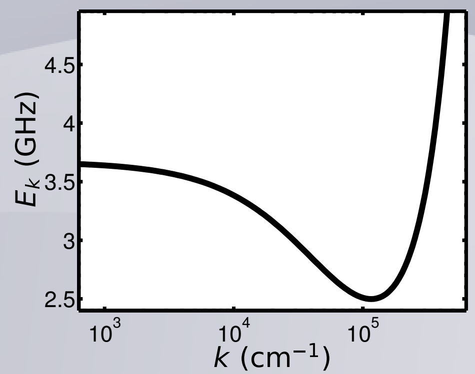
$$\rho(\mathsf{r}) = |\phi^a(\mathsf{r})|^2 = 4 \sum_n |\psi_n|^2 \cos^2(n\mathsf{q} \cdot \mathsf{r})$$

- vertices of YIG
  → solve discrete GPE



2.2 Recapitulation

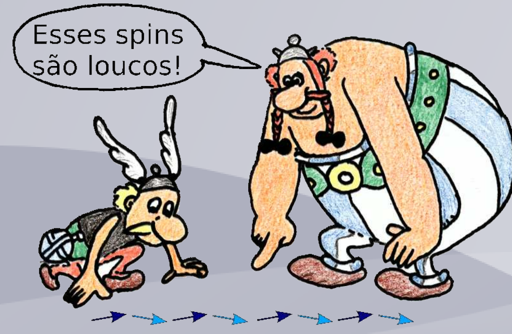
- development of **interacting spin-wave theory** with dipole dipole interactions (straightforward)
- interesting properties of the energy dispersion
- interactions: possible **condensation** of bosons at finite wave-vectors and integer multiples

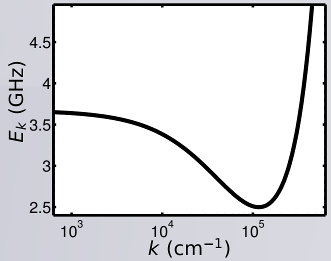


3 Summary

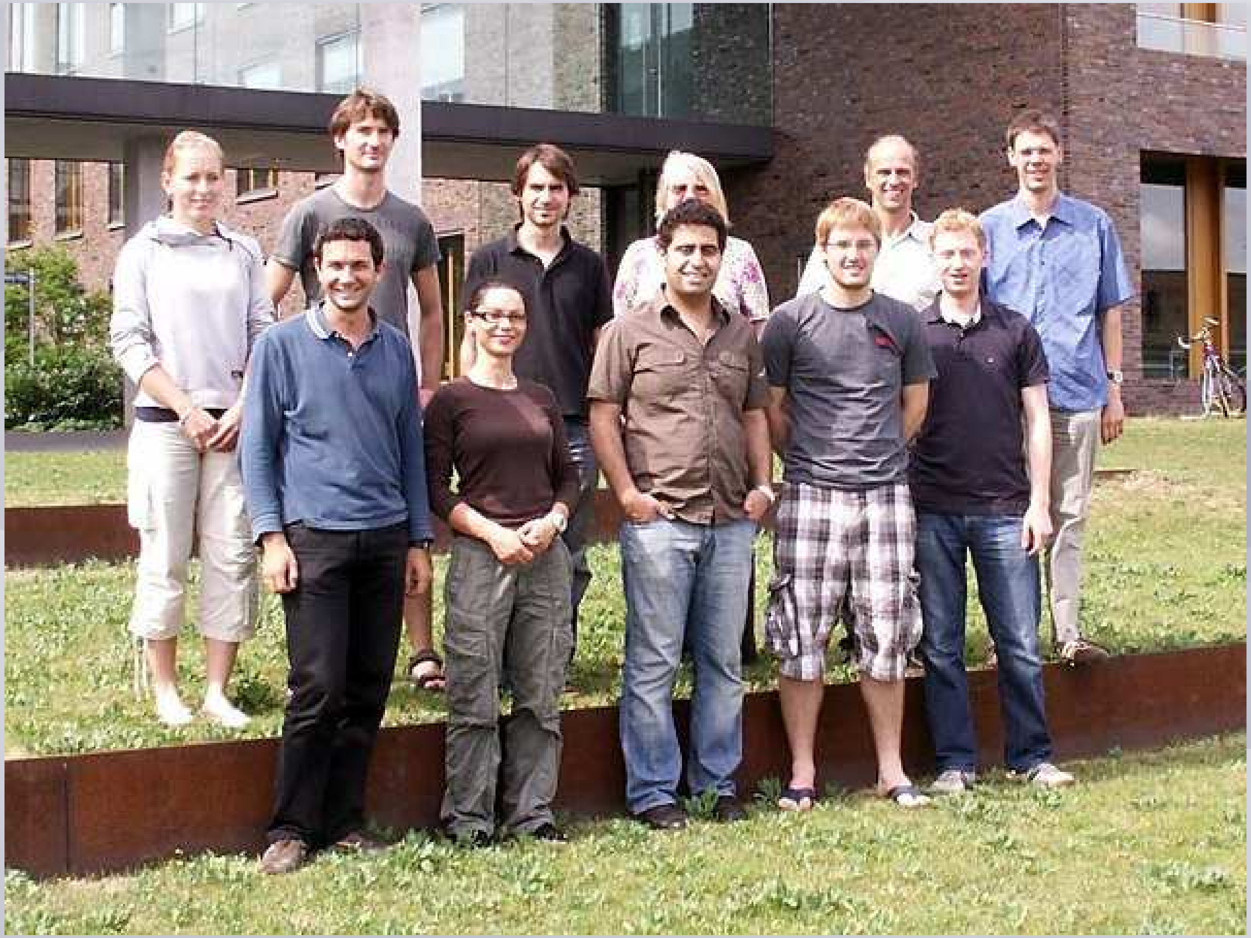
- Description of magnetic insulators: Spin-wave theory

- Applications
  - Hybrid approach for Quantum Antiferromagnets in a magnetic field nonanalytic contributions to the spin-wave velocity
  - Spin-wave theory for thin film ferromagnets





4 Acknowledgement



Group of Peter Kopietz at ITP in Frankfurt (Germany)

www.itp.uni-frankfurt.de/~kreisel/en

2.1 Euclidean action of NLSM with uniform magnetic field

- $$S_{\mathrm{NLSM}}[\vec{\Omega}] = \frac{\rho_s}{2}\int_0^\beta d\tau \int d^D r \Big[\sum_{\mu=1}^{D}(\partial_\mu \vec{\Omega})^2 + c^{-2}(\partial_\tau \vec{\Omega} - i\vec{h}\times\vec{\Omega})^2\Big],$$

  spin field

- spin stiffness and spin wave velocity at T=0

$$\rho_s = JS^2 a^{2-D}$$
$$c = 2JSa\sqrt{D}$$

- effect of the magnetic field $\partial_\tau \rightarrow \partial_\tau - i\vec{h}\times$
- but the magnon dispersions can not be characterized by a single $c(h)$!