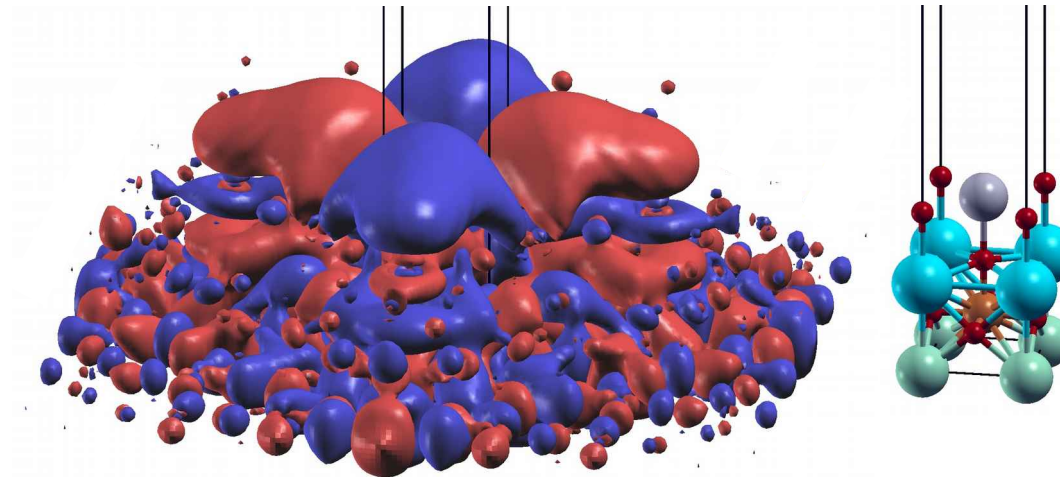


# Wannier based theory of unconventional superconductors

**Andreas Kreisel**

Institut für Theoretische Physik, Universität Leipzig, Germany



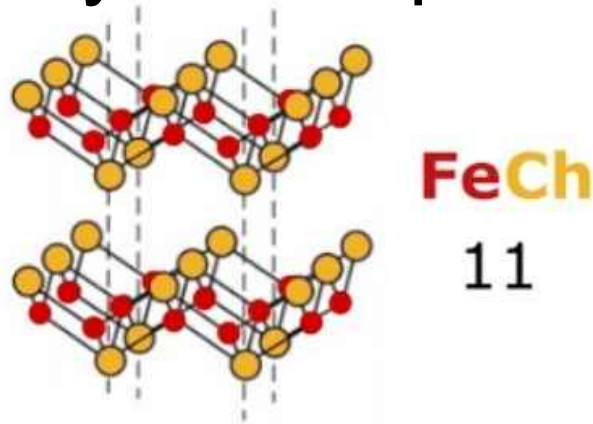
A. Kreisel, *et al.*, Phys. Rev. Lett. **114**, 217002 (2015)

**A. Kreisel**, *et al.*, Phys. Rev. B **94**, 224518 (2016)

P. Choubey, AK, *et al.* Phys. Rev. B **96**, 174523 (2017)

# Theoretical simulation of STM

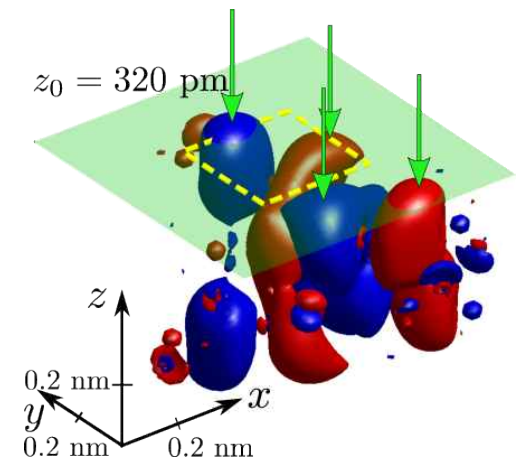
- Scope: layered superconductors, complications



- using wavefunction information in layered superconductors: Wannier method

- Applications

- LiFeAs (multiband, s-wave)
- Cuprates:  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ ,  $\text{Ca}_2\text{CuO}_2\text{Cl}_2$
- FeSe (nematic state, orbital order)

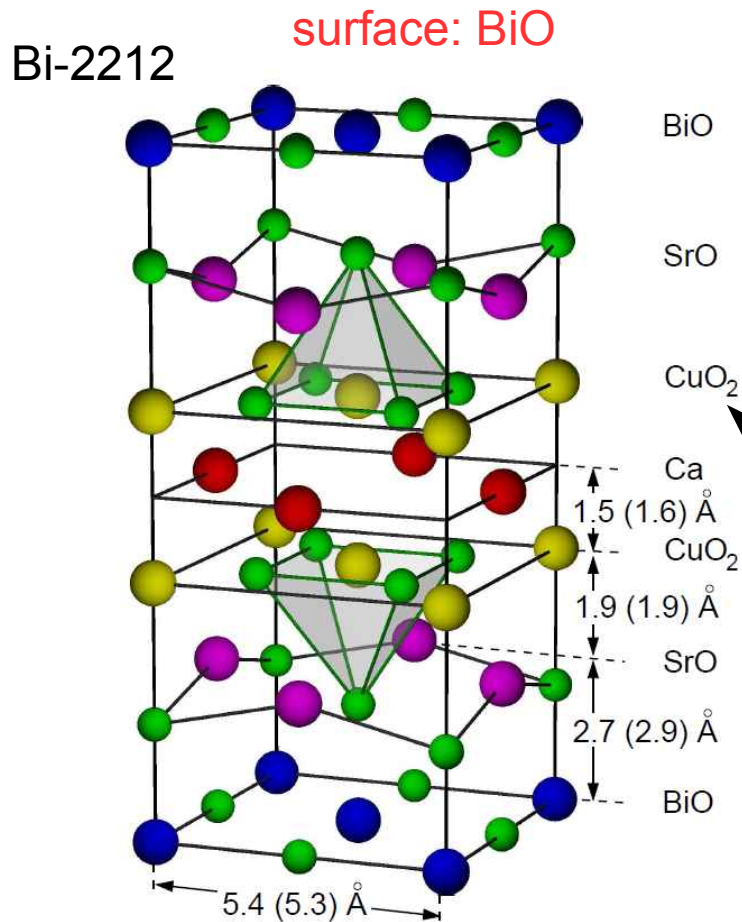


# Layered superconductors

- 2 examples: surface atoms  $\neq$  superconducting layer

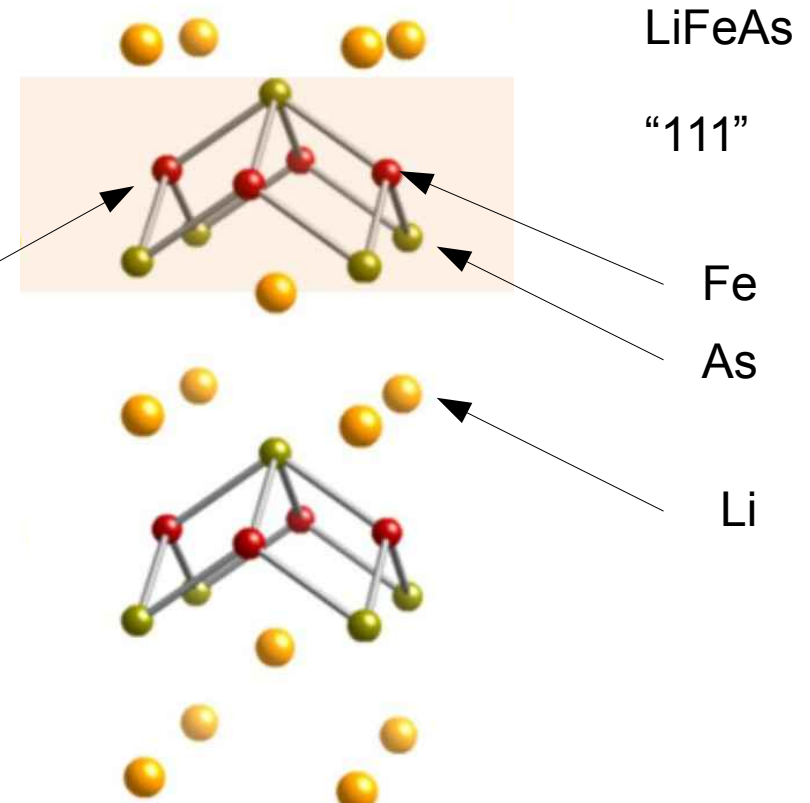
Cuprates

Iron based superconductors



states at the  
Fermi level  
Cu-plane  
Fe-plane

surface: Li, As?

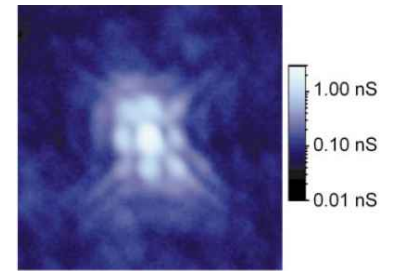


# Theoretical approaches: Cuprates

- LDOS: impurity in d-wave superconductor → bound state

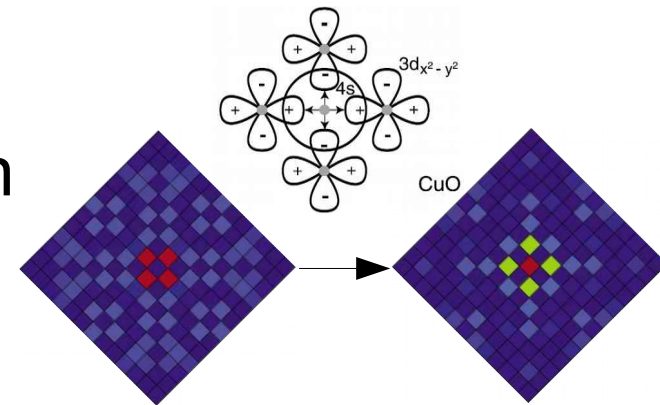
$$\Omega \equiv \Omega' + i\Omega'' = \Delta_0 \frac{\pi c/2}{\ln(8/\pi c)} \left[ 1 + \frac{i\pi}{2} \frac{1}{\ln(8/\pi c)} \right]$$

J. M. Byers, M. E. Flatté, and D. J. Scalapino Phys. Rev. Lett. **71**, 3363 (1993)  
 A. V. Balatsky, M. I. Salkola, and A. Rosengren Phys. Rev. B **51**, 15547 (1995)



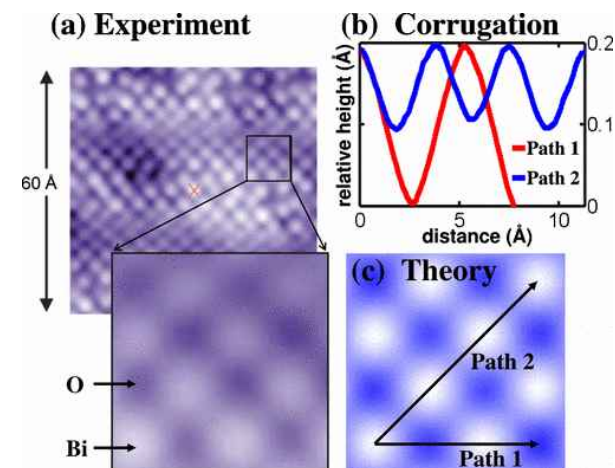
- “Filter function”: STM tip probes states in the superconducting layer by tunneling matrix elements

Martin *et al.*, PRL **88**, 097003 (2002)



- Large tight binding basis set of orbitals + Greens function method to calculate tunneling matrix elements

J. Nieminen, *et al.*, PRB **80**, 134509 (2009)



# Approaches: multiband systems

- Identification of nature of impurities in FeSe monolayer (non-SC) by ab-initio calculations

Dennis Huang et al., Nano Lett., **16** (7), 4224 (2016)

- Ab initio calculations of effects of orbital order on the surface layer

Kim et al., Sci. Adv. 2017; **3**: eaao0362

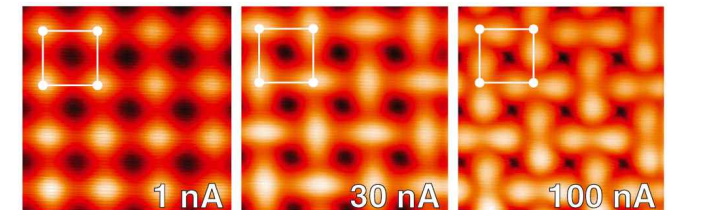
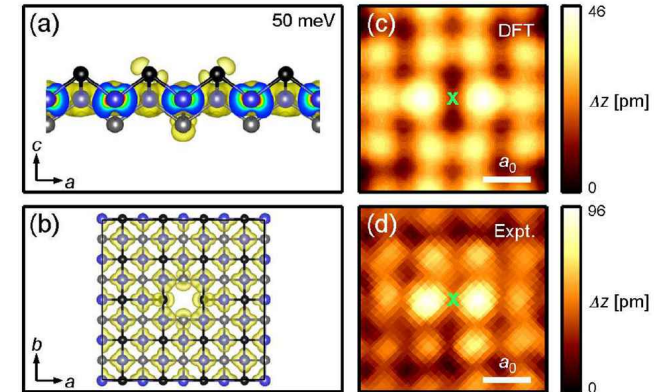
- Wannier method (this talk)

See also: “holographic maps”

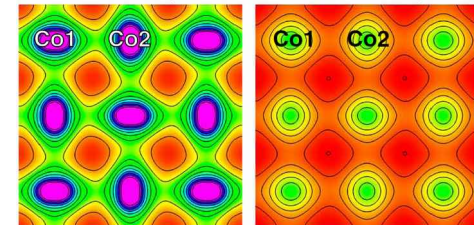
Dalla Torre, He, Demler  
Nat. Phys., **12**, 1052 (2016)

$$h_G(\mathbf{q}, V) = g(\mathbf{q}, V)g^*(\mathbf{q} + \mathbf{G}, V)$$

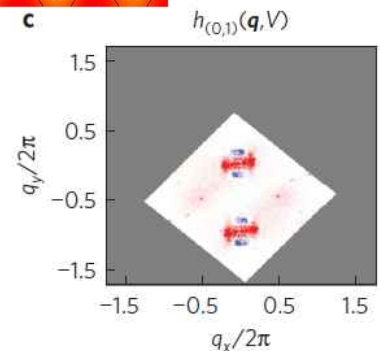
unravel intra-unitcell information



D) 1 Å above Co plane E) 2.5 Å above Co plane



CeCoIn5



# Wannier method: example LiFeAs

- Ab-initio calculation

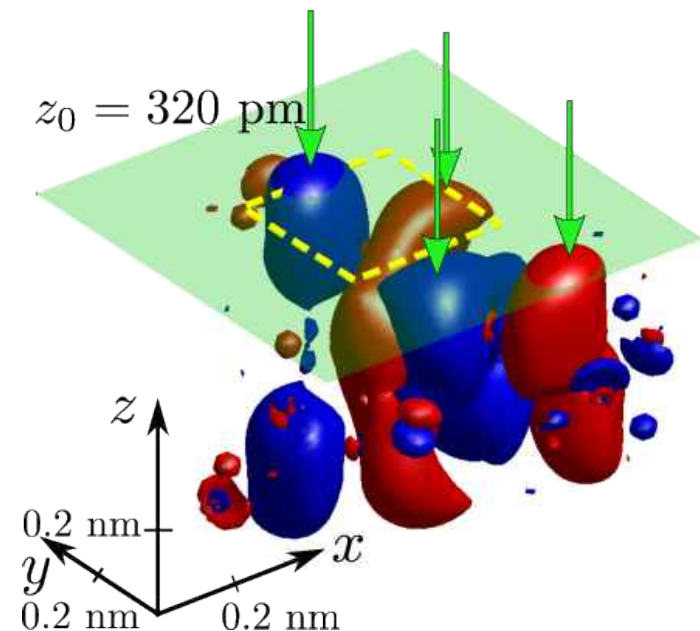
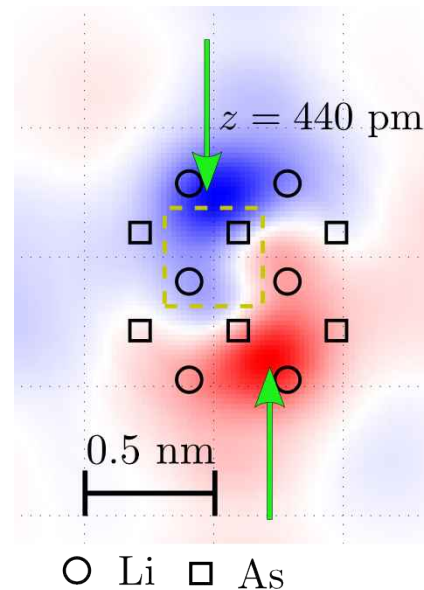
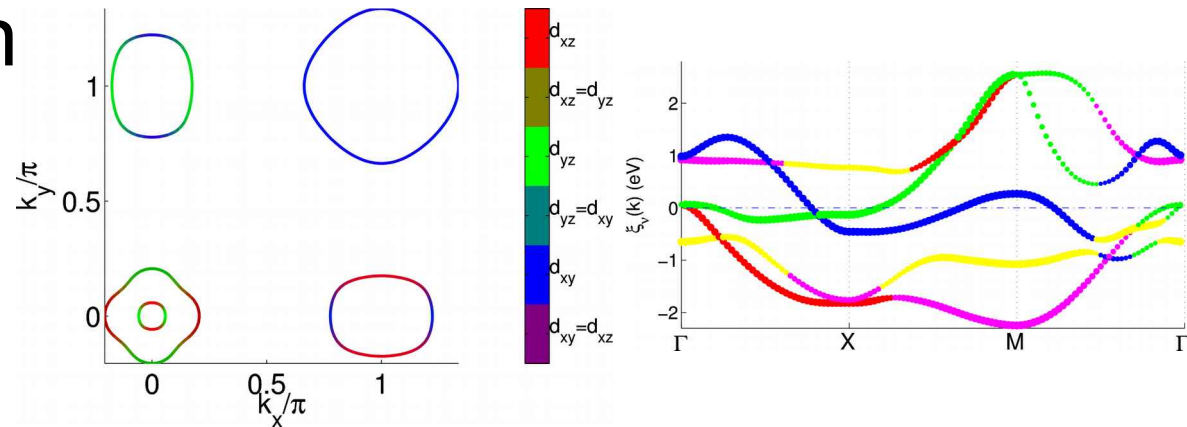
- band structure

5 band model

$$H_0 = \sum_{\mathbf{R}\mathbf{R}',\sigma} t_{\mathbf{R}\mathbf{R}'} c_{\mathbf{R}\sigma}^\dagger c_{\mathbf{R}'\sigma}$$

$$- \mu_0 \sum_{\mathbf{R},\sigma} c_{\mathbf{R}\sigma}^\dagger c_{\mathbf{R}\sigma}$$

- Wannier functions (including glide plane symmetry)



# Superconductivity

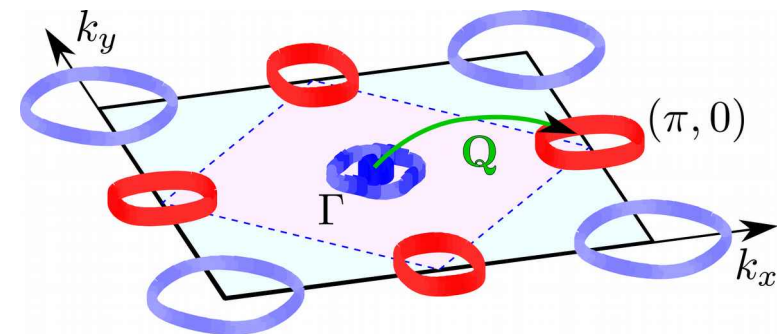
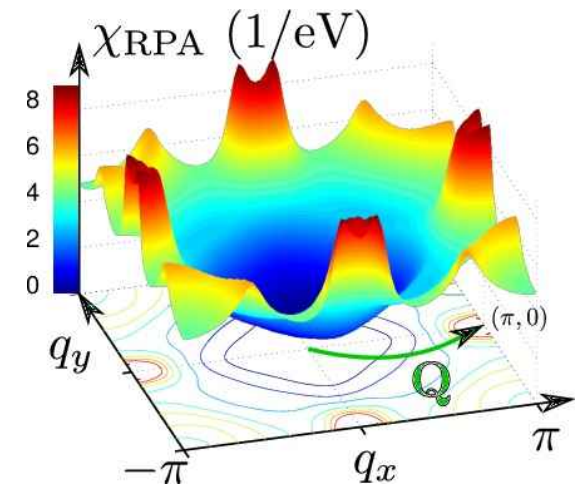
- superconducting order parameter from spin- fluctuation theory

$$H_{\text{BCS}} = - \sum_{\mathbf{R}, \mathbf{R}'} \Delta_{\mathbf{R}\mathbf{R}'} c_{\mathbf{R}\uparrow}^\dagger c_{\mathbf{R}'\downarrow}^\dagger + H.c.,$$

- calculate Greens function in superconducting state

$$H_{\text{Nambu}} = \begin{pmatrix} H_k & \Delta_k \\ \Delta_k^\dagger & -H_{-k} \end{pmatrix}$$

$$G_0(\mathbf{k}, \omega) = [\omega - H_{\text{Nambu}} + i0^+]^{-1}$$



Real space Greens function by Fourier transform

# Impurity

- ab-initio calculation of impurity potential for Co, Ni, Mn in LiFeAs (engineered impurity)

$$H_{\text{imp}} = \sum_{\sigma} V_{\text{imp}} c_{\mathbf{R}^* \sigma}^{\dagger} c_{\mathbf{R}^* \sigma} \quad H = H_0 + H_{\text{BCS}} + H_{\text{imp}}$$

- T-matrix approach to obtain Greens function

other methods also possible

- BdG
- Gutzwiller mean field

Kreisel et al., Phys. Rev. Lett. **114**, 217002 (2015)  
Choubey et al., New J. Phys. **19**, 013028 (2017)

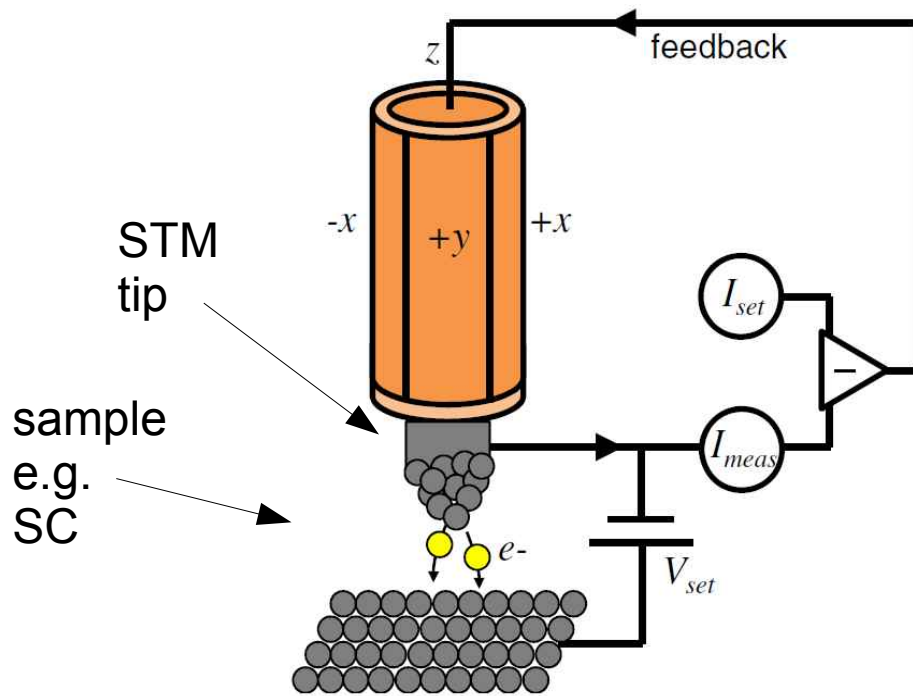
$$\underline{\hat{G}}_{\mathbf{R}, \mathbf{R}'}(\omega) = \underline{\hat{G}}_{\mathbf{R}-\mathbf{R}'}^0(\omega) + \underline{\hat{G}}_{\mathbf{R}}^0(\omega) \underline{\hat{T}}(\omega) \underline{\hat{G}}_{-\mathbf{R}'}^0(\omega)$$

$$\underline{\hat{T}}(\omega) = [1 - \underline{\hat{V}}_{\text{imp}} \underline{\hat{G}}(\omega)]^{-1} \underline{\hat{V}}_{\text{imp}}$$

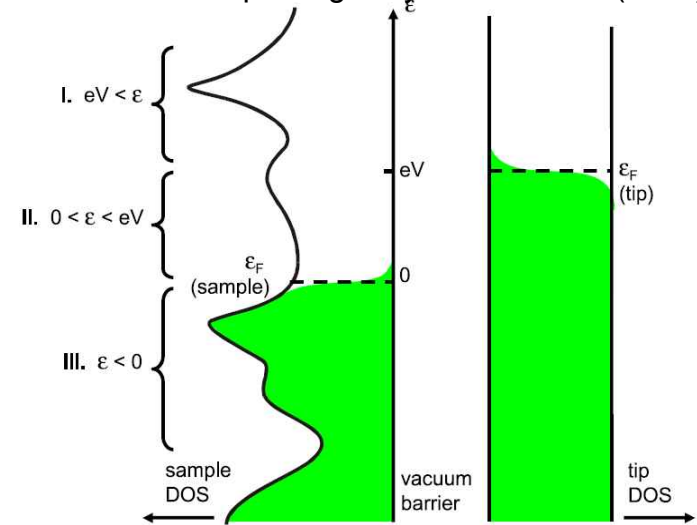
lattice Green function  
(state of the art)



# Scanning tunneling microscopy



J. Hoffman Rep. Prog. Phys. **74** 124513 (2011)



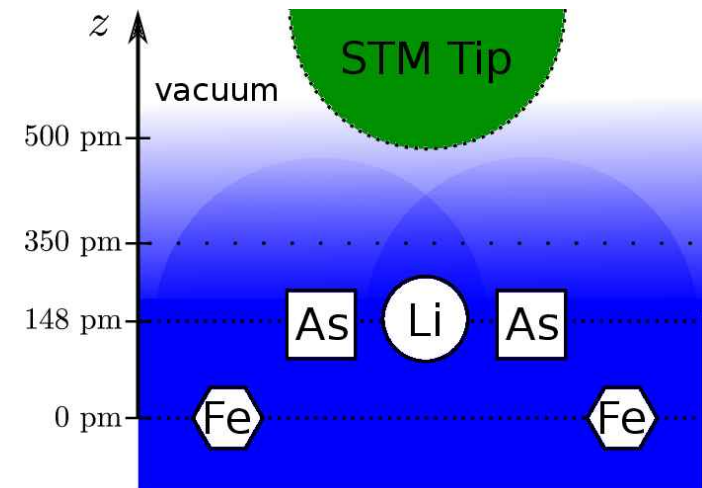
Tunnelling current:

J. Tersoff and D. R. Hamann, PRB **31**, 805 (1985)

$$I(V, x, y, z) = -\frac{4\pi e}{\hbar} \rho_t(0) |M|^2 \int_0^{eV} \rho(x, y, z, \epsilon) d\epsilon$$

Local Density Of States (LDOS)  
of sample at given energy **at the tip position**

$$\rho(x, y, z, \omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{r}, \mathbf{r}; \omega)$$



# continuum LDOS (cLDOS)

- Basis transformation  $\psi_\sigma(\mathbf{r}) = \sum_{\mathbf{R}\mu} c_{\mathbf{R}\mu\sigma} w_{\mathbf{R}\mu}(\mathbf{r})$

$$\underline{\hat{G}}_{\mathbf{R},\mathbf{R}'}(\omega) = \underline{\hat{G}}_{\mathbf{R}-\mathbf{R}'}^0(\omega) + \underline{\hat{G}}_{\mathbf{R}}^0(\omega) \underline{\hat{T}}(\omega) \underline{\hat{G}}_{-\mathbf{R}'}^0(\omega)$$

surface Wannier  
function with phases

$$\underline{G}(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mathbf{R}, \mathbf{R}', \mu\nu} \underline{\hat{G}}_{\mathbf{R}, \mathbf{R}'}^{\mu, \nu}(\omega) w_{\mathbf{R}\mu}(\mathbf{r}) w_{\mathbf{R}'\nu}(\mathbf{r}')$$

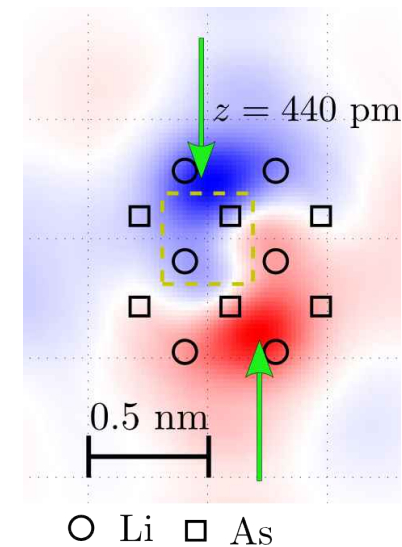
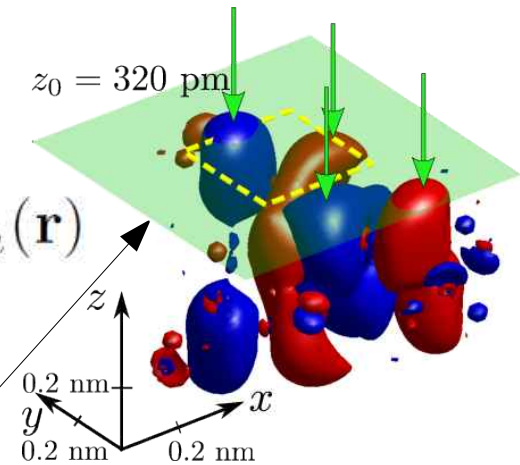
continuum position

nonlocal contributions

lattice Green function

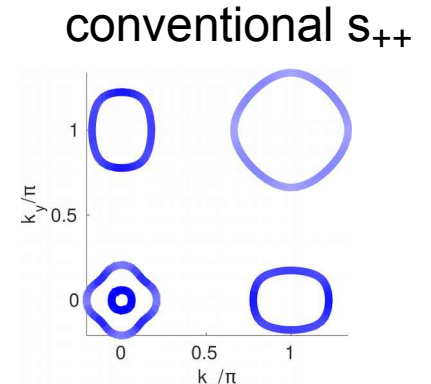
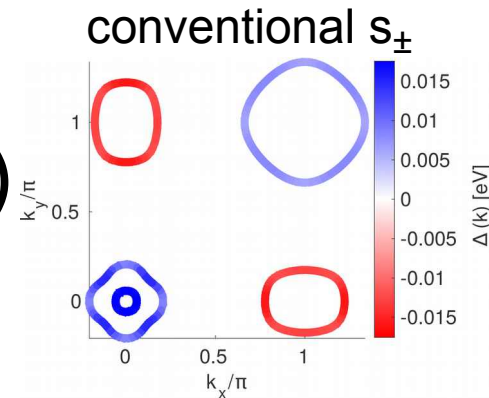
continuum Local Density Of States (cLDOS)  
of sample at given energy **at the tip position**

$$\rho(r, \omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{r}, \mathbf{r}; \omega)$$



# LiFeAs: Questions

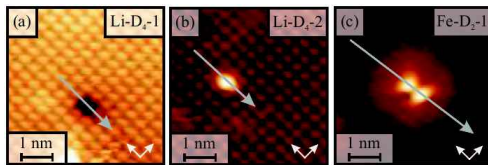
- Properties of the order parameter (sign-change)
- Interpretation of



Y. Wang, A. Kreisel, et al. Phys. Rev. B **88**, 174516 (2013)  
 Z. P. Yin, K. Haule, G. Kotliar Nature Physics **10**, 845 (2014)  
 T. Saito, et al. Phys. Rev. B **90**, 035104 (2014)  
 F. Ahn, et al. Phys. Rev. B **89**, 144513 (2014)

- impurity shapes

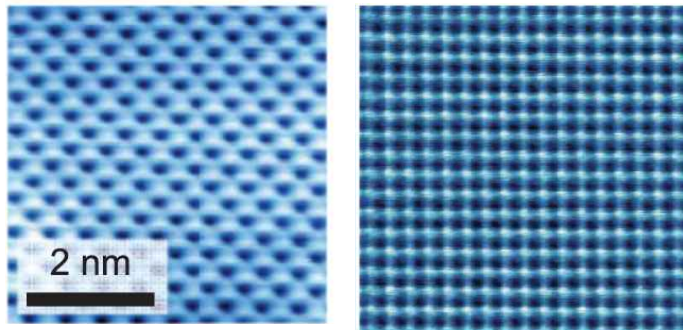
R. Schlegel, et al., Phys. Status Solidi B, **254**: 1600159 (2017)



Hanaguri, unpublished (KITP 2011)

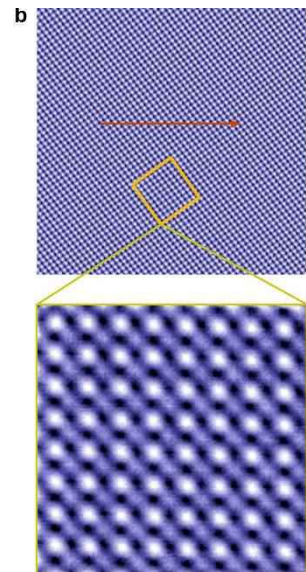


- registered “surface lattice” in STM



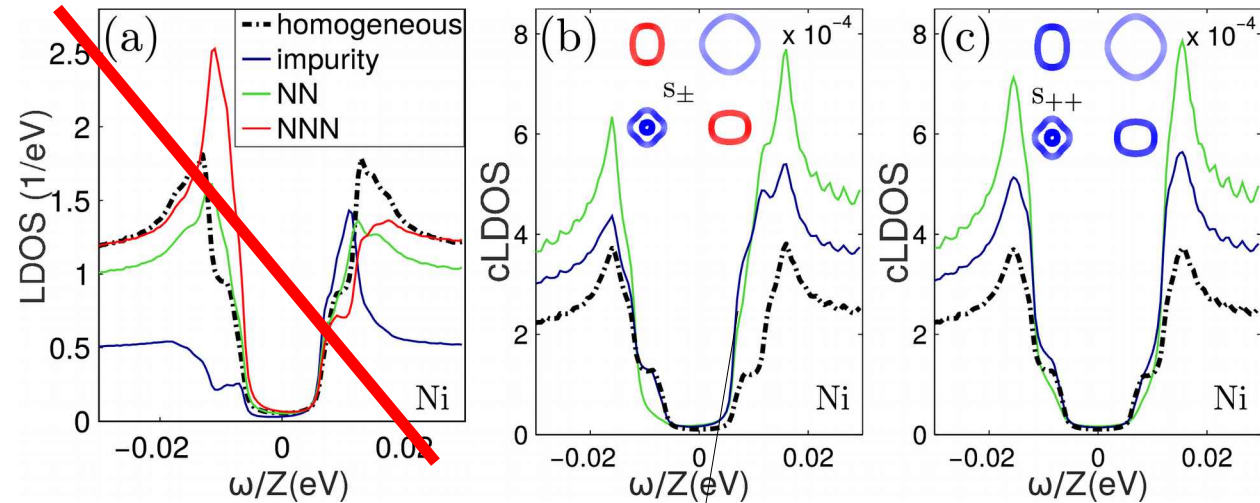
LiFeAs: Li or As lattice?

Shun Chi, et al., PRL **109**, 087002 (2012)  
 T. Hanaguri, et al. PRB **85**, 214505 (2012)  
 S. Grothe, et al., PRB **86**, 174503 (2012)  
 J. -X. Yin, et al., arXiv, 1602.04949 (2016)



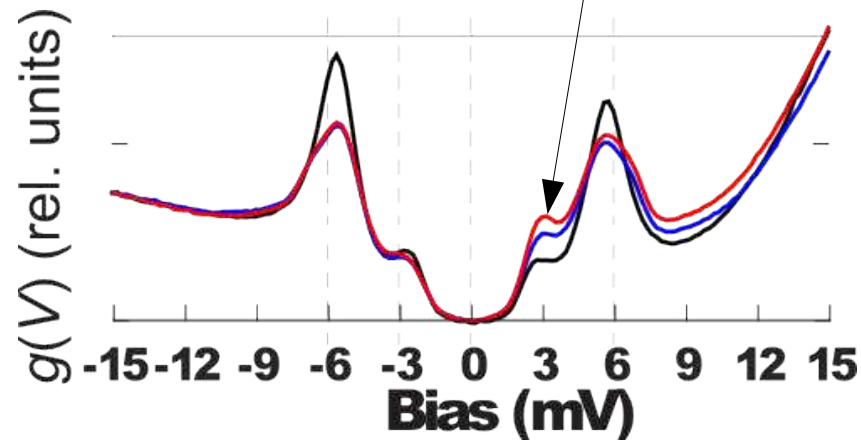
# LiFeAs: spectra

- evidence for sign-changing order parameter by in-gap state with known impurity (Ni)



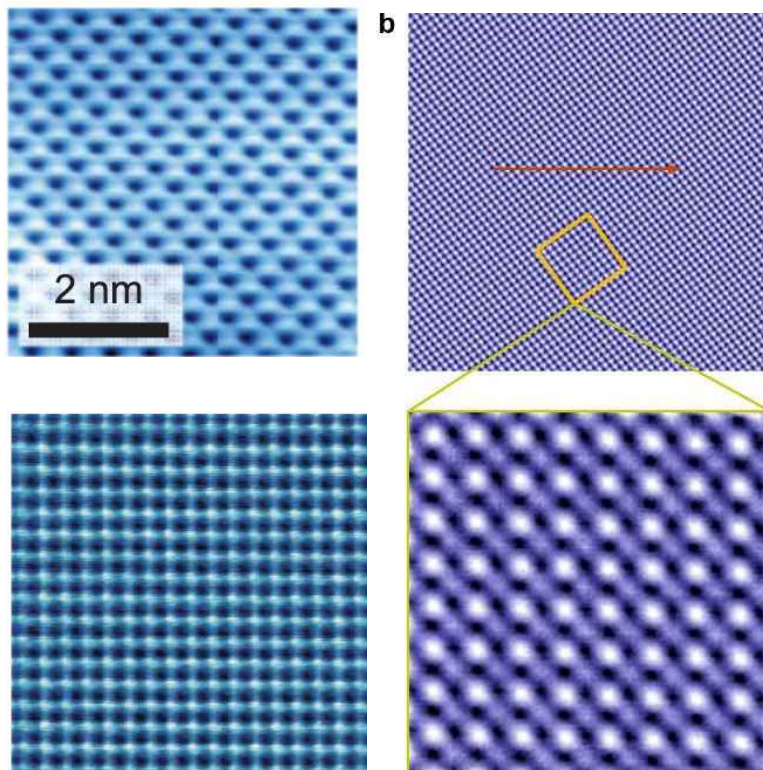
lattice LDOS: strong response at negative bias

$$0 \approx 1 - V_{\text{imp}}^{\mu\mu} G_{\mathbf{R}=0}^0(\omega)^{\mu\mu}$$



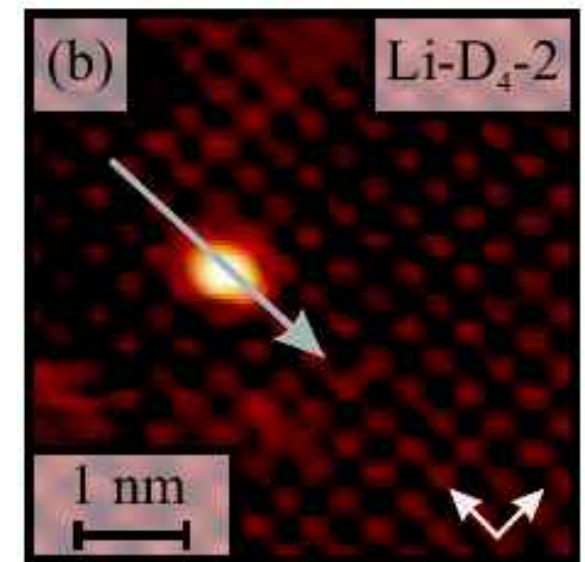
# Height and current dependence of topographs

- experiment: Li or As lattice?



Shun Chi, et al., PRL **109**, 087002 (2012)  
T. Hanaguri, et al. PRB **85**, 214505 (2012)  
S. Grothe, et al., PRB **86**, 174503 (2012)  
J. -X. Yin, et al., arXiv, 1602.04949 (2016)

height maxima at Li positions!?  
counter-intuitive from chemistry point of view



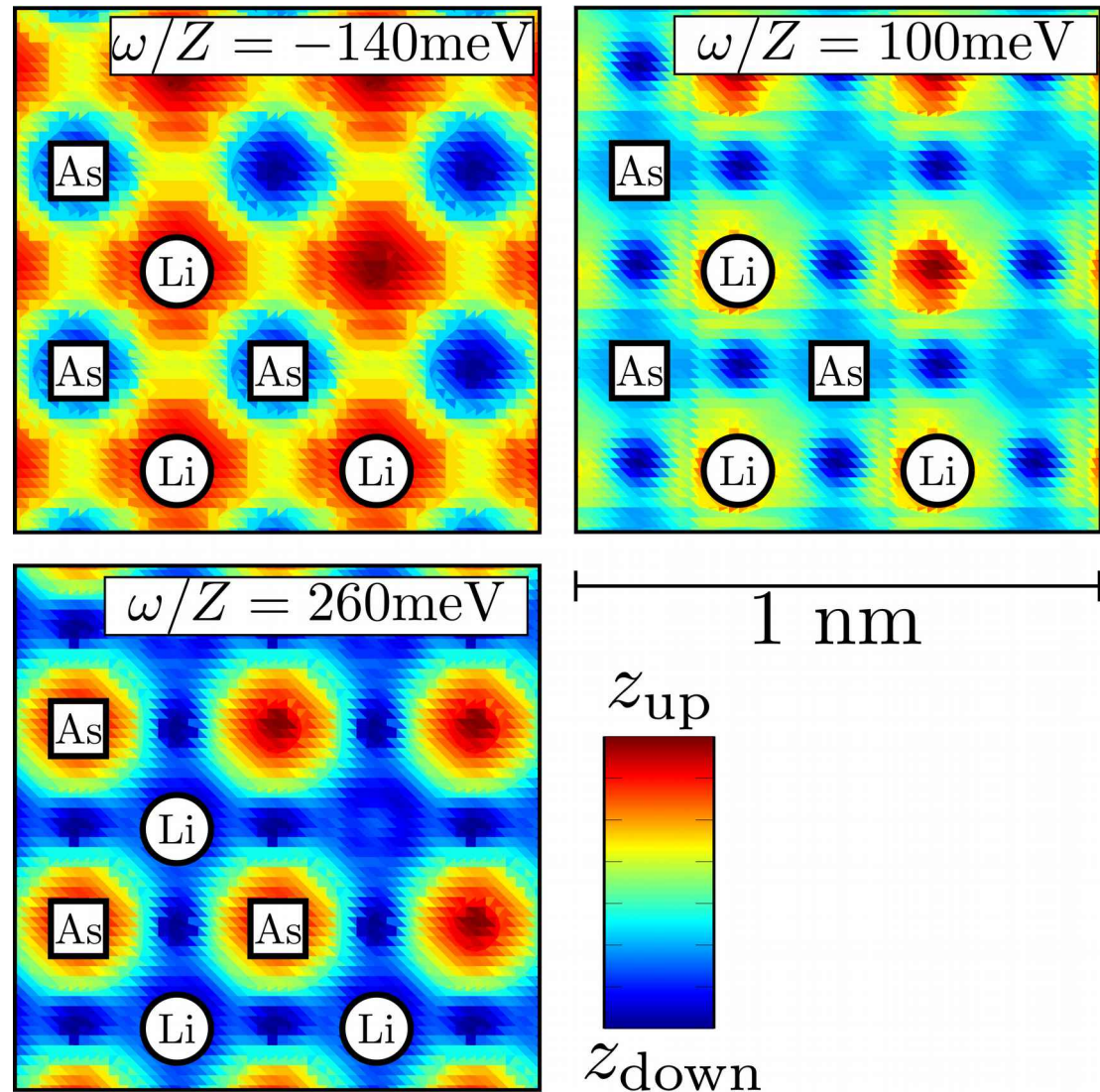
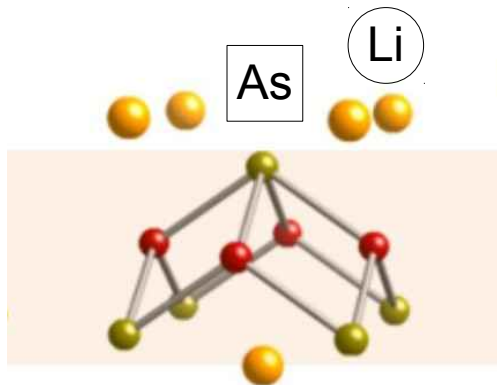
R. Schlegel, et al., Phys. Status Solidi B, **254**: 1600159 (2017)

# Simulation of topographs

- solve for  $z(x, y)$

$$I_0 = \frac{4\pi e}{\hbar} \rho_t(0) |M|^2 \int_0^{eV} d\omega \rho(x, y, z(x, y), \omega)$$

- switching of height maxima as a function of bias voltage



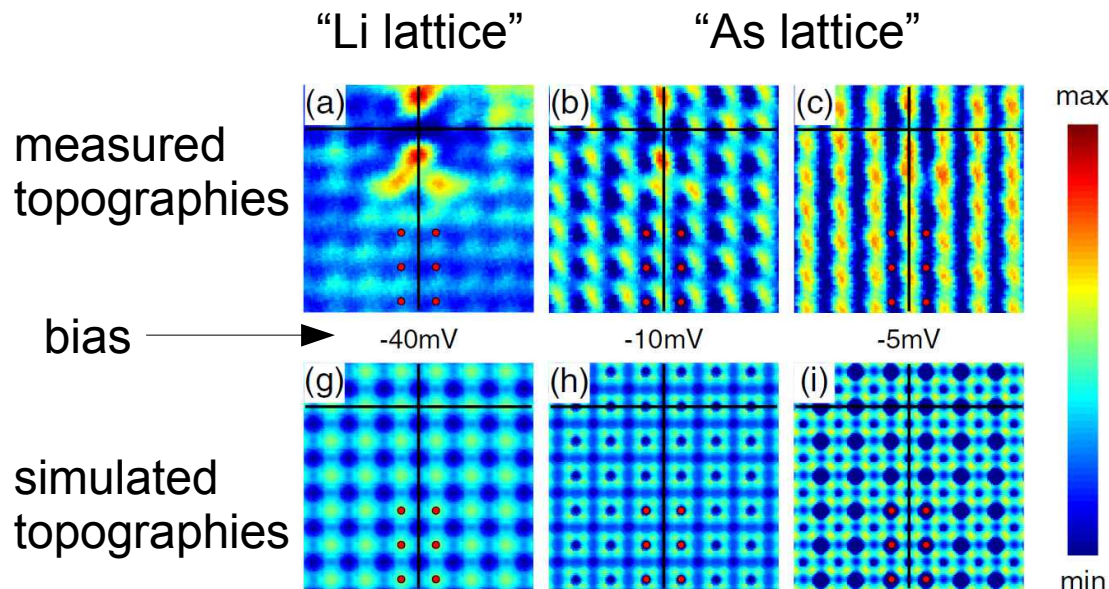
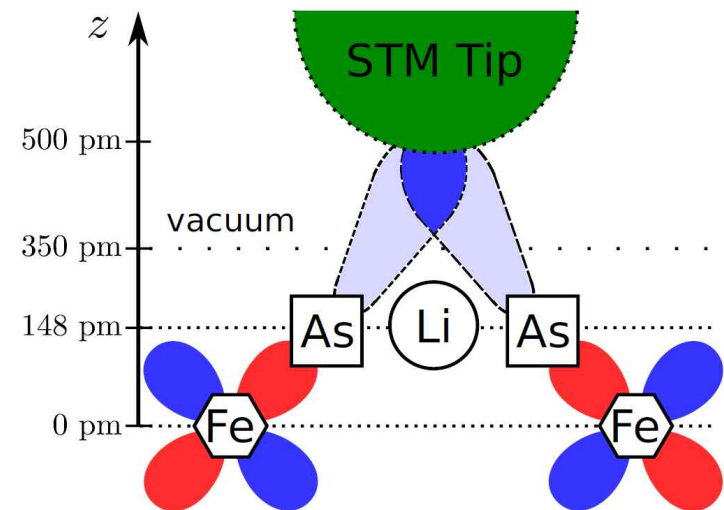
# Results

## registered surface lattice in STM

- tunneling into states described by Wannier functions

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mu, \nu, \mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mu, \mathbf{R}', \nu; \omega) w_{\mathbf{R}, \mu}(\mathbf{r}) w_{\mathbf{R}', \nu}^*(\mathbf{r}')$$

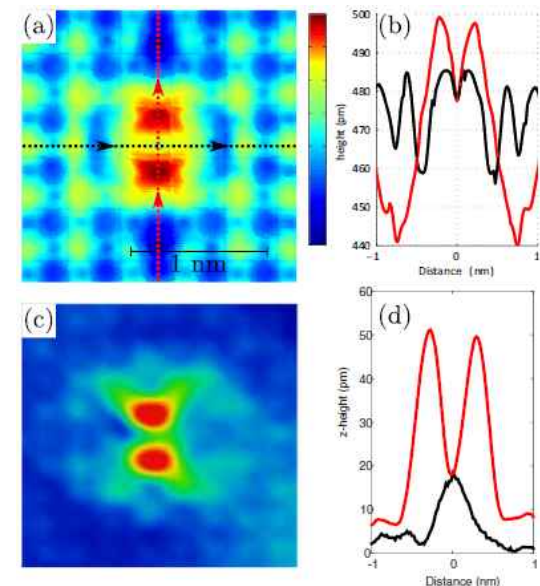
- registered lattice switches as function of bias and current



A. Kreisel, *et al.* Phys. Rev. B **94**, 224518 (2016)

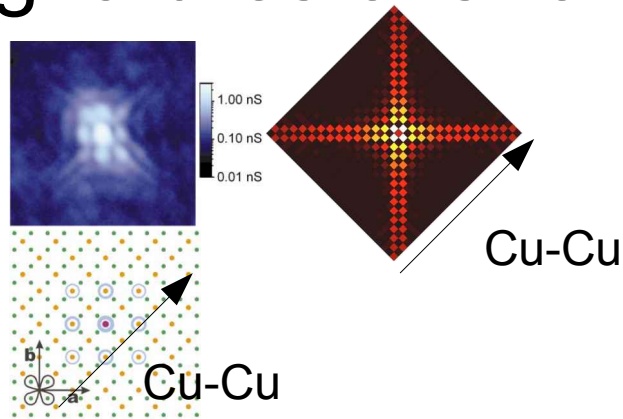
simulated topography close to strong imp.

measured topography close to Ni



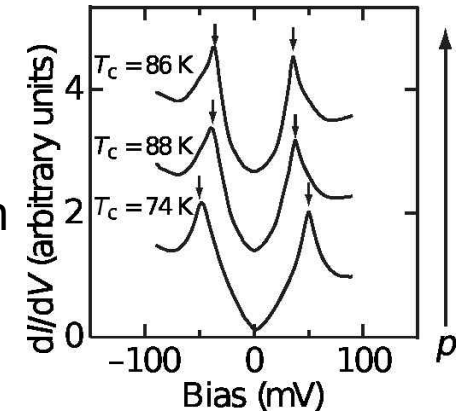
# Cuprates: Questions

- signatures of strong impurity in d-wave SC

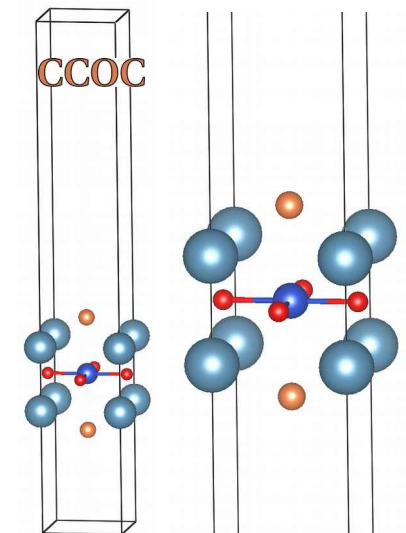
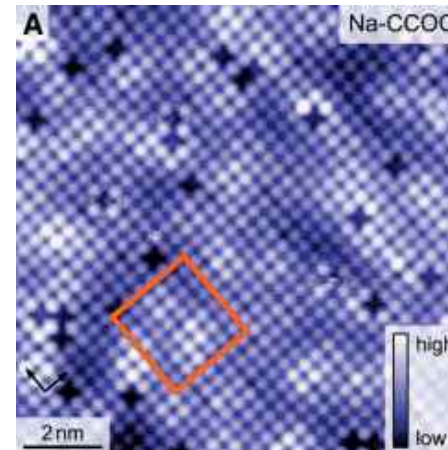
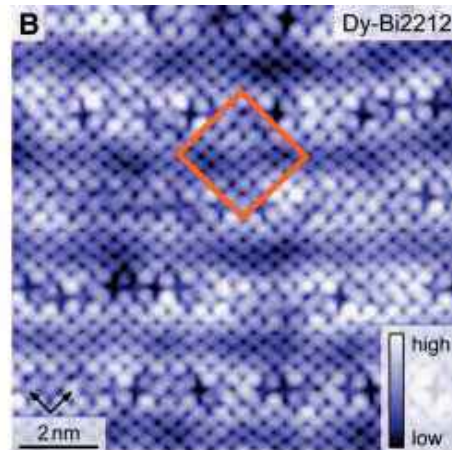
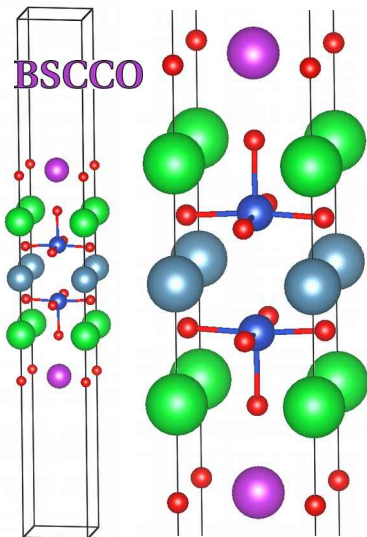


Kohsaka et al. Nature, **454**, 1072 (2008)

Crossover between U-shaped and V-shaped spectra



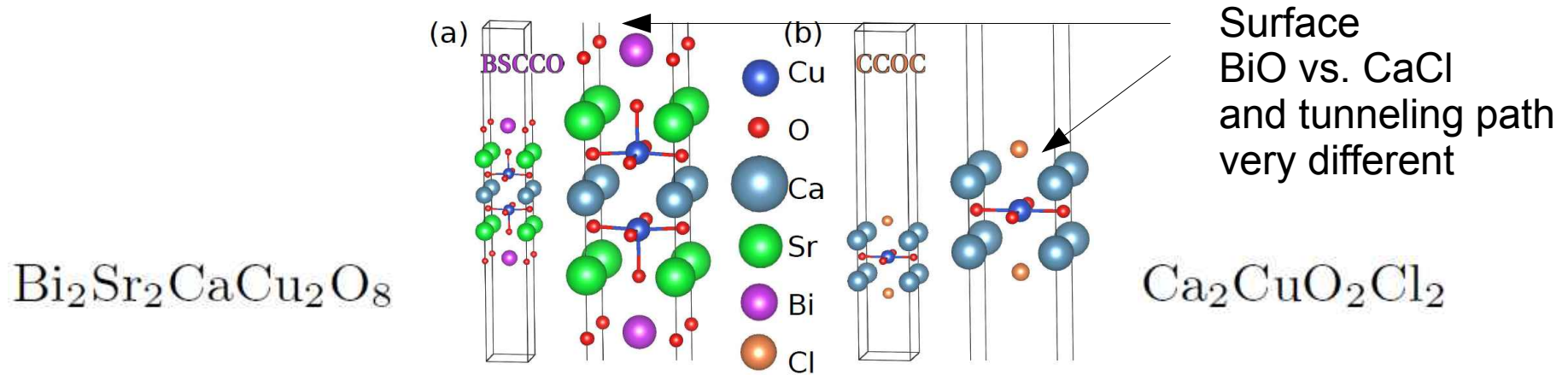
- universalities across materials



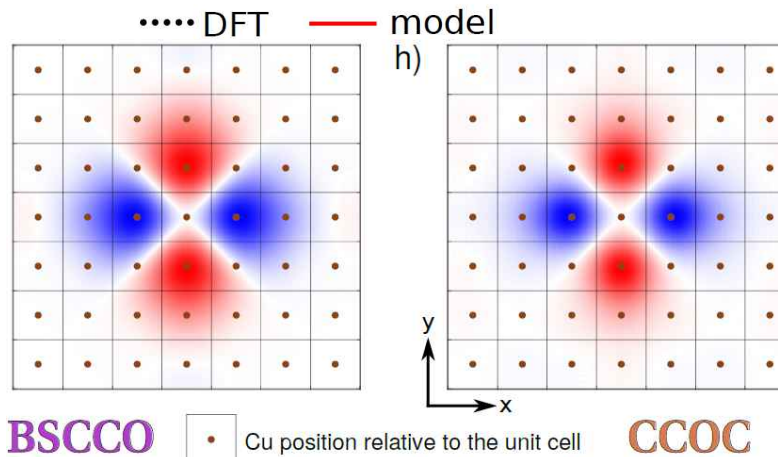
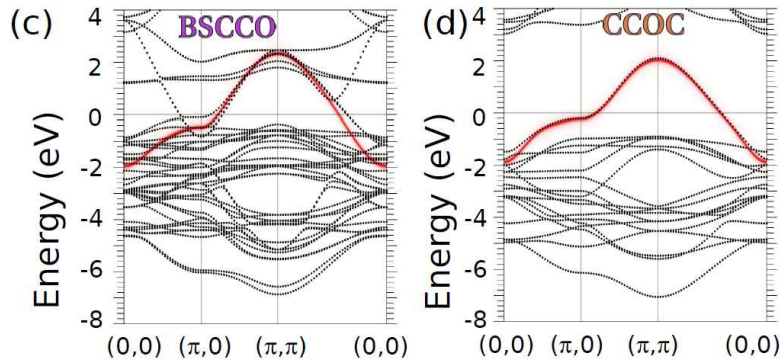
same properties for tunneling!



# Wannier method: Cuprates



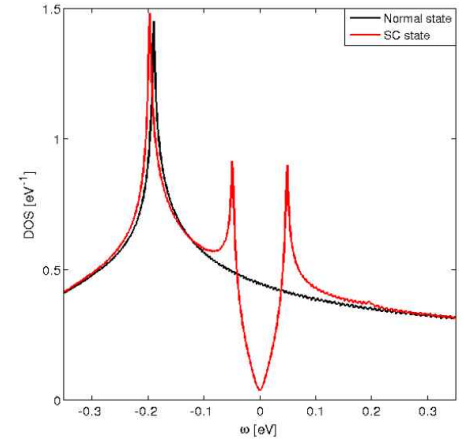
Ab initio calculation:  
1 band model  
+Wannier function



similar properties  
dictated by  
crystal symmetry

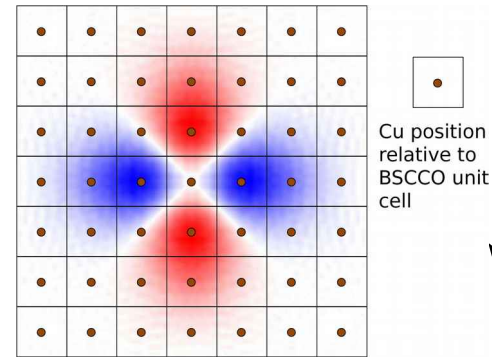
# Superconductivity

- superconducting order parameter (d-wave) (phenomenology or calculation: mean-field)



- continuum Green function

$$\psi_{\sigma}(\mathbf{r}) = \sum_{\mathbf{R}\mu} c_{\mathbf{R}\mu\sigma} w_{\mathbf{R}\mu}(\mathbf{r})$$



surface Wannier function with phases

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mathbf{R}'; \omega) w_{\mathbf{R}}(\mathbf{r}) w_{\mathbf{R}'}^*(\mathbf{r}')$$

lattice Green function

local density of states (LDOS)

$$\rho(r, \omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{r}, \mathbf{r}; \omega)$$

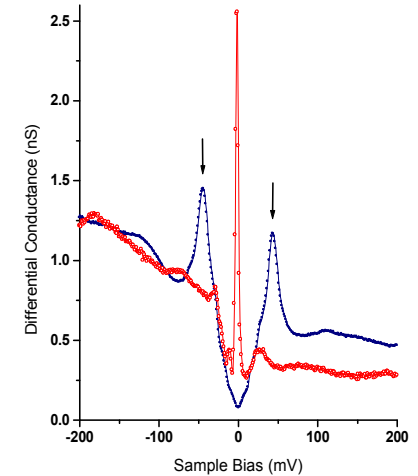
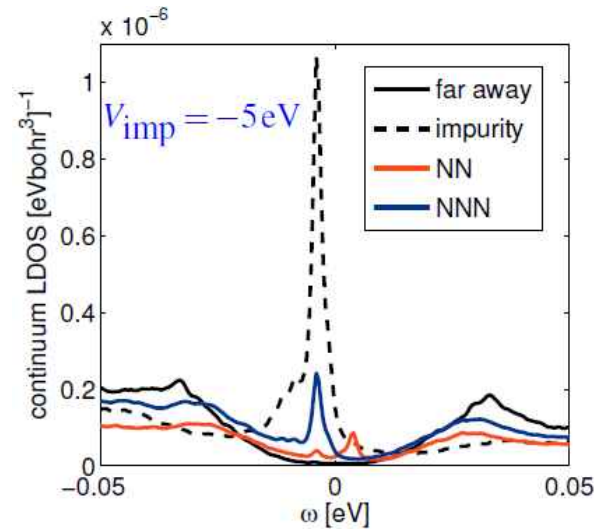
continuum position

nonlocal contributions

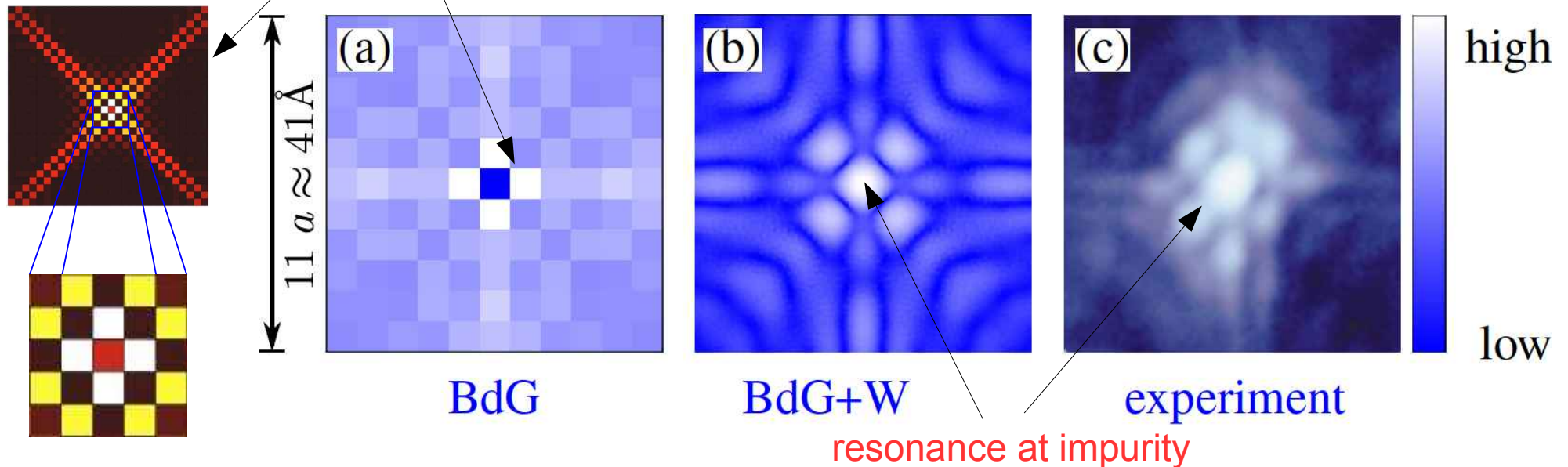
# BSCCO: Results STM maps and spectra

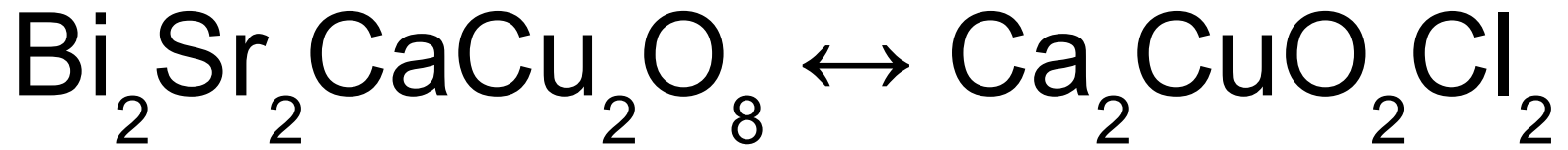
- d-wave order parameter
- Zn impurity:  
 $V_{\text{imp}} = -5 \text{ eV}$   
resonance:  $-3.6 \text{ meV}$

Pan et al., Nature  
403, 746 (2000)



Zhu et al., PRB  
67, 094508  
(2003)

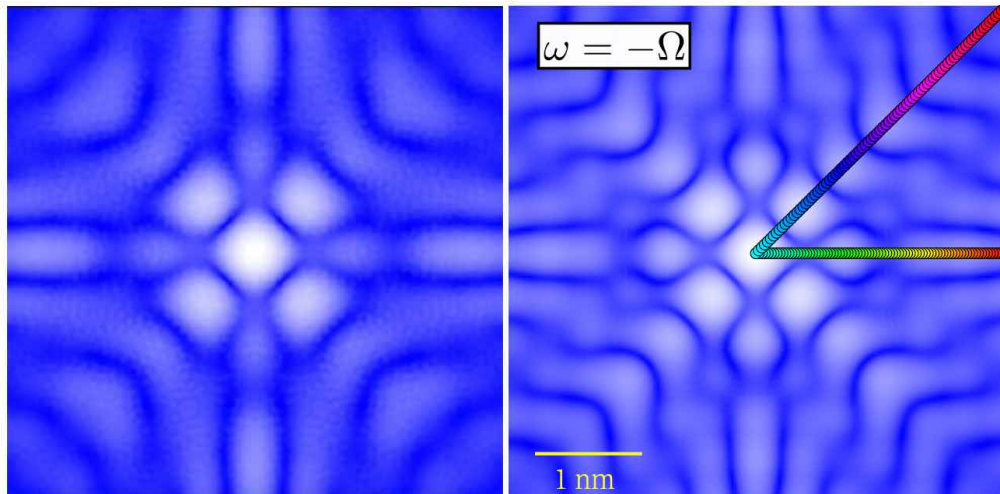
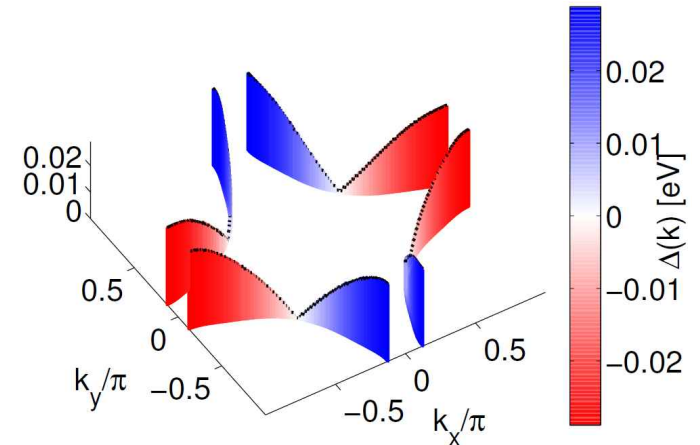




- superconductivity:  
d-wave order parameter
- T-matrix calculation+ Wannier method

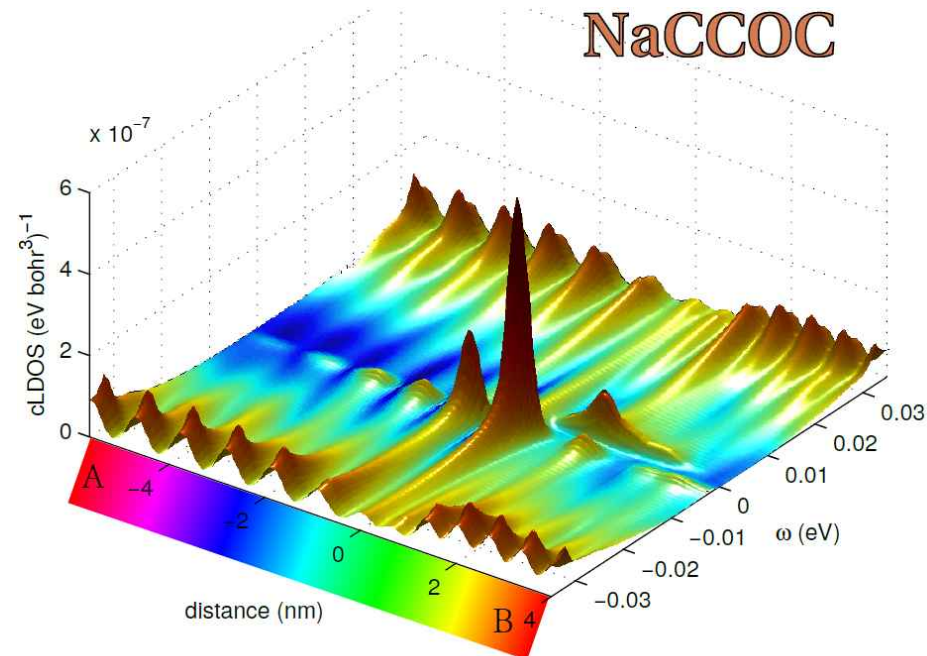
$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mu, \nu, \mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mu, \mathbf{R}', \nu; \omega) w_{\mathbf{R}, \mu}(\mathbf{r}) w_{\mathbf{R}', \nu}^*(\mathbf{r}')$$

- strong impurity  
spectra + conductance map



$\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$

$\text{Ca}_2\text{CuO}_2\text{Cl}_2$

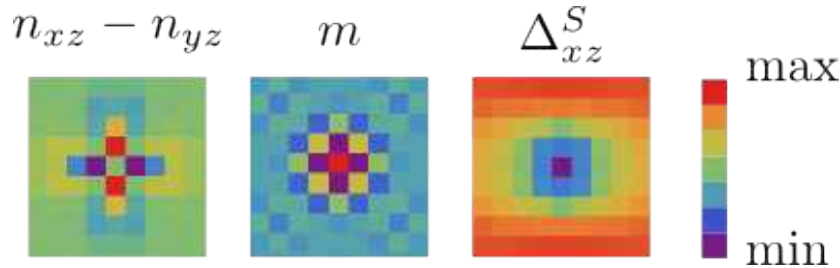


Kreisel et al., Phys. Rev. Lett. **114**, 217002 (2015)  
 Choubey et al., New J. Phys. **19**, 013028 (2017)  
 P. Choubey, et al., Phys. Rev. B **96**, 174523 (2017)

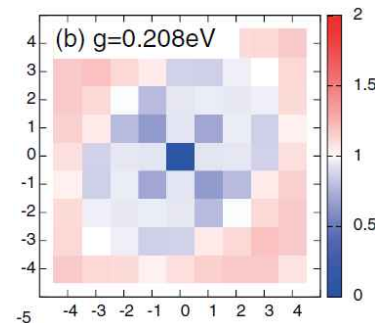
# Chiral defects

- from a symmetry perspective not compatible to impurities on any single site in FeSC
- multiple impurities?
- local order?

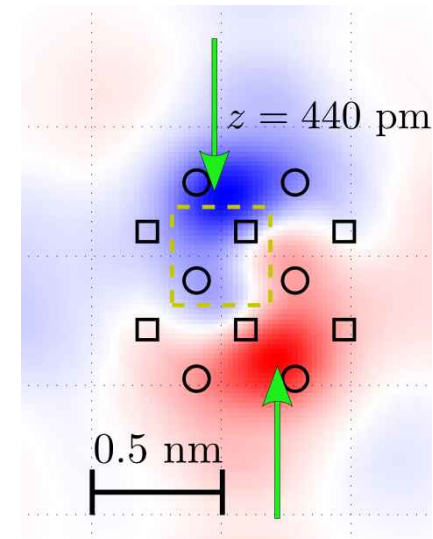
local orbital order  
 + Wannier function  
 → chiral defect structure



Gastiasoro, Andersen, J. Supercond Nov. Magn., **26**, 2651 (2013)

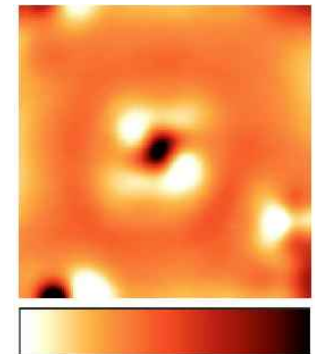
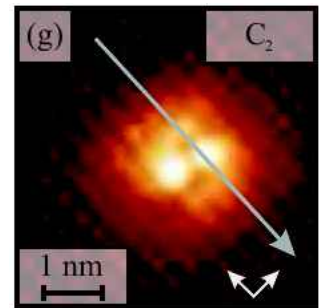
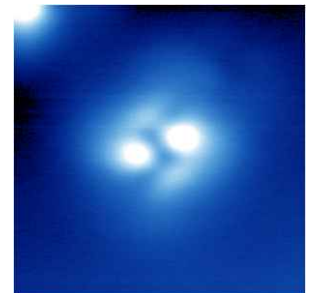


Inoue, Yamakawa, Kontani PRB 85, 224506 (2012)



○ Li □ As

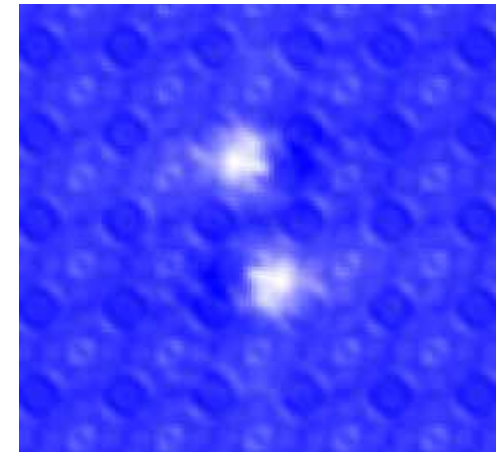
$d_{xz}$  Wannier function on Fe(2)



high low

# FeSe: orbital selective electronic structure

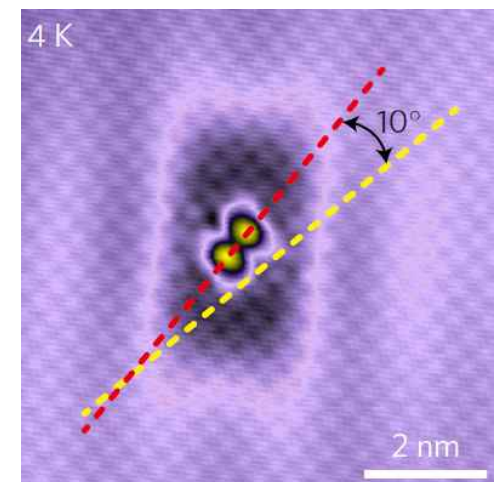
- Imprints on impurity states
  - Point-like impurity in electronic structure with nematic order
  - Strongly directed quasiparticle scattering via reduced coherence



- Use Green's function in conjunction with Wannier transformation

$$\tilde{G}_{\ell\ell'}(\mathbf{k}, \omega_n) = \sqrt{Z_\ell Z_{\ell'}} \sum_{\mu} \frac{a_{\mu}^{\ell}(\mathbf{k}) a_{\mu}^{\ell'*}(\mathbf{k})}{i\omega_n - \tilde{E}_{\mu}(\mathbf{k})}$$

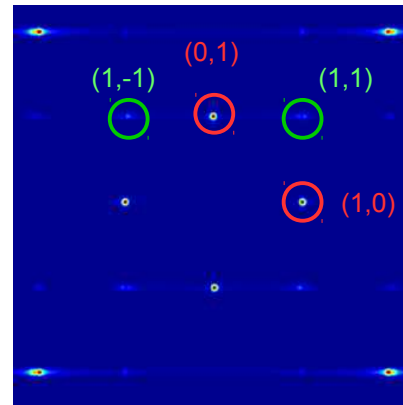
$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mu, \nu, \mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mu, \mathbf{R}', \nu; \omega) w_{\mathbf{R}, \mu}(\mathbf{r}) w_{\mathbf{R}', \nu}^*(\mathbf{r}')$$



# Imprints on cLDOS of homogeneous system

- Intra-unit cell information

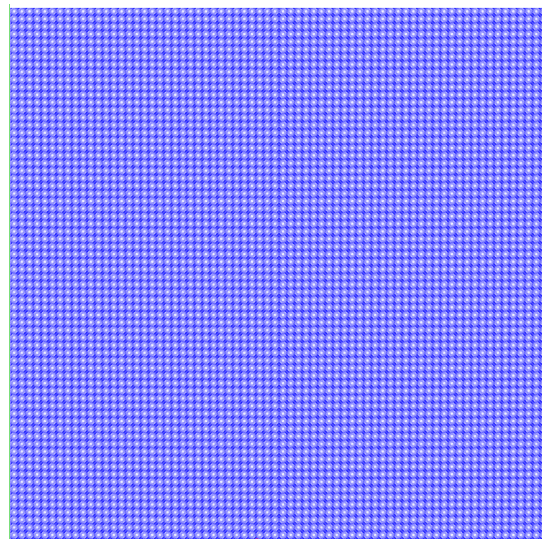
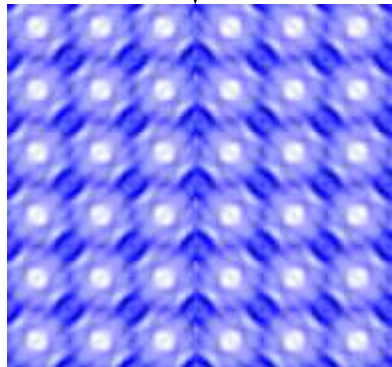
Fourier transform



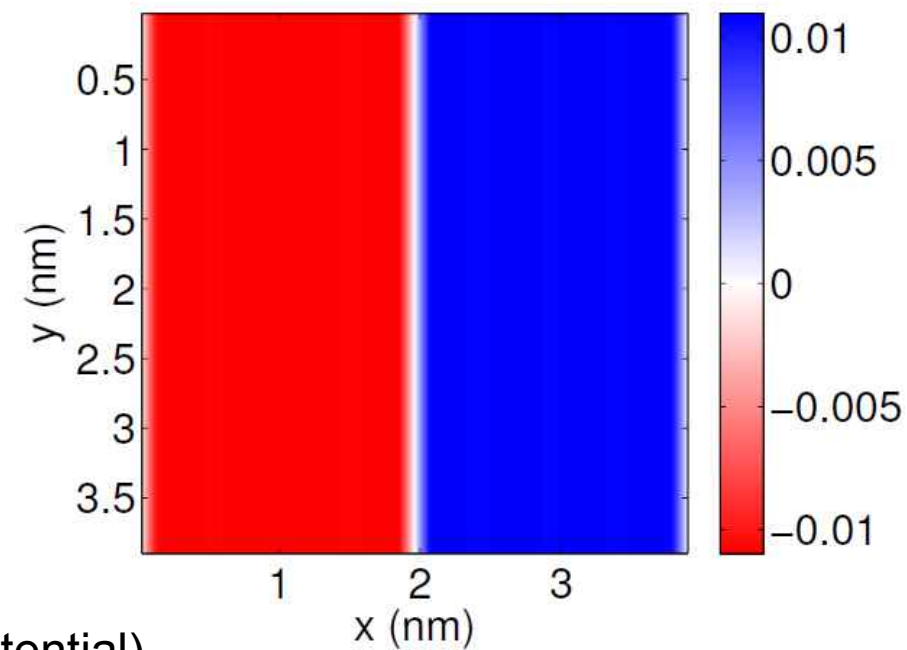
Mask (1,1) and (1,-1)  
Fourier transform back, take difference

$\omega = -0.02$  eV

Blow-up:  
Twin-boundary



Twin-boundary (no scattering potential)



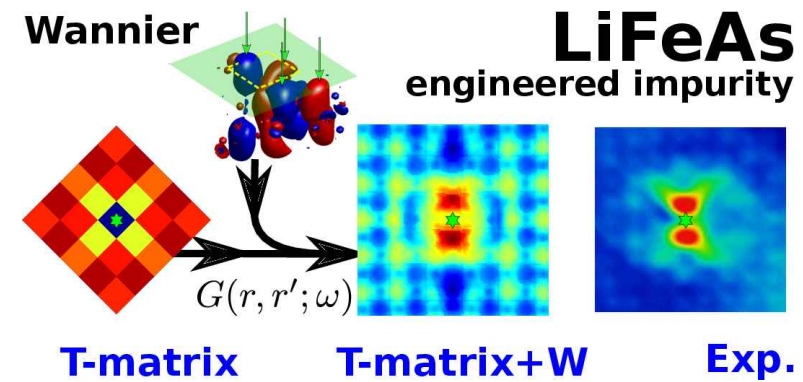
# Summary

- Wannier method: basis transformation of the lattice Green function
- Qualitative correct (symmetry) and quantitative predictive results
- Impurities and homogeneous lattice in LiFeAs
- Universality in cuprates

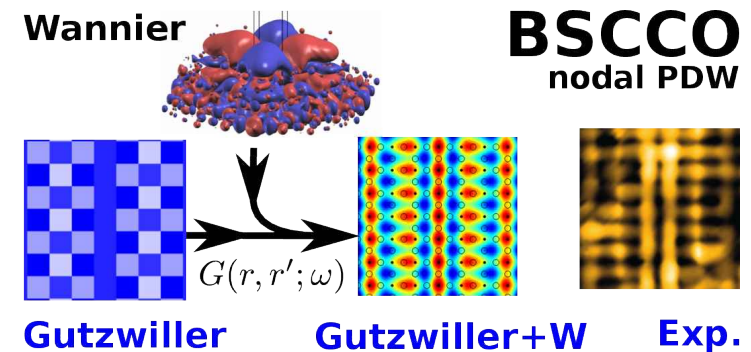
Kreisel et al., Phys. Rev. Lett. **114**, 217002 (2015)  
 Choubey, et al., Phys. Rev. B **96**, 174523 (2017)

- Nematicity in Fe-based SC

Sprau, et al. Science, **357**, 75 (2017)  
 A. Kostin, et al., arXiv:1802.02266



S. Chi, (...) , A. Kreisel, et al. Phys. Rev. B **94**, 134515 (2016)  
 A. Kreisel, et al. Phys. Rev. B **94**, 224518 (2016)



Choubey et al., New J. Phys. **19**, 013028 (2017)

## Acknowledgments



Tight binding models and Wannier functions available

[www.physik.uni-leipzig.de/~kreisel/model.php](http://www.physik.uni-leipzig.de/~kreisel/model.php)