

# Towards a realistic simulation of disorder in unconventional superconductors

**Andreas Kreisel, Brian Andersen**

Niels Bohr Institute, University of Copenhagen, 2100 København, Denmark

**Peayush Choubey, Peter Hirschfeld**

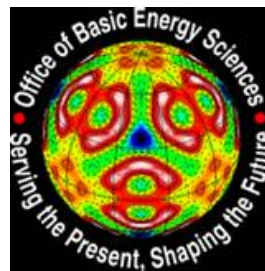
Department of Physics, University of Florida, Gainesville, FL 32611, USA

**Tom Berlijn**

Center for Nanophase Materials Sciences and Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA

**Wei Ku**

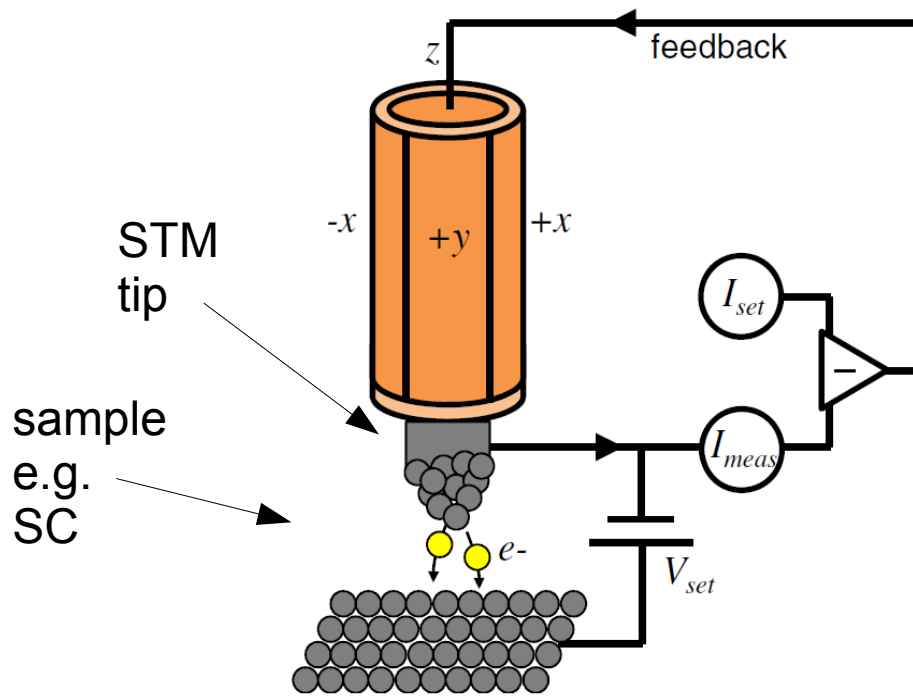
Condensed Matter Physics and Materials Science Department, Brookhaven National Laboratory, Upton, New York 11973, USA



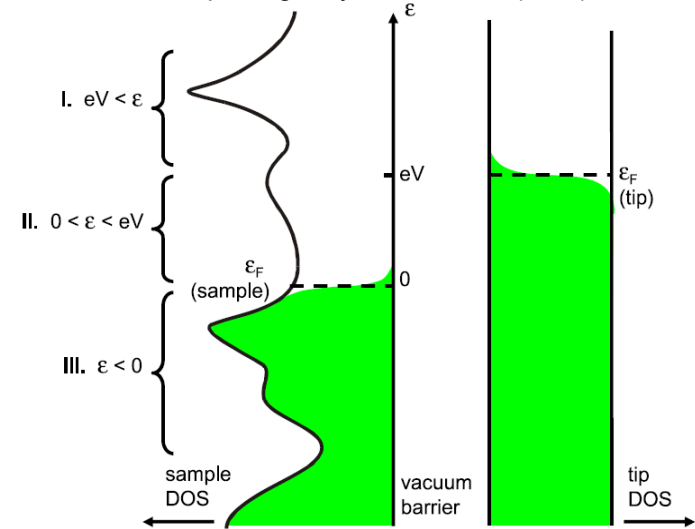
# Outline

- Motivation
  - STM: impurities as probe for electronic structure, order parameter
  - layered superconductors, complications
- Theoretical methods to investigate impurity physics in superconductors
- Using wavefunction information in layered superconductors
- Applications
  - BiSrCaCuO (single band, d-wave)
  - LiFeAs (multiband, s-wave)

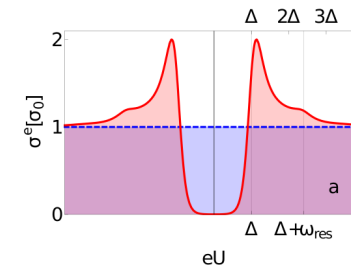
# Scanning tunnelling microscopy



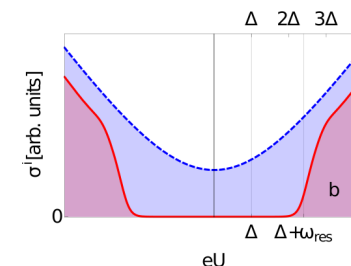
J. Hoffman Rep. Prog. Phys. **74** 124513 (2011)



Inelastic tunneling:  
coupling to bosonic mode



superconducting  
state



normal state

J. R. Kirtley and D. J. Scalapino, PRL **65**, 798 (1990)  
J. R. Kirtley, PRB **47**, 11379 (1993)  
Patrik Hlobil, et al., arXiv:1603.05288 (2016)

Tunnelling current:

$$I(V, x, y, z) = -\frac{4\pi e}{\hbar} \rho_t(0) |M|^2 \int_0^{eV} \rho(x, y, z, \epsilon) d\epsilon$$

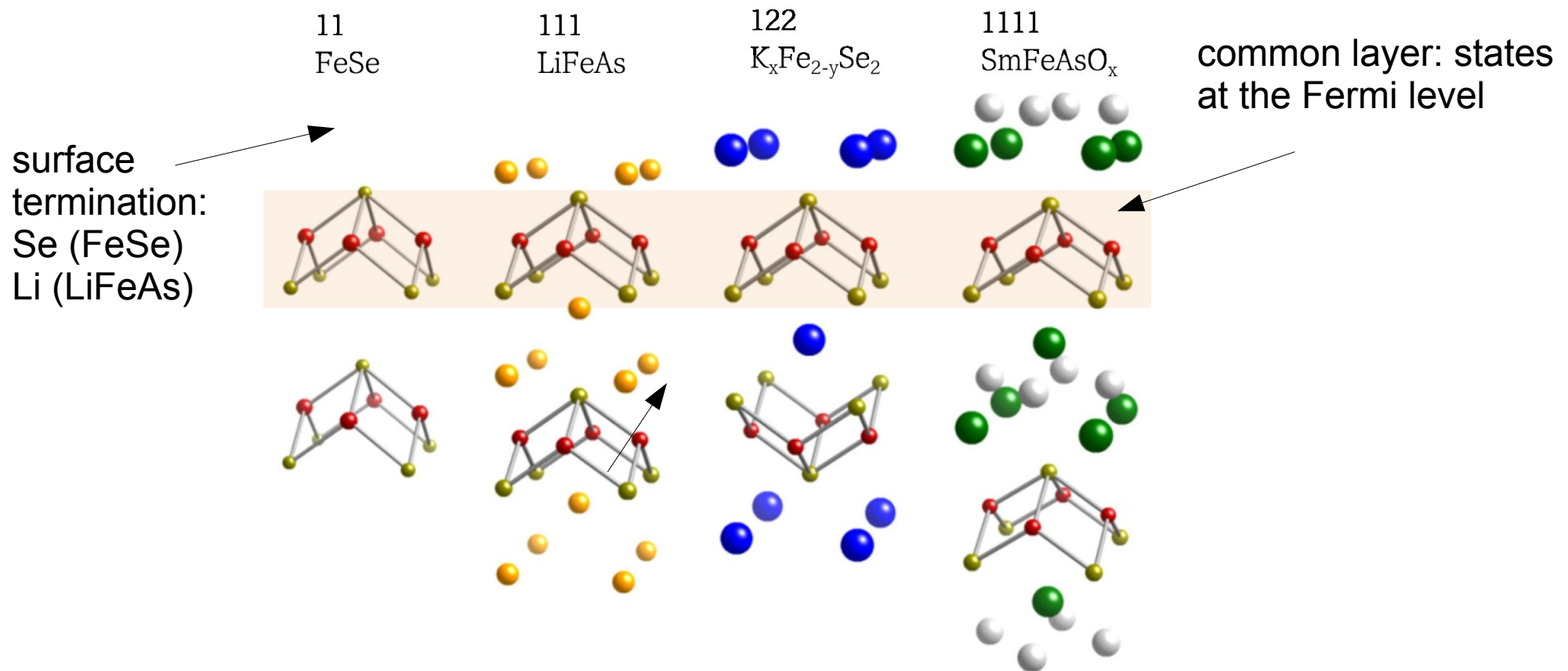
Local Density Of States (LDOS)  
of sample at given energy **at the tip position**

J. Tersoff and D. R. Hamann, PRB **31**, 805 (1985)

# Layered superconductors

LDOS of sample at given energy **at the tip position**

- Iron based superconductors





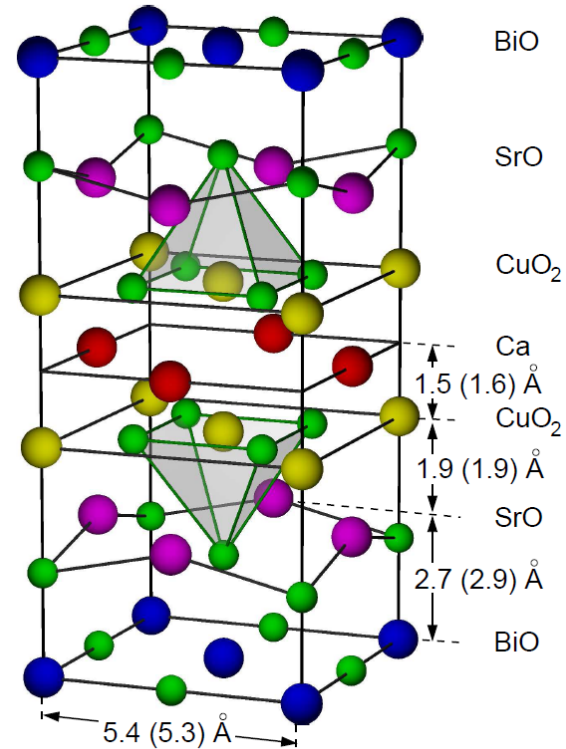
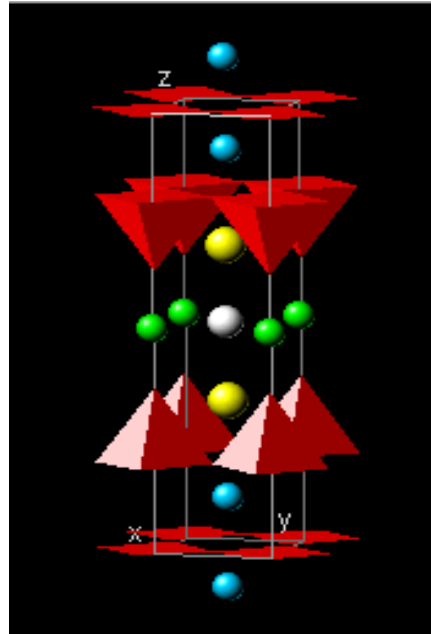
# Layered superconductors

- Cuprates

Hg1Ba2Ca2Cu3O8

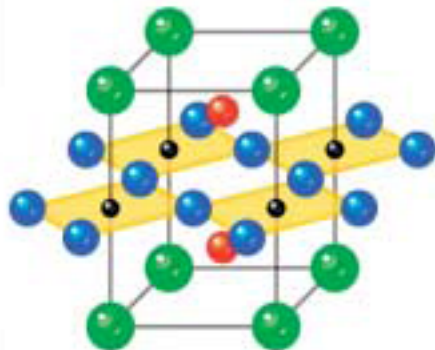
$T_c = 135$  K  
under pressure: 153 K

 Cu/O



Bi-2212

 Cu  
 Ca/Na  
 O  
 Cl

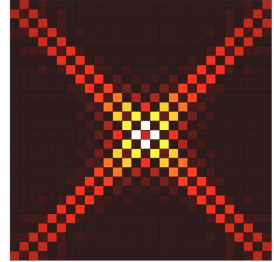


Na-CCOC

# Bound states of nonmagnetic impurity

- d-wave superconductors
  - local LDOS: 4 fold pattern
  - low energy bound state

J. M. Byers, M. E. Flatté, and D. J. Scalapino  
Phys. Rev. Lett. **71**, 3363 (1993)



A. V. Balatsky, M. I. Salkola, and A. Rosengren  
Phys. Rev. B **51**, 15547 (1995)

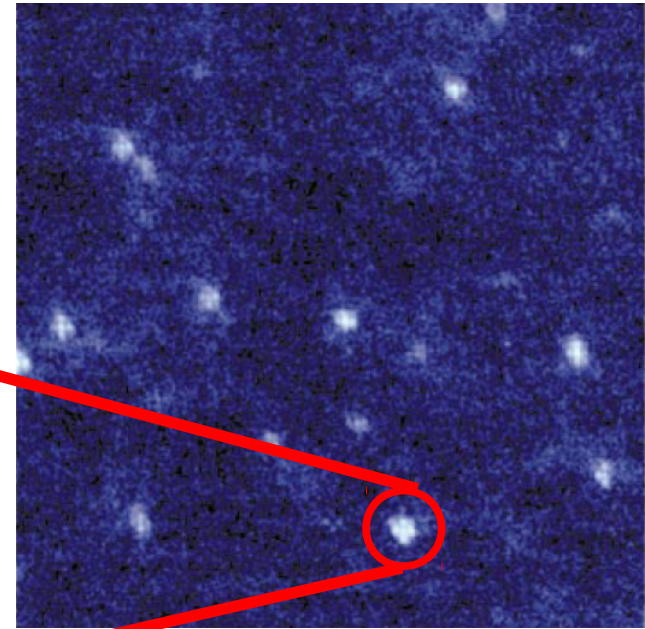
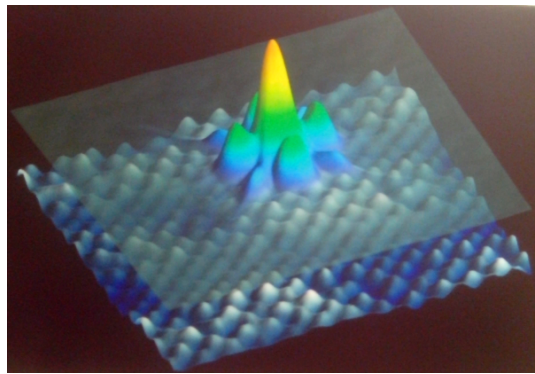
Stamp, Journal of Magnetism and Magnetic Materials,  
**63**, 429 - 431 (1987) (p-wave)

$$\Omega \equiv \Omega' + i\Omega'' = \Delta_0 \frac{\pi c/2}{\ln(8/\pi c)} \left[ 1 + \frac{i\pi}{2} \frac{1}{\ln(8/\pi c)} \right]$$

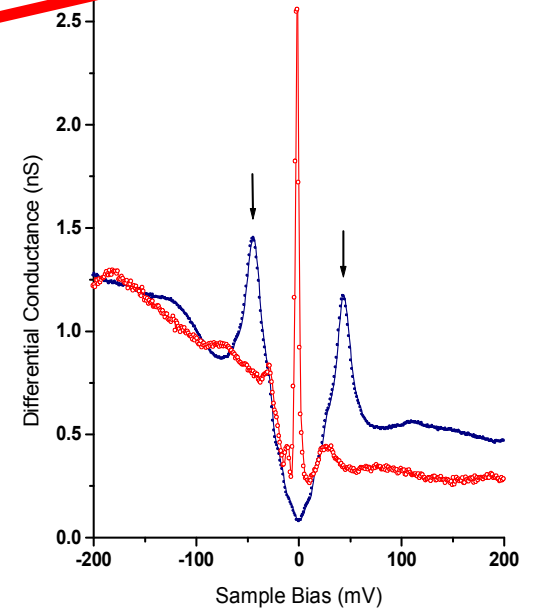
# STM experiment on Bi-2212

- LDOS map at  $-1.5$  meV  
~20 Zn atoms in field of view
  - 4 fold pattern

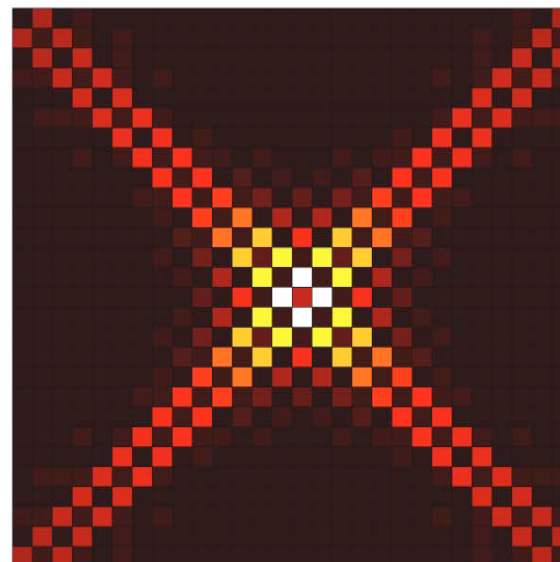
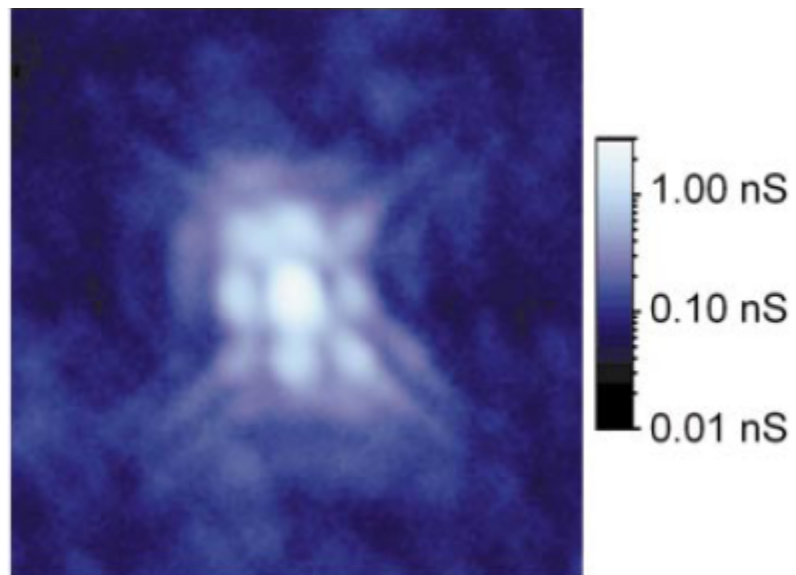
Pan et al., Nature  
403, 746 (2000)



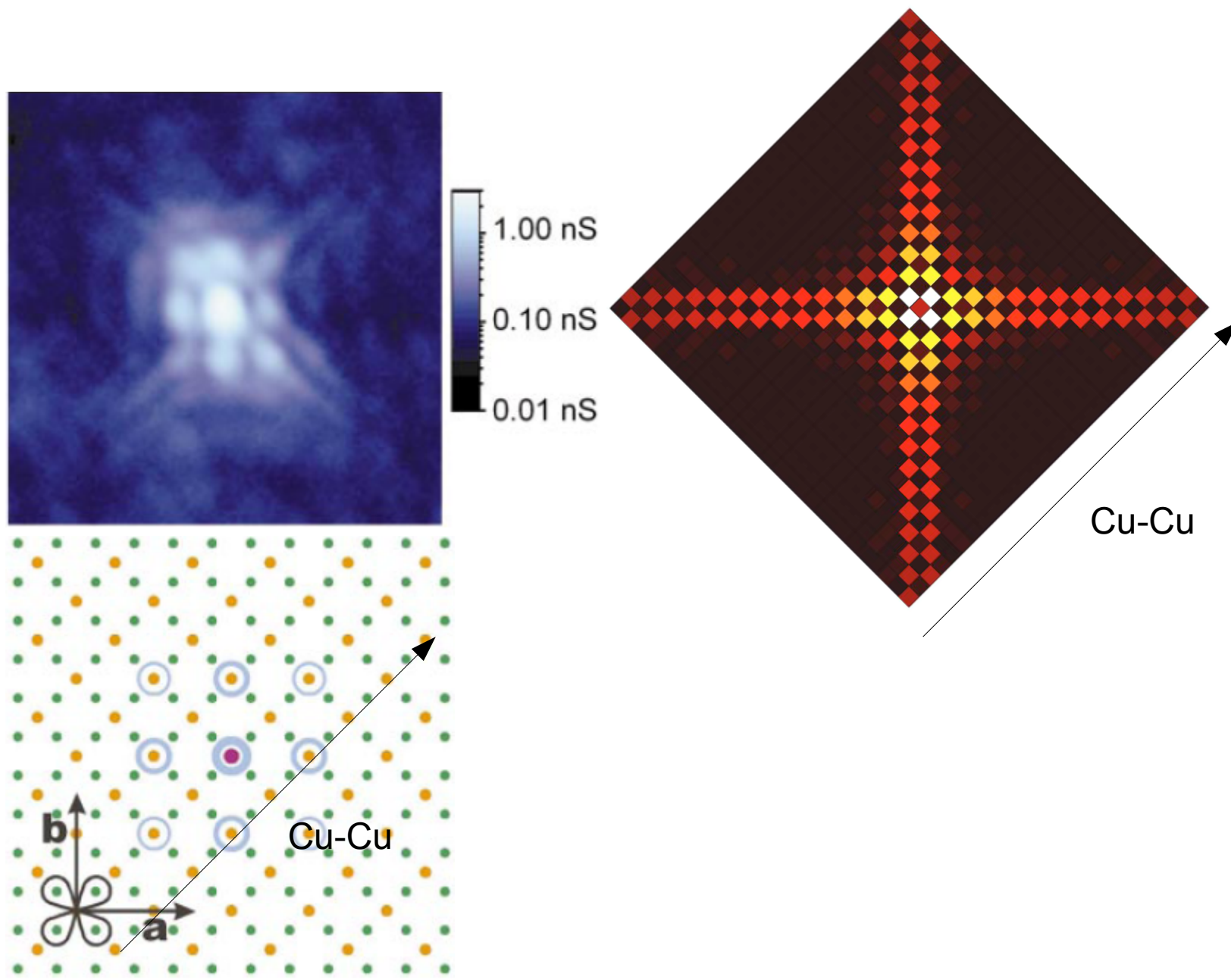
- low energy bound state in on site spectrum



# comparison to theory



# comparison to theory



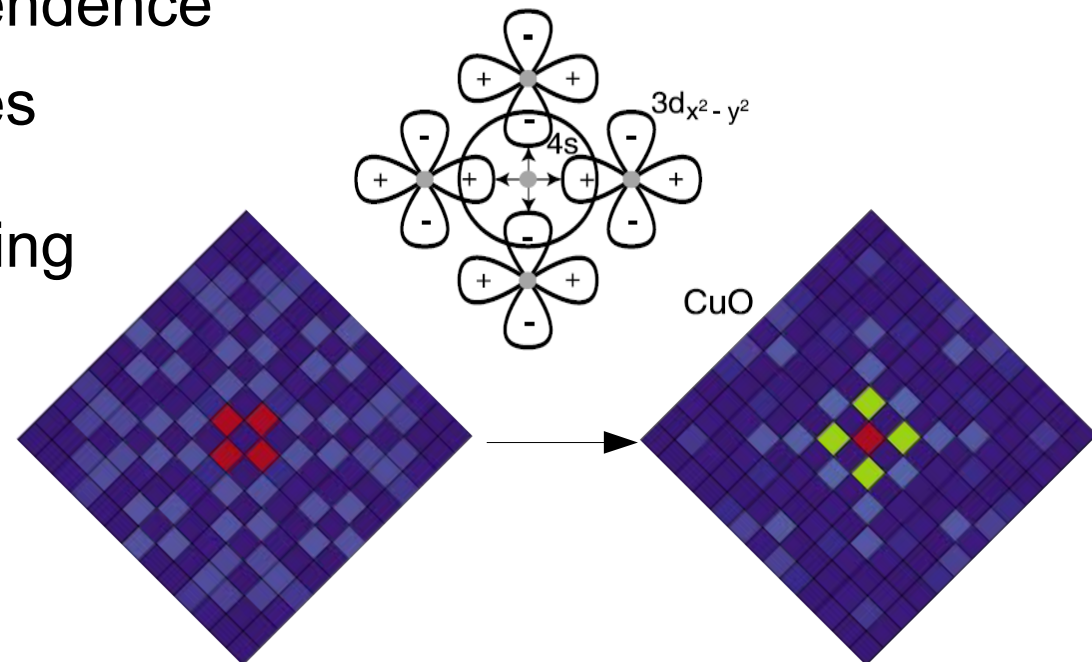
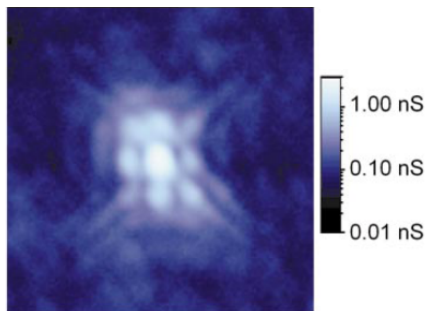


# Theories for spatial pattern

- extended impurity potentials (magnetic Ni impurity)
- Correlations: “Kondo screening” (magnetic impurity), “modifications of the theory for the case of a nonmagnetic impurity” → consistent with experimental spatial dependence
- Bi-O filter function: STM probes neighbouring Cu states due to momentum dependent tunnelling matrix elements

Jian-Ming Tang and Michael E. Flatté PRB **66**, 060504(R) (2002)

Anatoli Polkovnikov PRB **65**, 064503 (2002)



Martin *et al.*, PRL **88**, 097003 (2002)

# Theory: State of the art methods

- Hamiltonian

$$H = H_0 + H_{\text{BCS}} + H_{\text{imp}}$$

$H_0 = \sum_{R,R',\sigma} t_{RR'} c_{R,\sigma}^\dagger c_{R',\sigma} - \mu_0 \sum_{R,\sigma} c_{R,\sigma}^\dagger c_{R,\sigma}$ 
band structure kinetic energy

$H_{\text{BCS}} = - \sum_{R,R'} \Delta_{RR'} c_{R,\uparrow}^\dagger c_{R',\downarrow}^\dagger + H.c.,$ 
superconductivity gap function / pairing

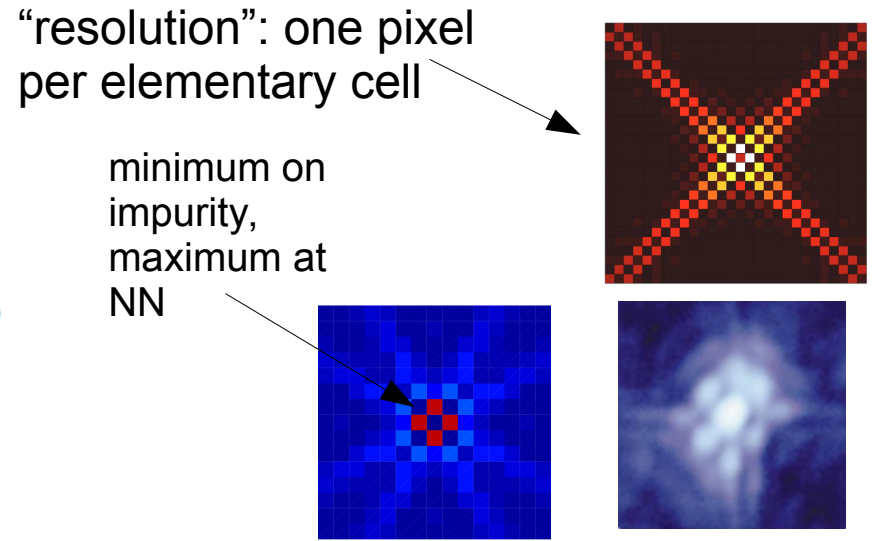
$H_{\text{imp}} = \sum_{\sigma} V_{\text{imp}} c_{R^*,\sigma}^\dagger c_{R^*,\sigma}$ 
impurity scatterer (non)magnetic potential /  $T_2$  scatterer

- T-matrix calculations
- self-consistent mean field theory (BdG)
- lattice Green function
- Local Density of States (LDOS)

$$\hat{G}(\mathbf{r}, \mathbf{r}'; \omega) = \hat{G}_0(\mathbf{r} - \mathbf{r}', \omega) + \hat{G}_0(\mathbf{r}, \omega) \hat{T}(\omega) \hat{G}_0(\mathbf{r}', \omega)$$

$$N_{\text{imp}}(\mathbf{r}, \omega) = - \frac{1}{\pi} \text{Im}[\hat{G}_0(\mathbf{r}, \omega) \hat{T}(\omega) \hat{G}_0(\mathbf{r}, \omega)]_{11}$$

Zn impurity in BiSrCaCuO



T-matrix calculation  
 Bi-O filter function needed  
 Martin *et al.*, PRL **88**, 097003 (2002)

# Theory: State of the art methods

## Bogoliubov-de Gennes (BdG)

- Hamiltonian  $H = H_0 + H_{\text{BCS}} + H_{\text{imp}}$
- self-consistent solution in real space  
(NxN grid, determine gaps)  $\Delta_{R R'} = \Gamma_{R R'} \langle c_{R' \downarrow} c_{R \uparrow} \rangle$
- eigenvalues  $E_n$ , eigenvectors  $(u_n, v_n)$
- lattice Green function

$$G_\sigma(R, R'; \omega) = \sum_n \left( \frac{u_R^{n\sigma} u_{R'}^{n\sigma*}}{\omega - E_{n\sigma} + i0^+} + \frac{v_R^{n-\sigma} v_{R'}^{n-\sigma*}}{\omega + E_{n-\sigma} + i0^+} \right)$$

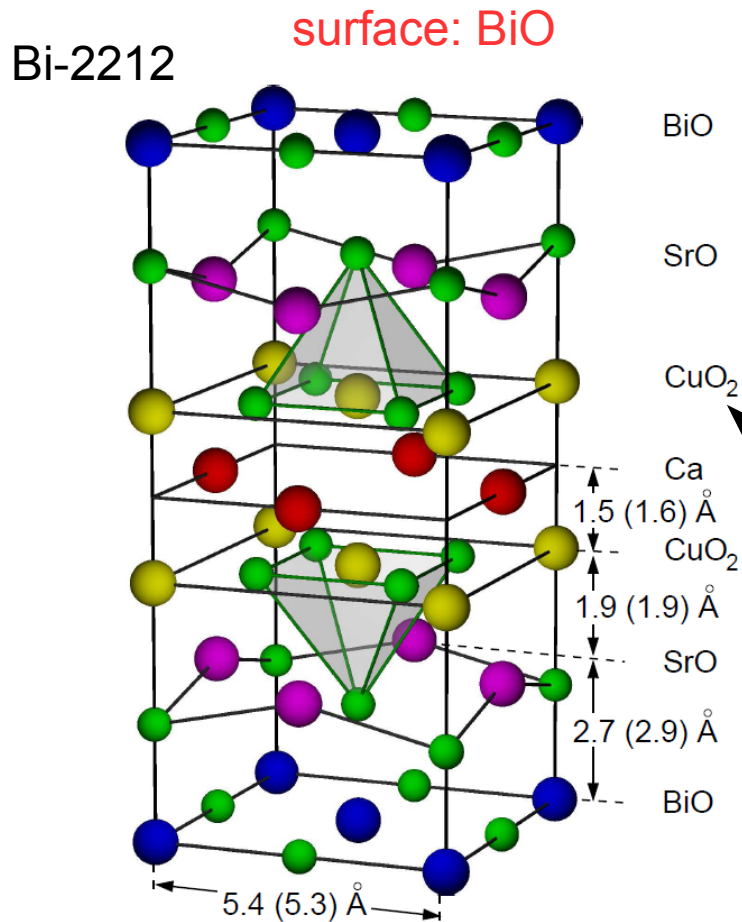


# Layered superconductors

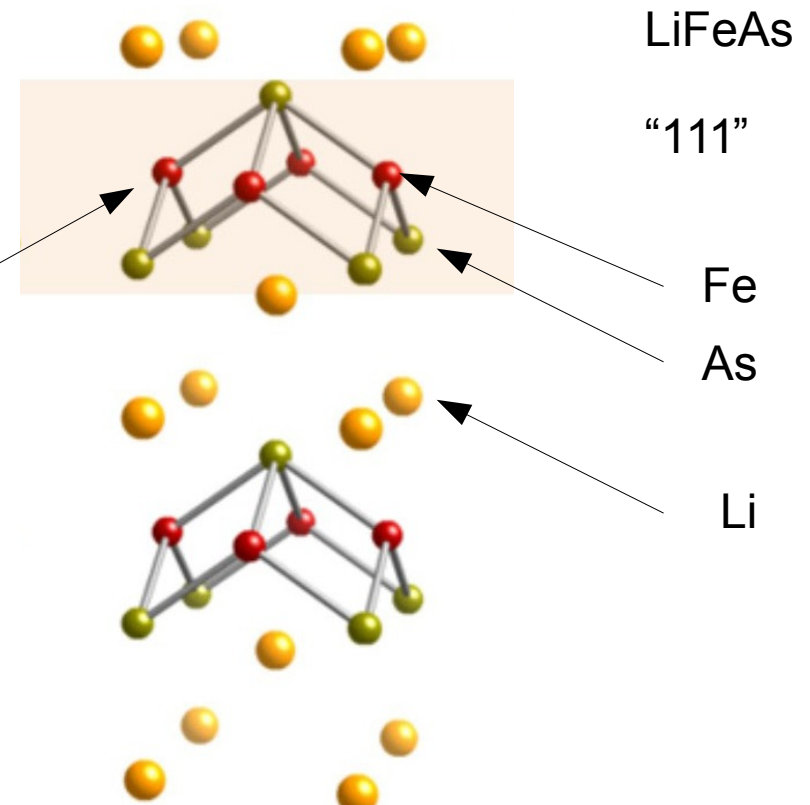
- 2 examples: surface atoms  $\neq$  superconducting layer

Cuprates

Iron based superconductors



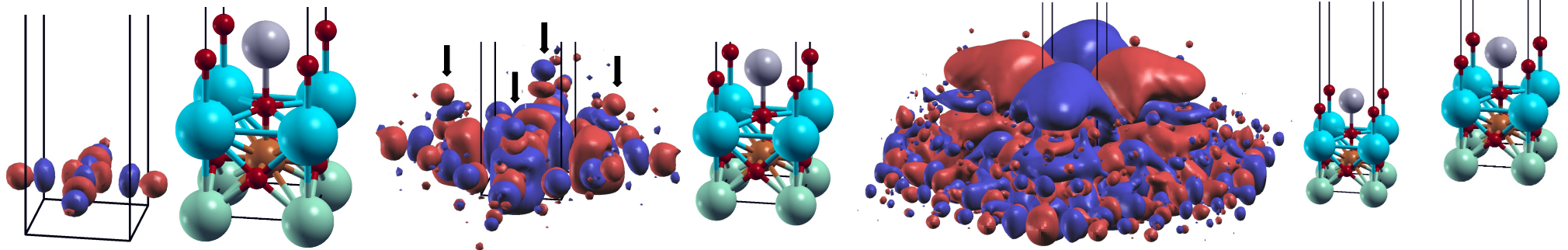
surface: Li, As?



states at the Fermi level  
Cu-plane  
Fe-plane

# Wannier method

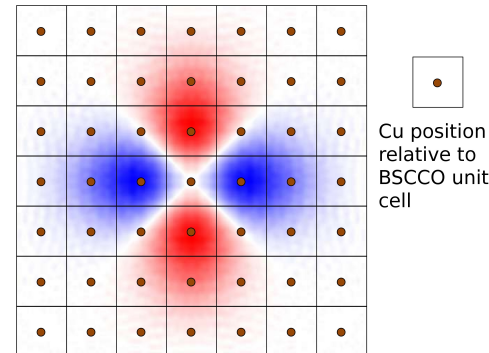
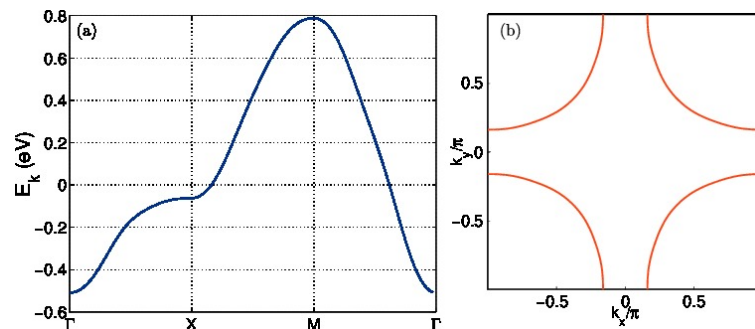
- first principles calculation (surface)
- 1 band tight binding model:  
1 Wannier function



Cu dxy

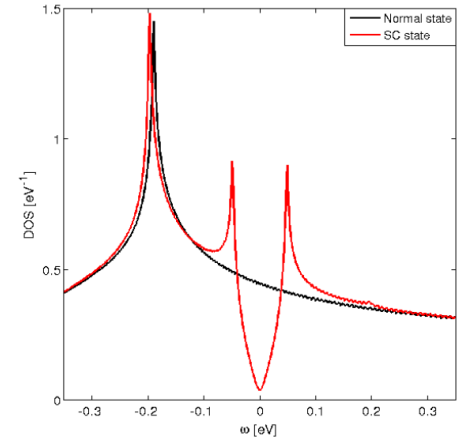
NN apical O tails

at surface: only contributions to NN



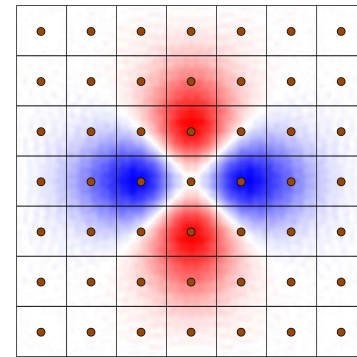
# Superconductivity

- superconducting order parameter (d-wave) (phenomenology or calculation fx. mean-field)



- continuum Green function

$$\psi_{\sigma}(\mathbf{r}) = \sum_{\mathbf{R}, \mu} c_{\mathbf{R}, \mu \sigma} w_{\mathbf{R}, \mu}(\mathbf{r})$$



Cu position relative to BSCCO unit cell

surface Wannier function with phases

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mathbf{R}, \mathbf{R}' } G(\mathbf{R}, \mathbf{R}'; \omega) w_{\mathbf{R}}(\mathbf{r}) w_{\mathbf{R}'}^*(\mathbf{r}')$$

lattice Green function

local density of states (LDOS)

$$\rho(\mathbf{r}, \omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{r}, \mathbf{r}; \omega)$$

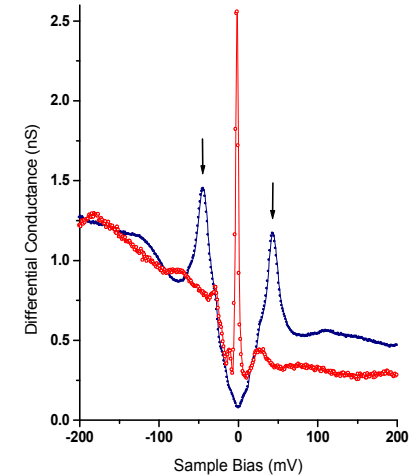
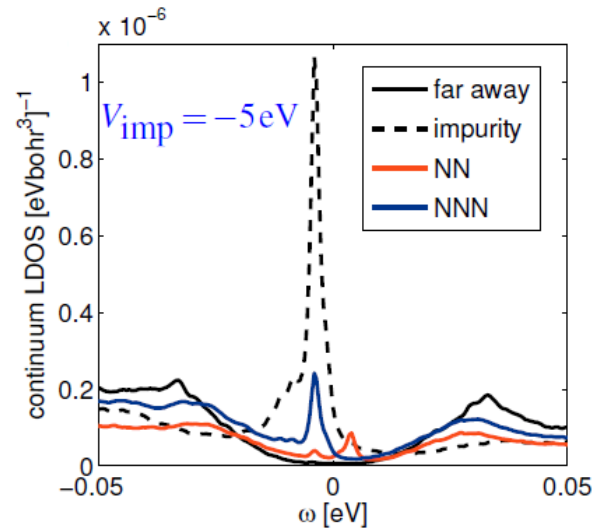
continuum position

nonlocal contributions

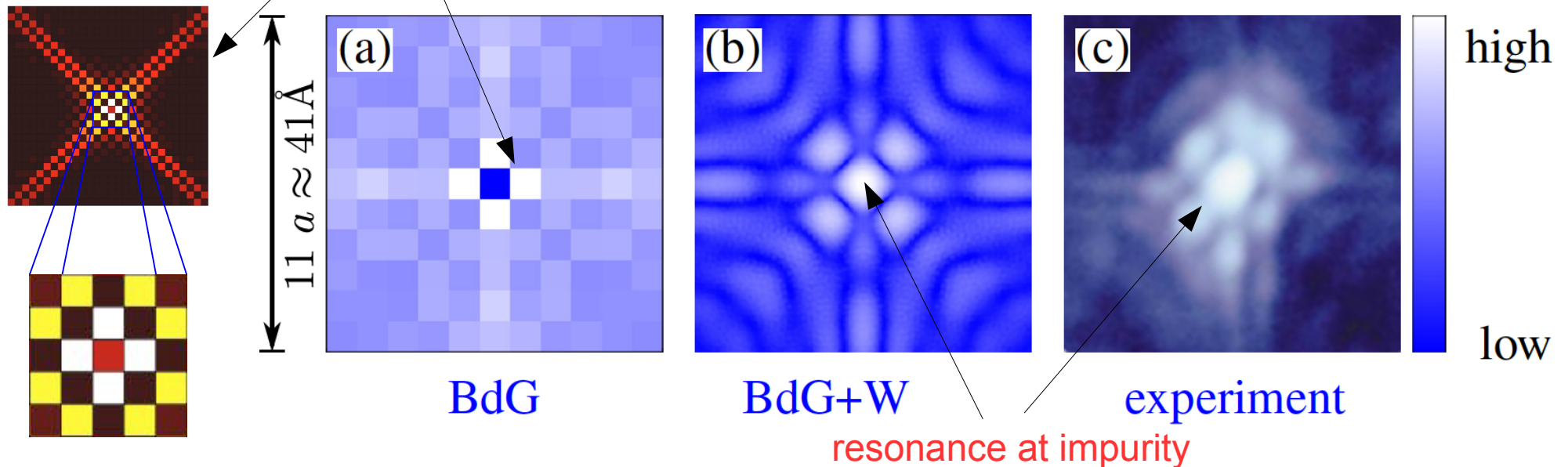
# BSCCO: Results STM maps and spectra

- d-wave order parameter
- Zn impurity:  
 $V_{\text{imp}} = -5 \text{ eV}$   
 resonance:  $-3.6 \text{ meV}$

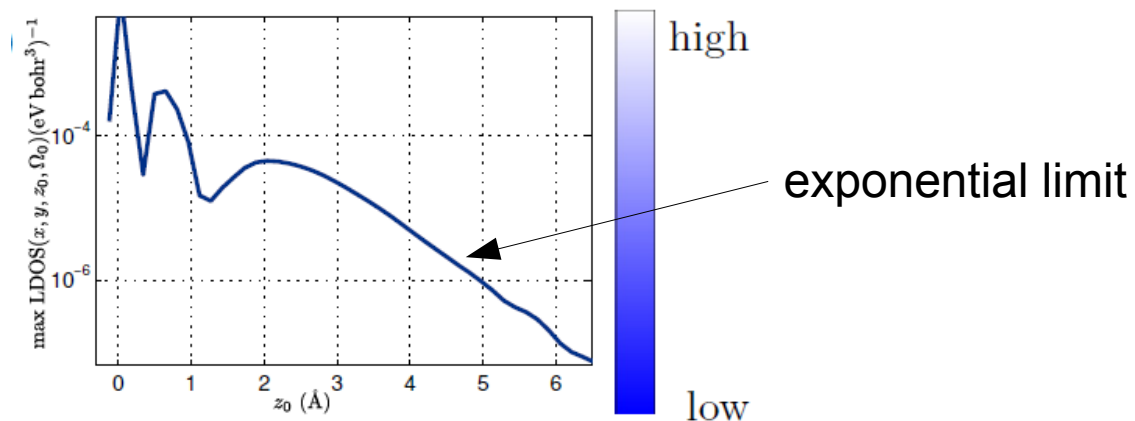
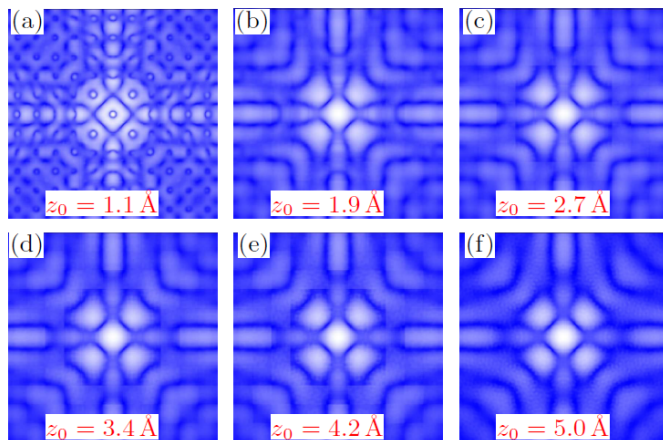
Pan et al., Nature  
403, 746 (2000)



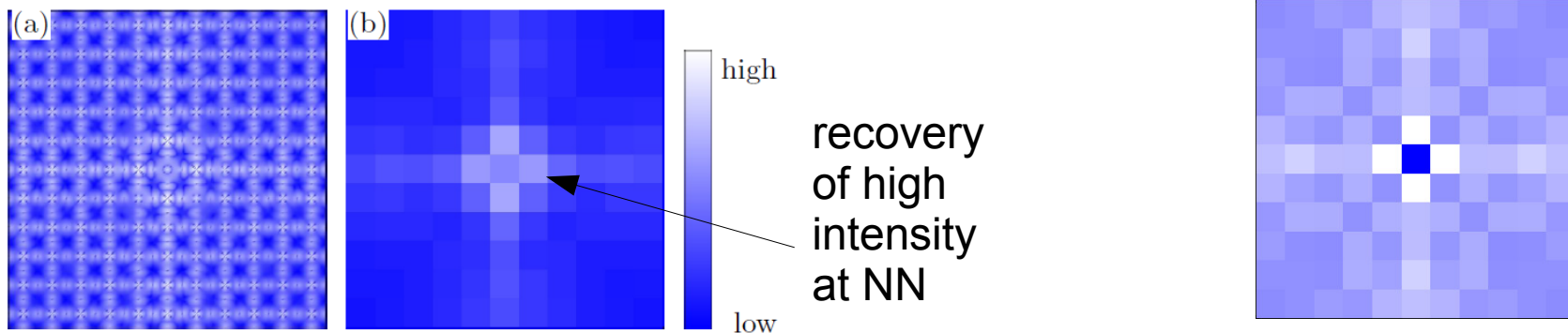
Zhu et al., PRB  
67, 094508  
(2003)



- dependence on tip height



- continuum LDOS in the Cu-plane

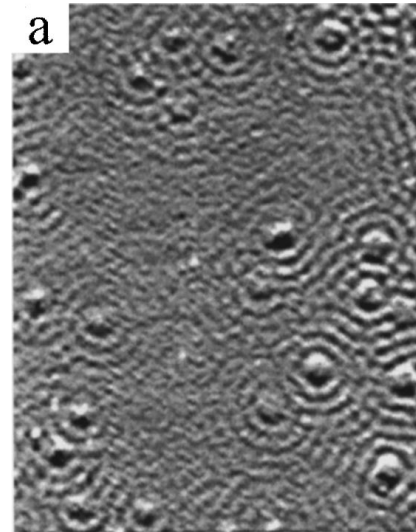


convolution with Gaussian blur  
of 1 pixel per elementary cell



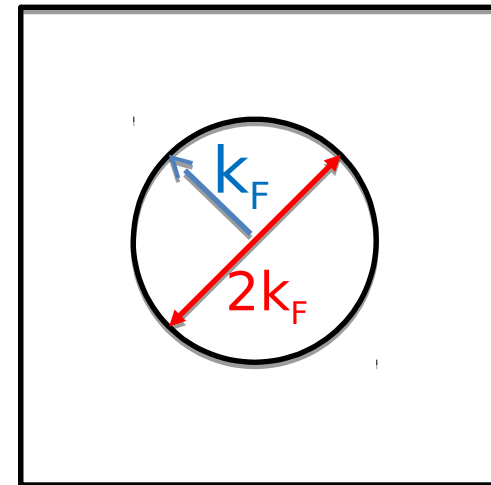
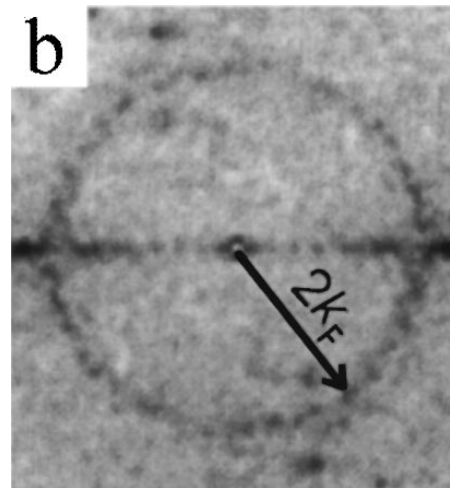
# Quasiparticle Interference (QPI)

- STM on normal metal (Cu)
  - impurities
  - Friedel oscillations



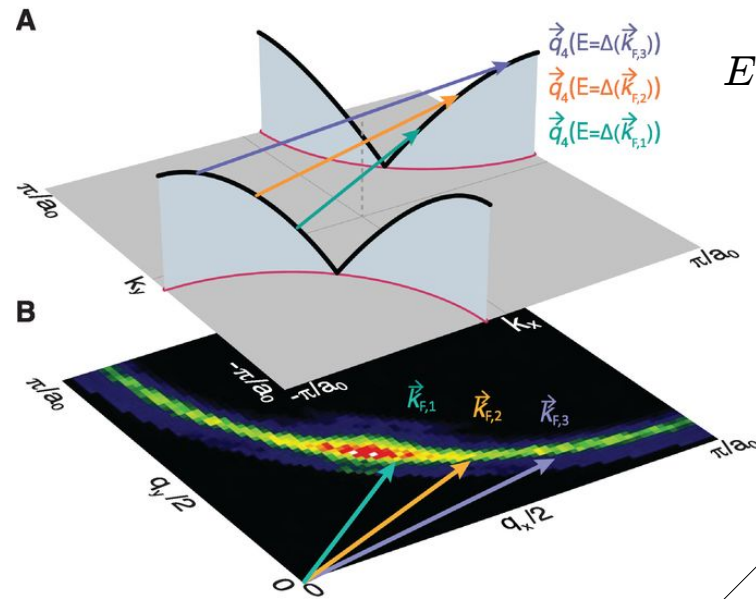
L. Petersen, et al.  
PRB **57**, R6858(R)  
(1998)

- Fourier transform of conductance map
  - mapping of constant energy contour



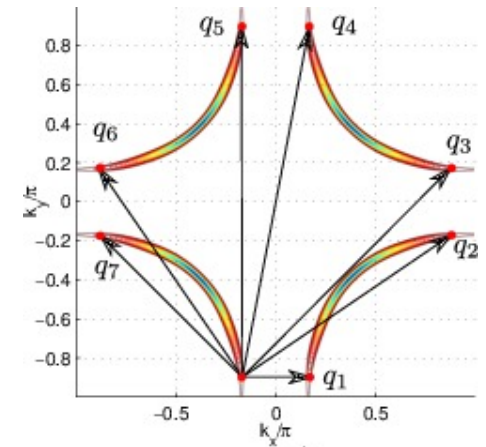
# QPI in superconductors

- Fourier transform of differential conductance maps

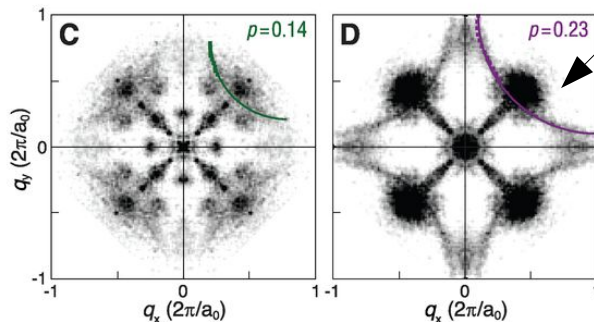
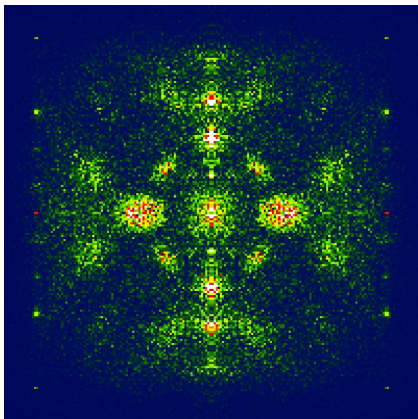


$$E_{\mathbf{k}} = \pm \sqrt{\epsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$$

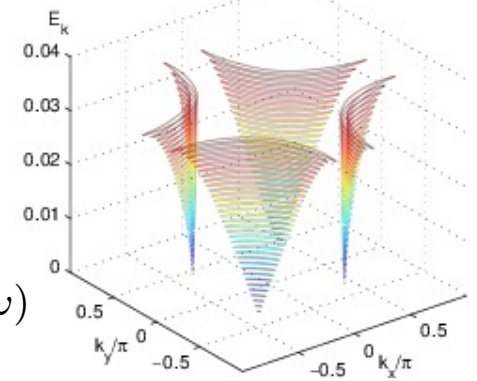
energy integrated maps: trace back Fermi surface



FT of conductance map at +22 meV



$$\Lambda(\mathbf{q}) = \int_0^{\Delta_0} d\omega Z(\mathbf{q}, \omega)$$

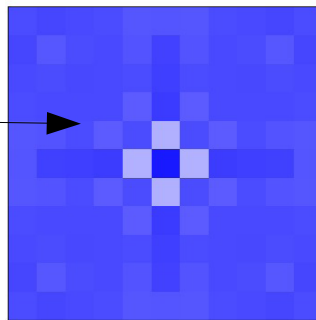


octet model: 7 scattering vectors between regions of high DOS

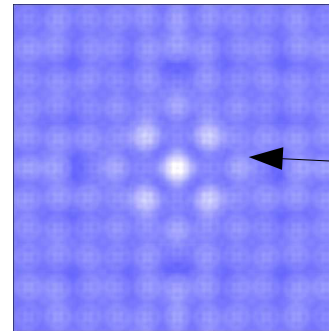
# Quasi Particle Interference (QPI)

- Fourier transform of conductance maps
- BSCCO: weak potential scatterer

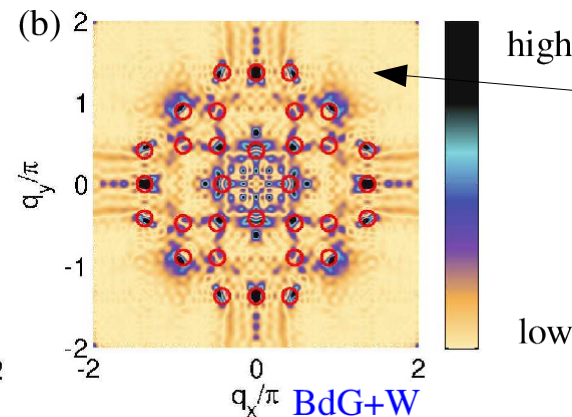
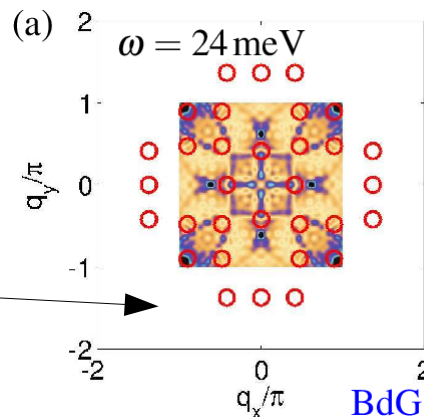
no intra-unitcell  
information  
1 pixel per  
elementary cell



atomic scale local  
density of states  
at STM tip  
position



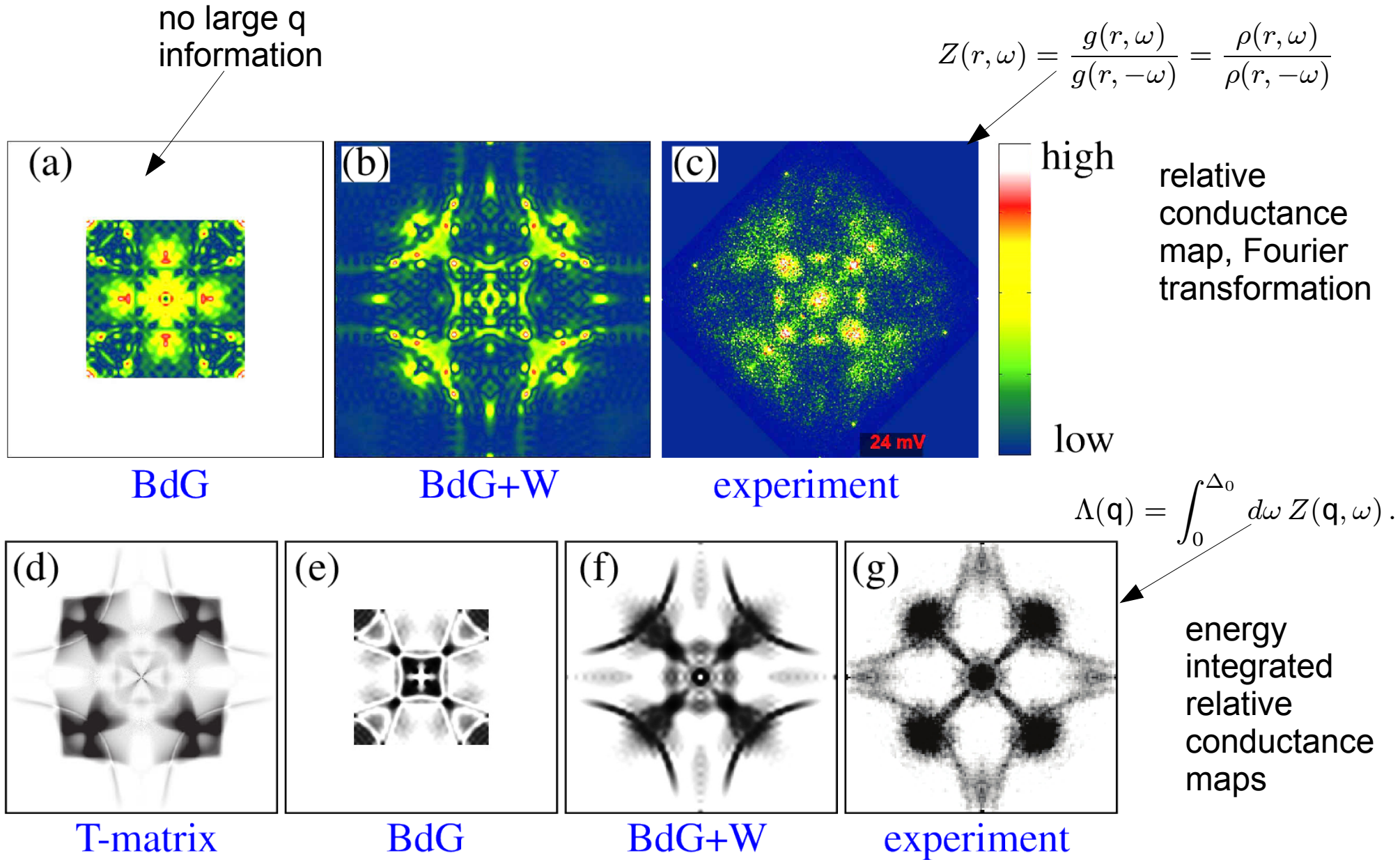
no information  
beyond first BZ



full  
information  
for all  
scattering  
vectors

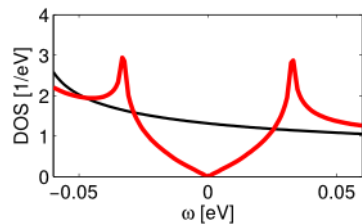


# Comparison to experiment



# Homogeneous superconductor

- phenomenological pairing interactions  
similar results from spin-fluctuation pairing



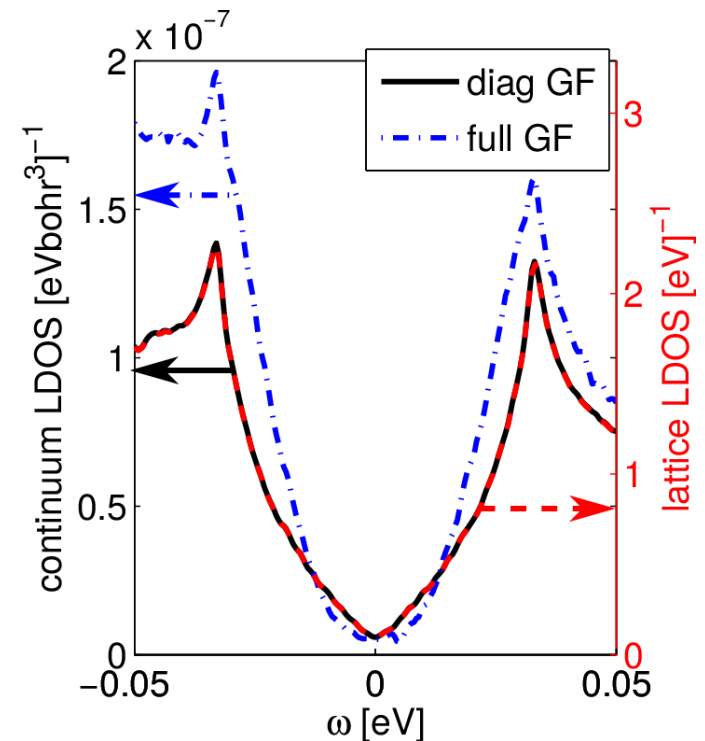
DOS of homogeneous superconductor

- spectra measured at the surface

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mathbf{R}'; \omega) w_{\mathbf{R}}(\mathbf{r}) w_{\mathbf{R}'}^*(\mathbf{r}')$$

local density of states (LDOS)

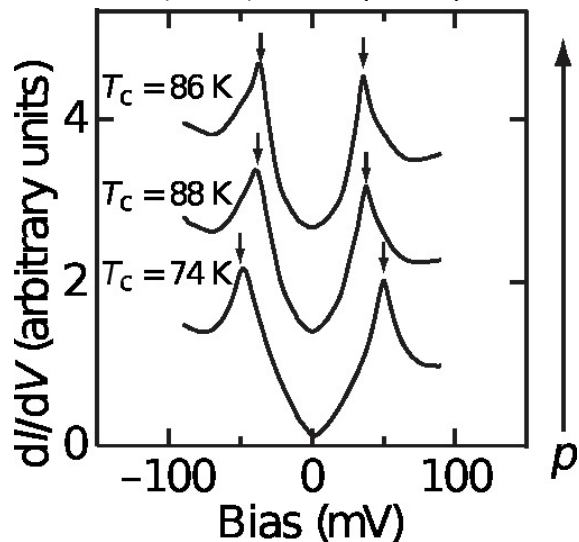
$$\rho(\mathbf{r}, \omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{r}, \mathbf{r}; \omega)$$



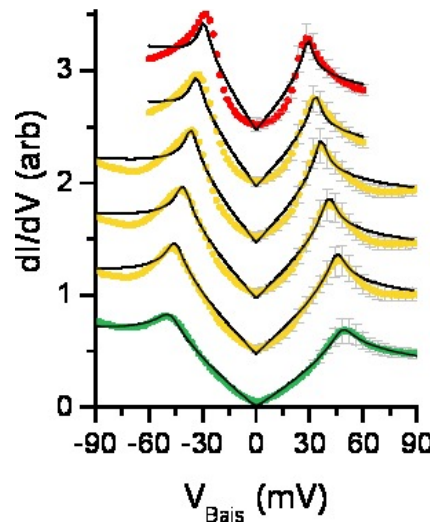
# STM Spectra: homogeneous SC

- overdoped: U-shape, lower doping: V-shape

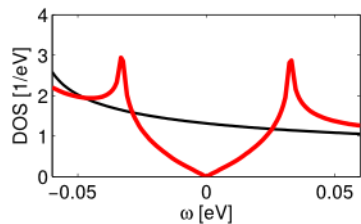
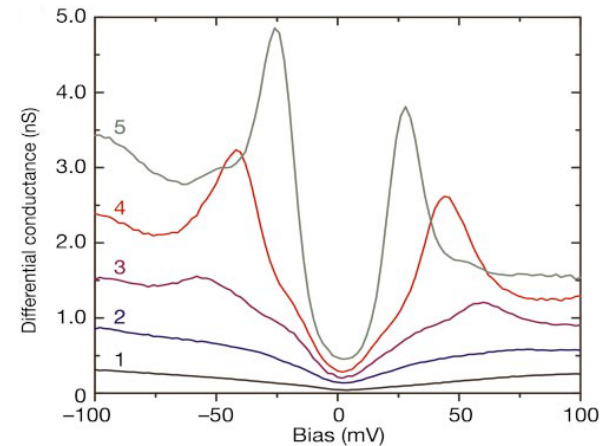
Kohsaka et al.  
Nature, 454, 1072 (2008)



Allredge et al.  
Nature Physics, 4, 319 (2008)

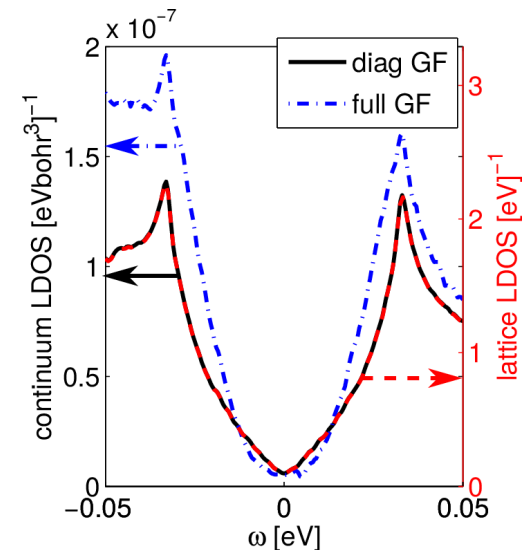


Pan et al. Nature, 413, 282 (2001)



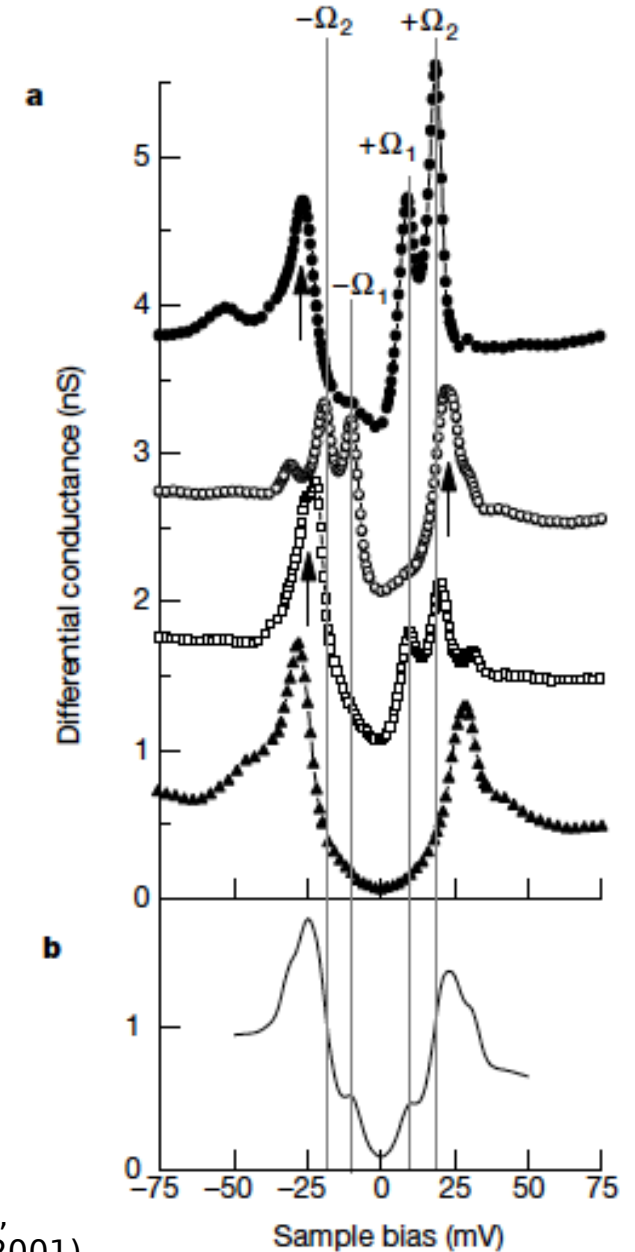
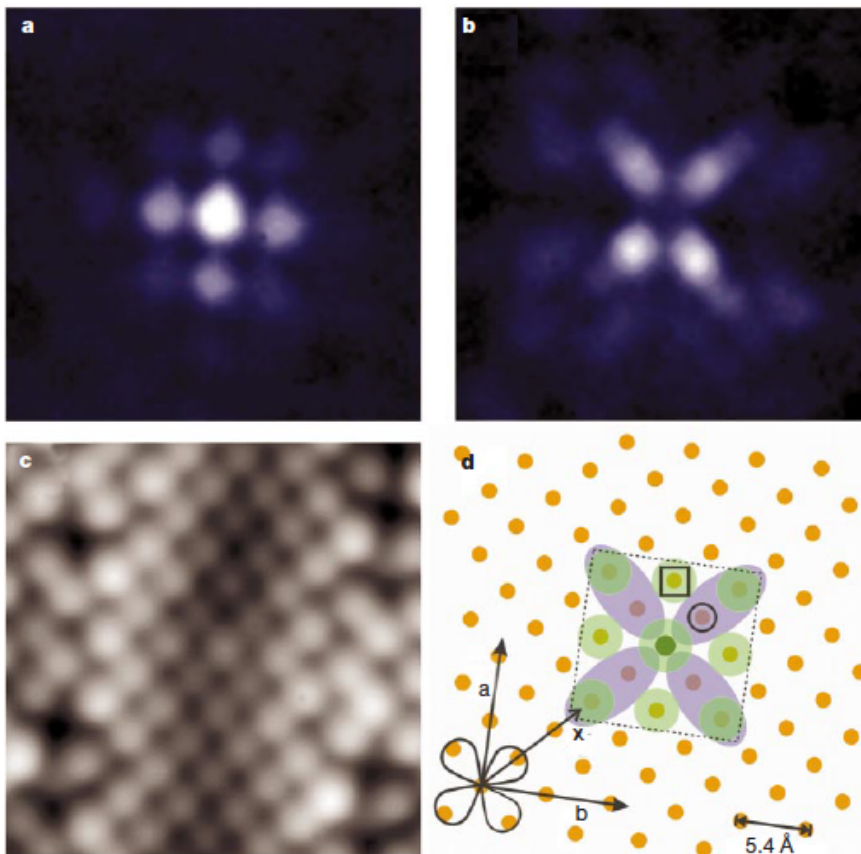
DOS of homogeneous superconductor

BdG+W: U shape enters naturally within our method, applicable to overdoped regime



# BSCCO: Magnetic impurity

- Ni on BSCCO:  
weak magnetic scatterer
- double resonance



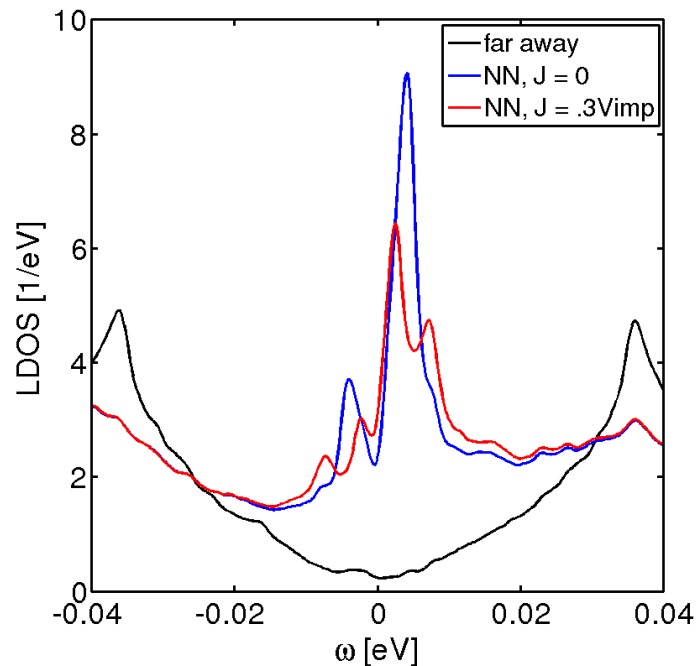
E. W. Hudson et al.,  
Nature **411**, 920 (2001)

# Ni impurity on BSCCO

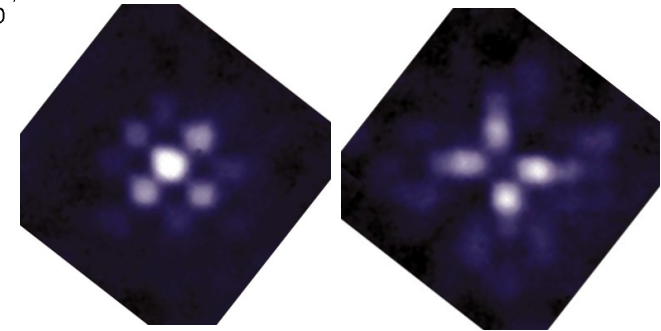
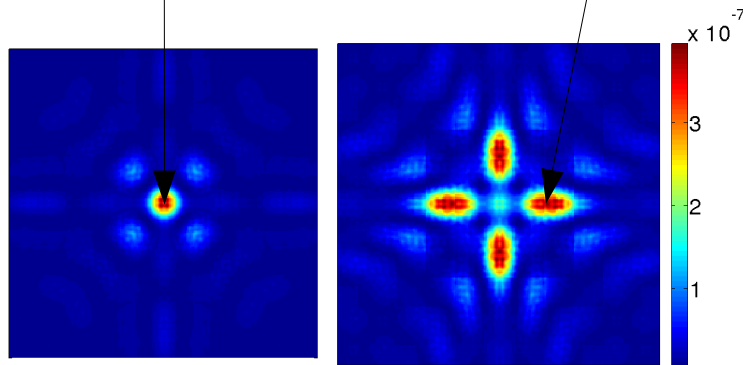
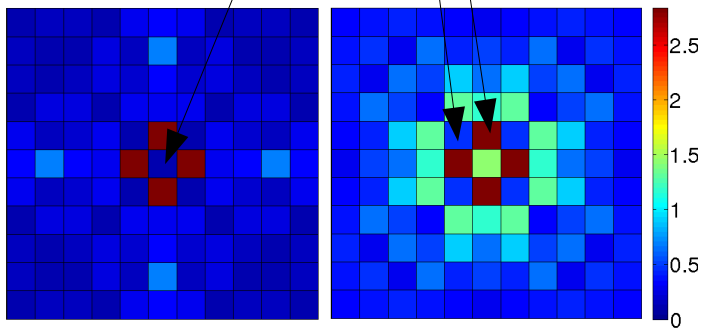
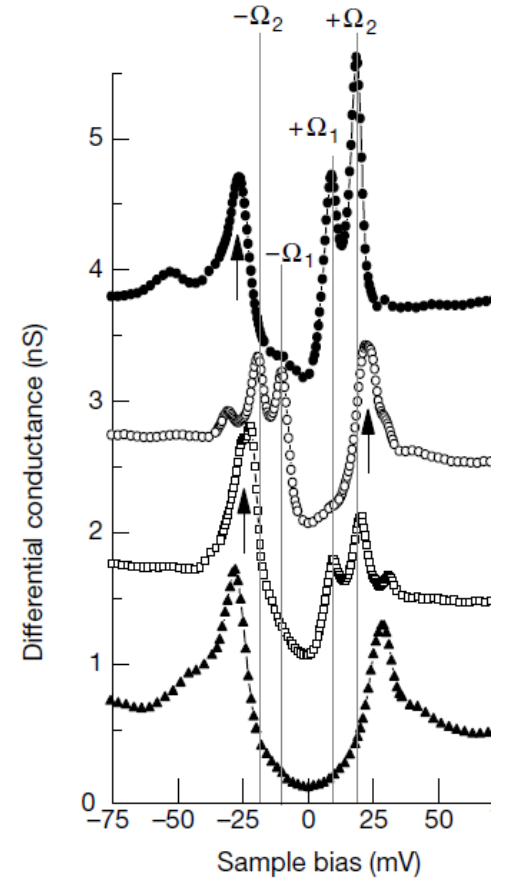
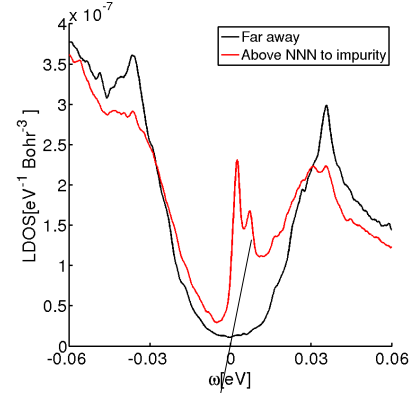
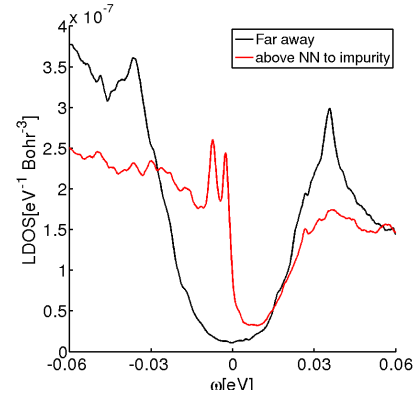
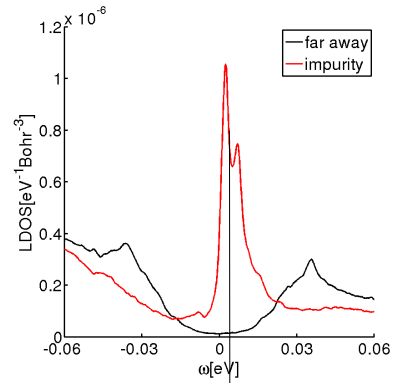
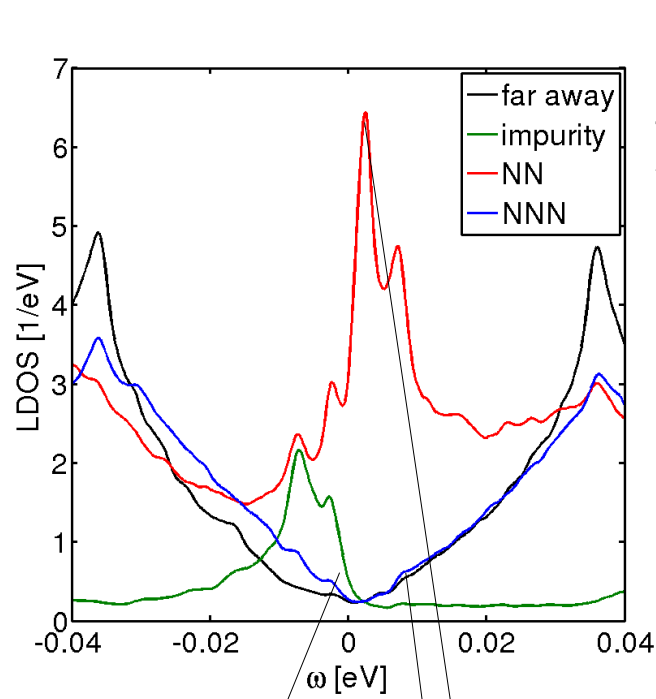
- chemistry: Ni 3d8 configuration → magnetic moment with  $S=1$
- Classical spin: additional magnetic potential

$$H_{\text{imp}}^{\text{mag}} = J(n_{R * \uparrow} - n_{R * \downarrow})$$

- resonances at  
+/- 2.4 meV (up)  
+/- 7.2 meV (down)



# Results: spectra, maps



$\Omega = -2.4$  meV

$\Omega = 2.4$  meV

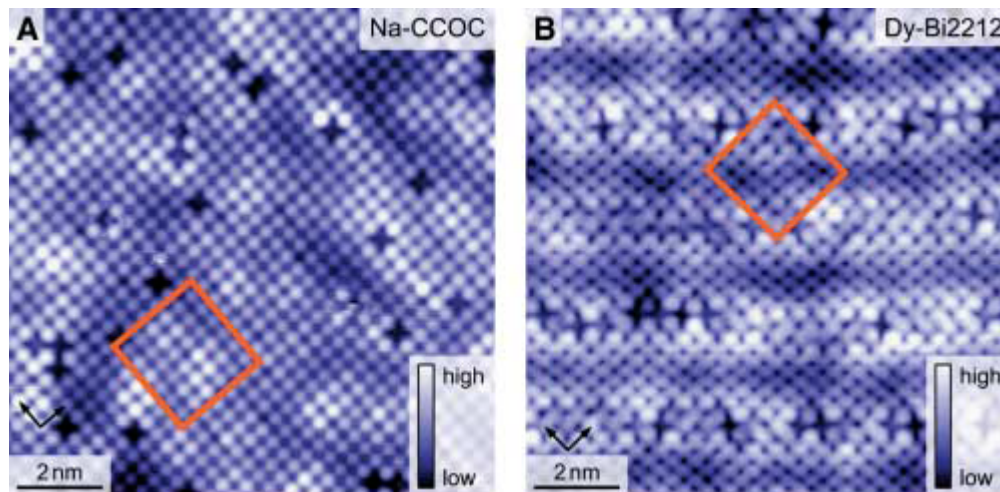
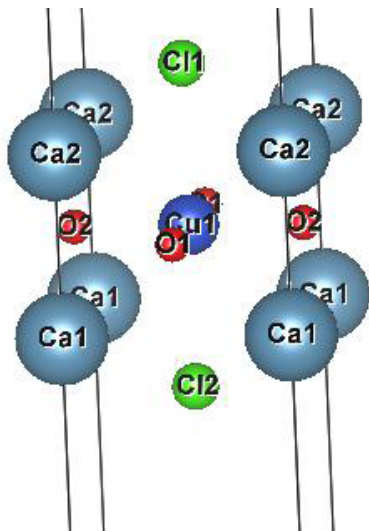


# Differences between materials?

- Different tunnelling layers, same physics

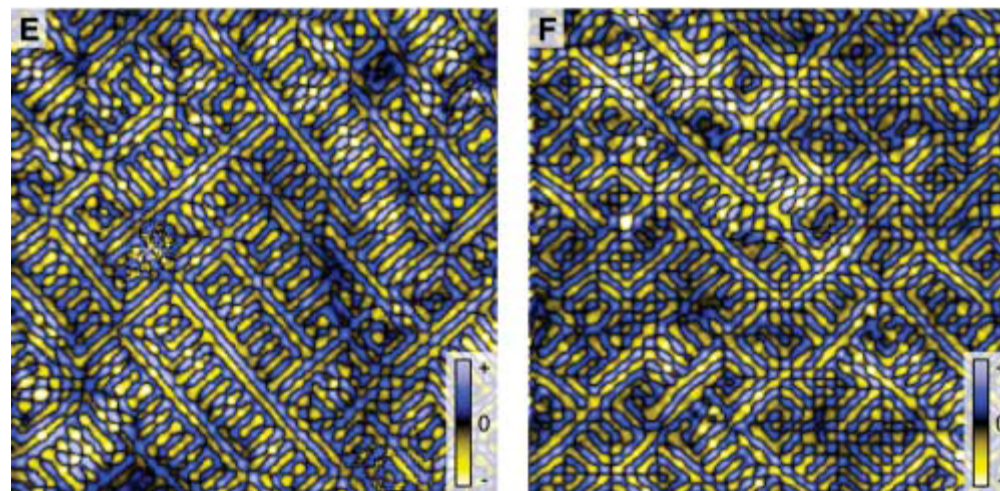
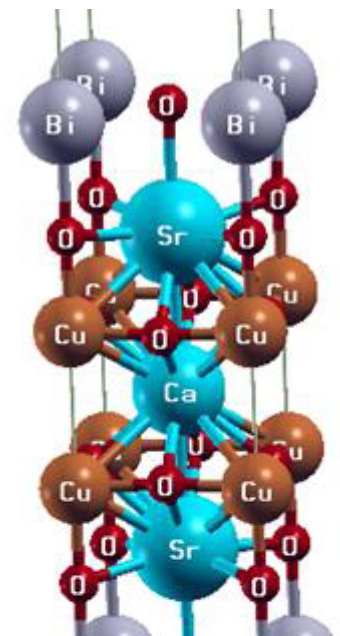
Na-CCOC

Ca-Cl surface

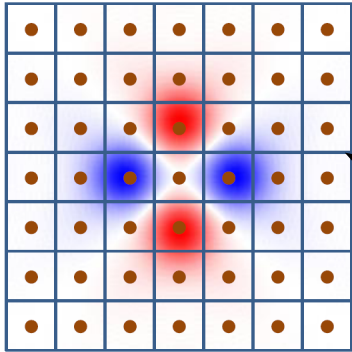


Dy-Bi2212

Bi-O surface  
(Sr-O in between)



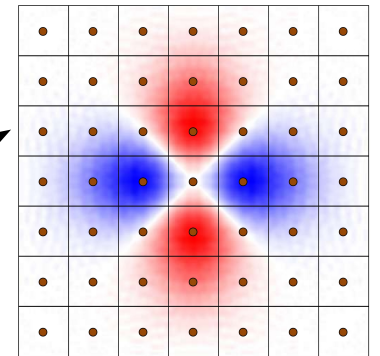
# Wannier functions



Na-CCOC

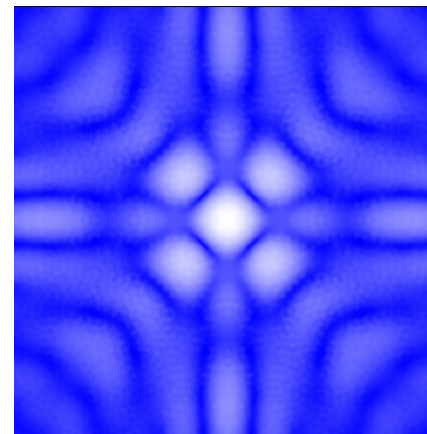
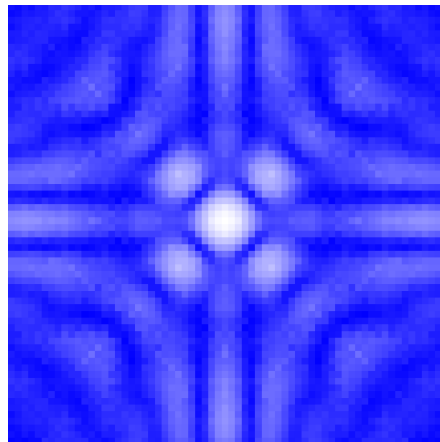
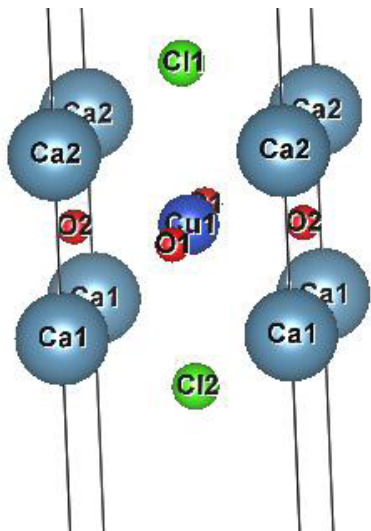
Ca-Cl surface

Wavefunctions very similar due to symmetry constraints on downfolding to  $dx^2-y^2$  band in conjunction with crystal symmetry

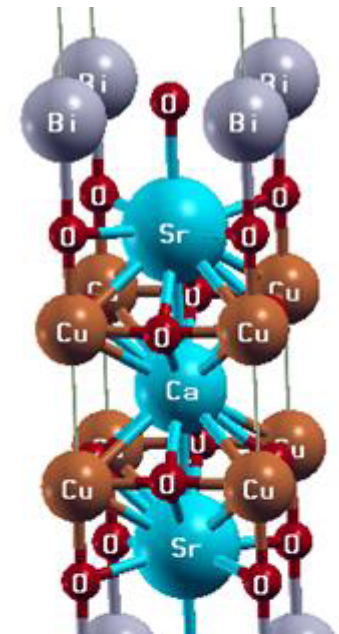


Dy-Bi2212

Cu position relative to BSCCO unit cell



Conductance maps for a strong potential scatterer (in the SC state, d-wave)



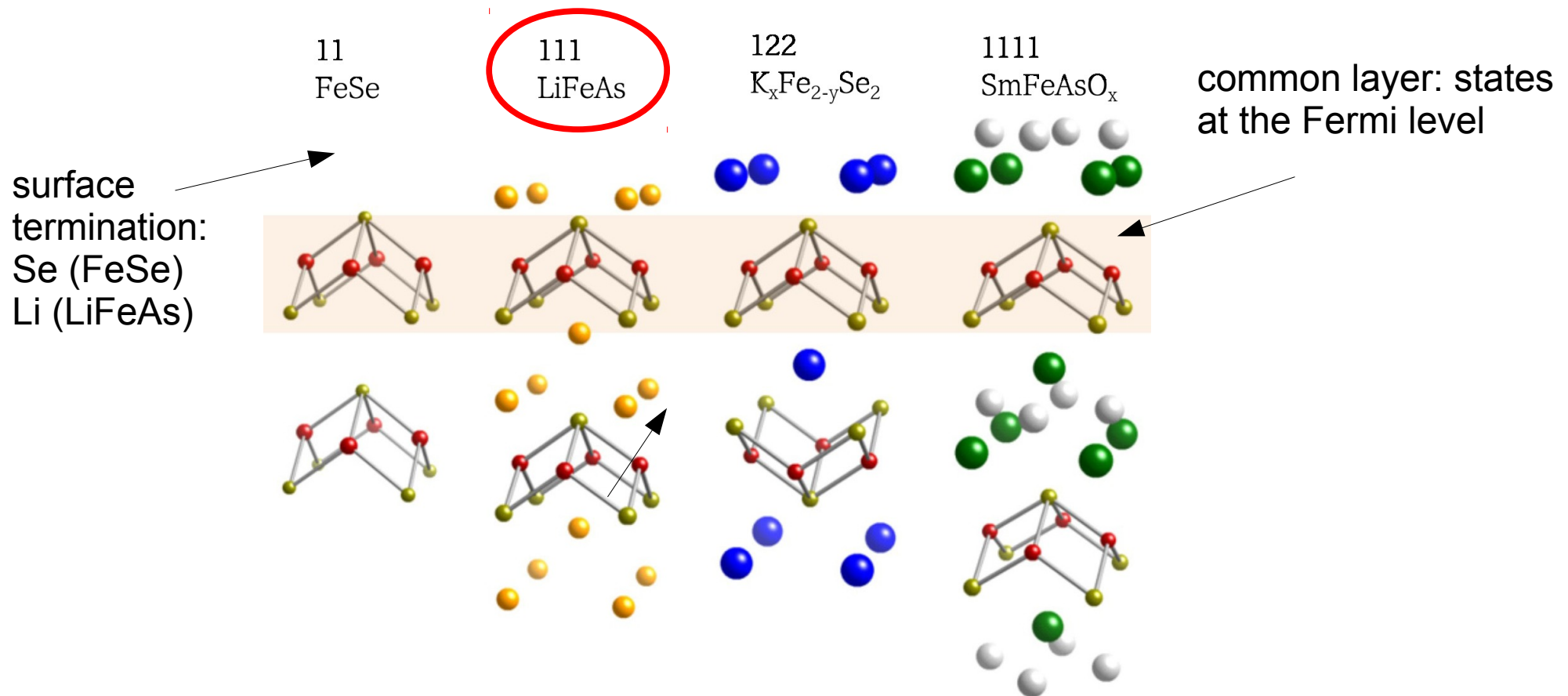
Bi-O surface (Sr-O in between)



# Layered superconductors

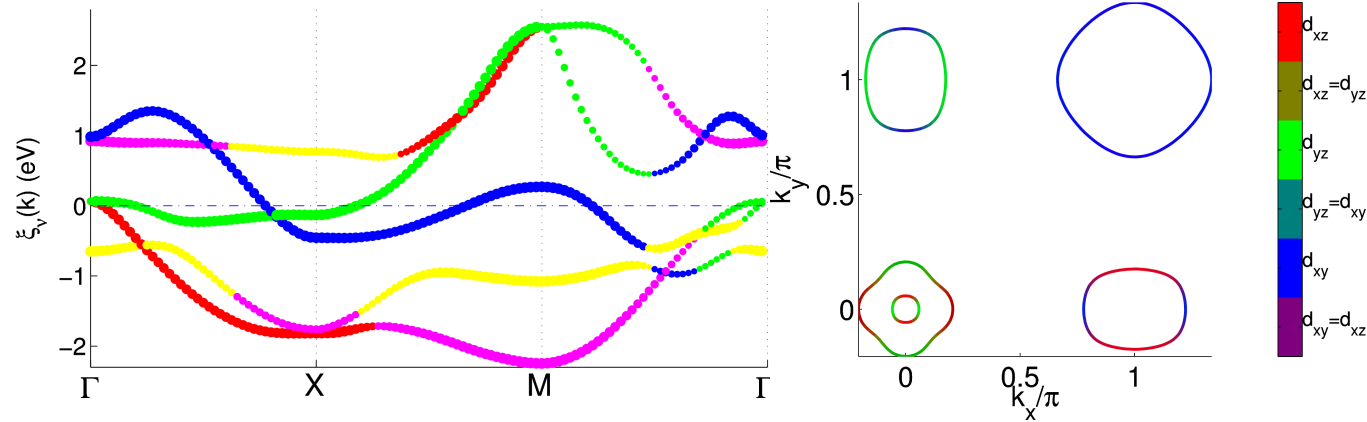
LDOS of sample at given energy **at the tip position**

- Iron based superconductors

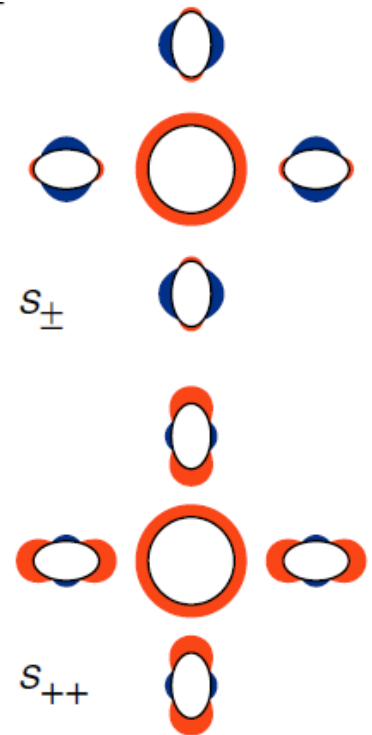
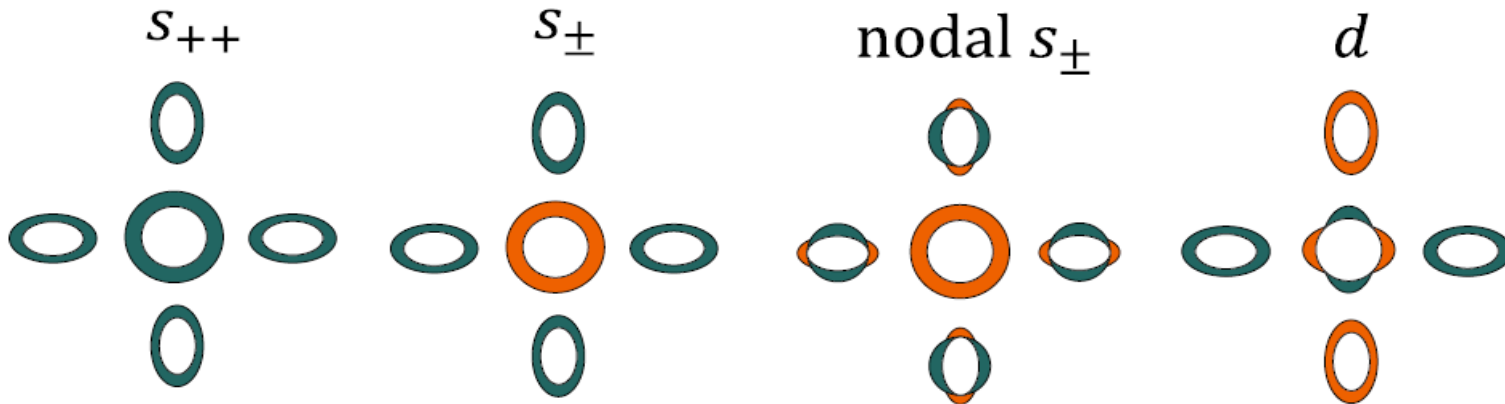


# Gap symmetries: FeSC

- Fermi surface  
5 band model



- Possible order parameters

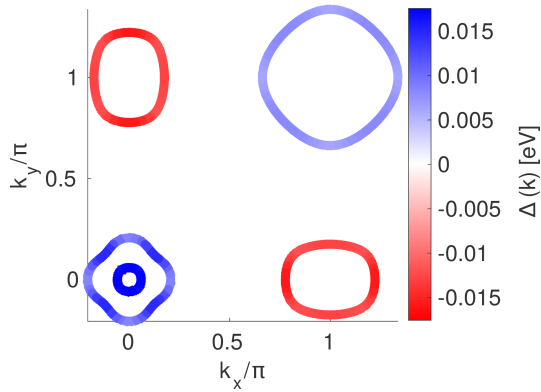


# LiFeAs: 18K superconductor

- proposed gap structures

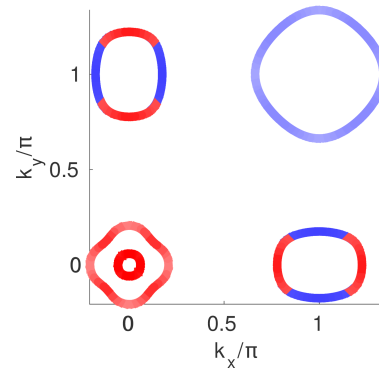
conventional  $s_{\pm}$

Y. Wang, A. Kreisel, et al.  
Phys. Rev. B 88, 174516 (2013)



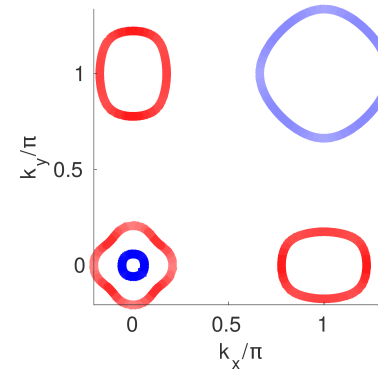
antiphase  $s_{\pm}$

Z. P. Yin, K. Haule, G. Kotliar  
Nature Physics 10, 845-850 (2014)



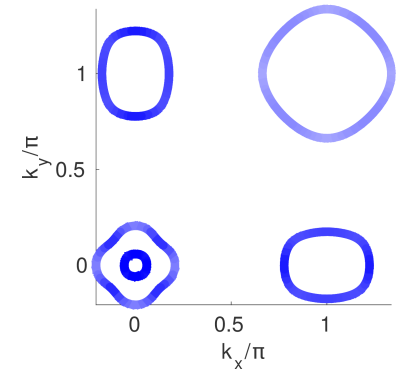
novel  $s_{\pm}$

F. Ahn, et al. Phys. Rev. B 89, 144513 (2014)

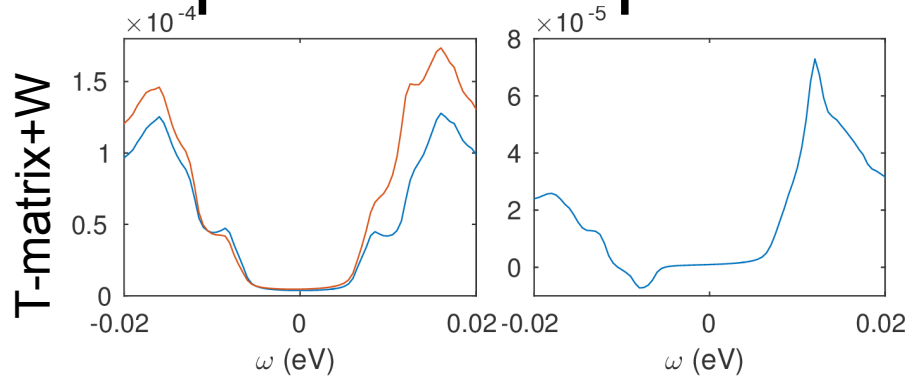


conventional  $s_{++}$

Tetsuro Saito, et al.  
Phys. Rev. B 90, 035104 (2014)

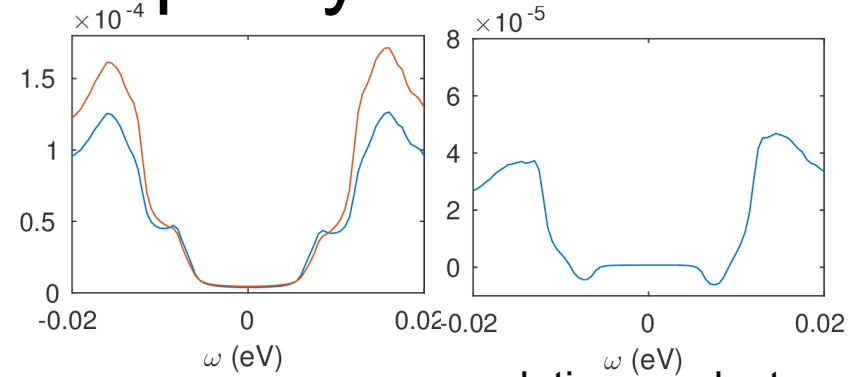


- experimental probes? → impurity bound states



conventional  $s_{\pm}$

relative conductance:  
peak-dip

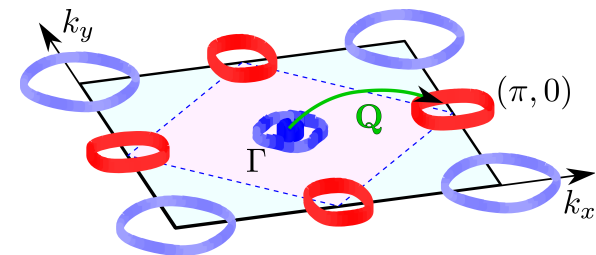
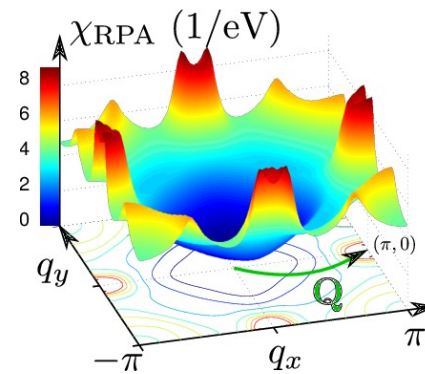
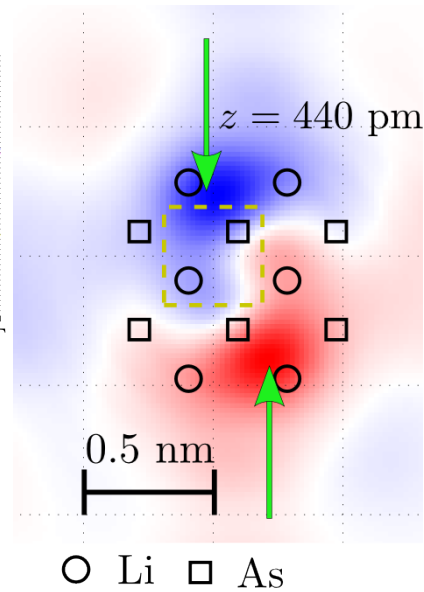
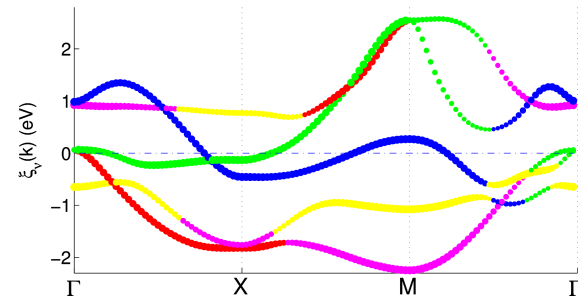


conventional  $s_{++}$

relative conductance:  
modification of  
normal state DOS

# Theory: T-matrix+Wannier

- ab-initio calculation for LiFeAs (surface)
  - tight-binding model (5 band)
  - Wannier functions (including glide plane symmetry)
- superconducting order parameter from spin-fluctuation theory
- ab-initio impurity potentials (Ni, Mn, Co) used in T-matrix calculation



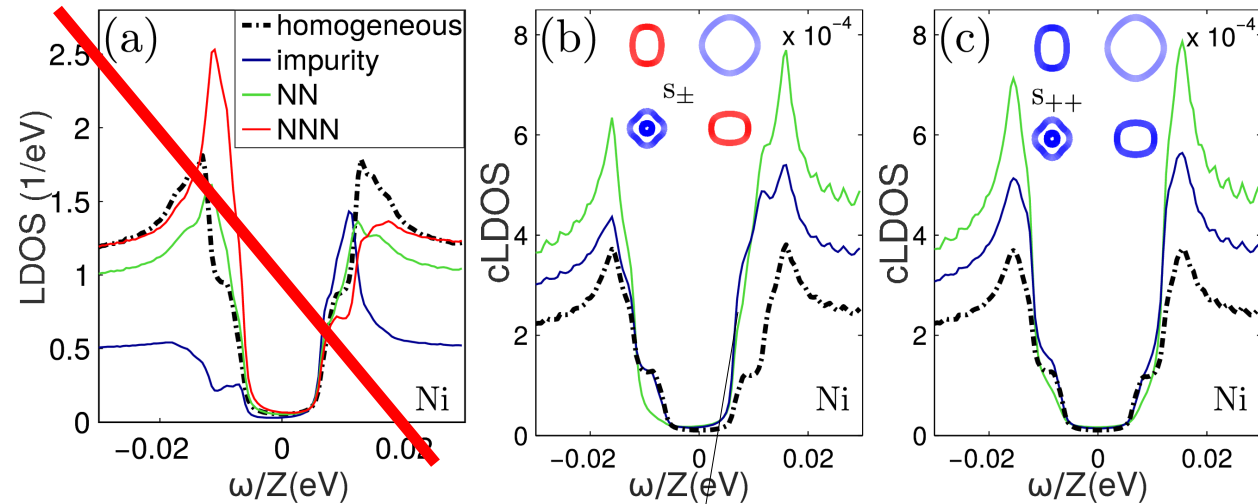
$$\hat{G}(\mathbf{r}, \mathbf{r}'; \omega) = \hat{G}_0(\mathbf{r} - \mathbf{r}', \omega) + \hat{G}_0(\mathbf{r}, \omega) \hat{T}(\omega) \hat{G}_0(\mathbf{r}', \omega)$$

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mathbf{R}'; \omega) w_{\mathbf{R}}(\mathbf{r}) w_{\mathbf{R}'}^*(\mathbf{r}')$$

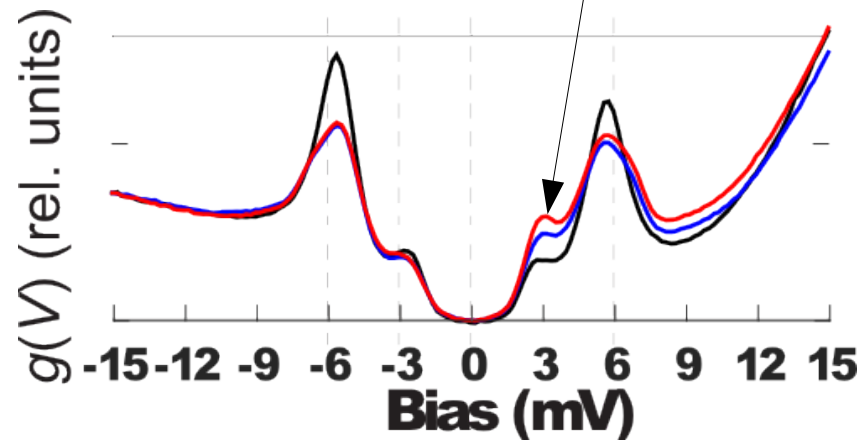
$$\rho(\mathbf{r}, \omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{r}, \mathbf{r}; \omega)$$

# LiFeAs: spectra

- evidence for sign-changing order parameter by in-gap state with engineered impurity

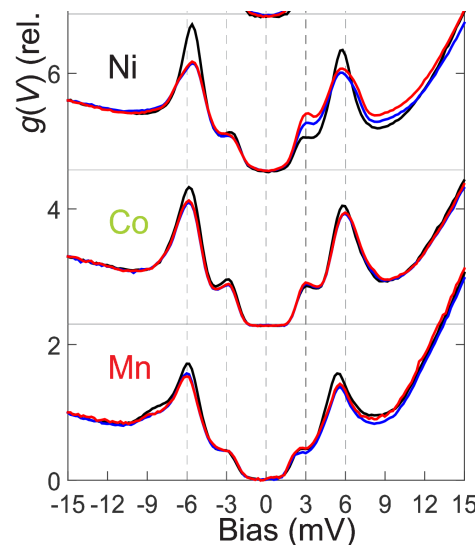
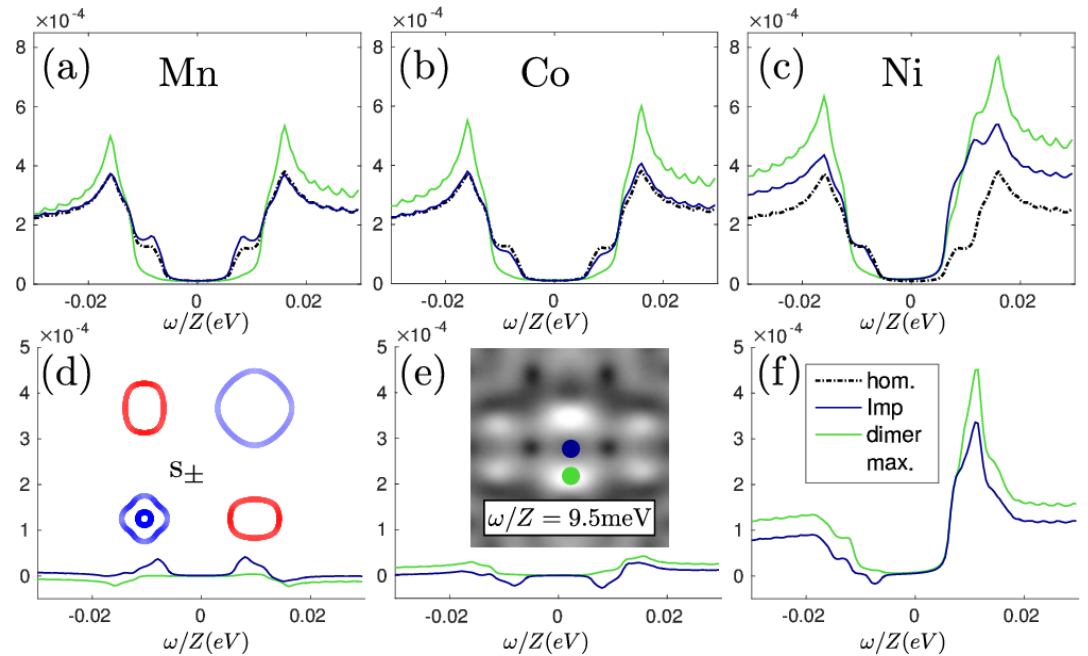


lattice LDOS: strong response at negative bias

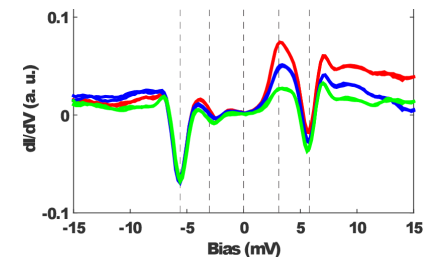


# LiFeAs: spectra

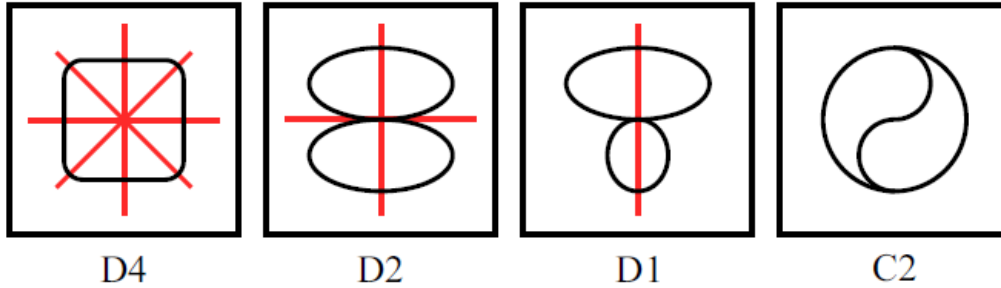
- sequence of impurity potentials from ab-initio calculation correct, but overall renormalization downwards required.



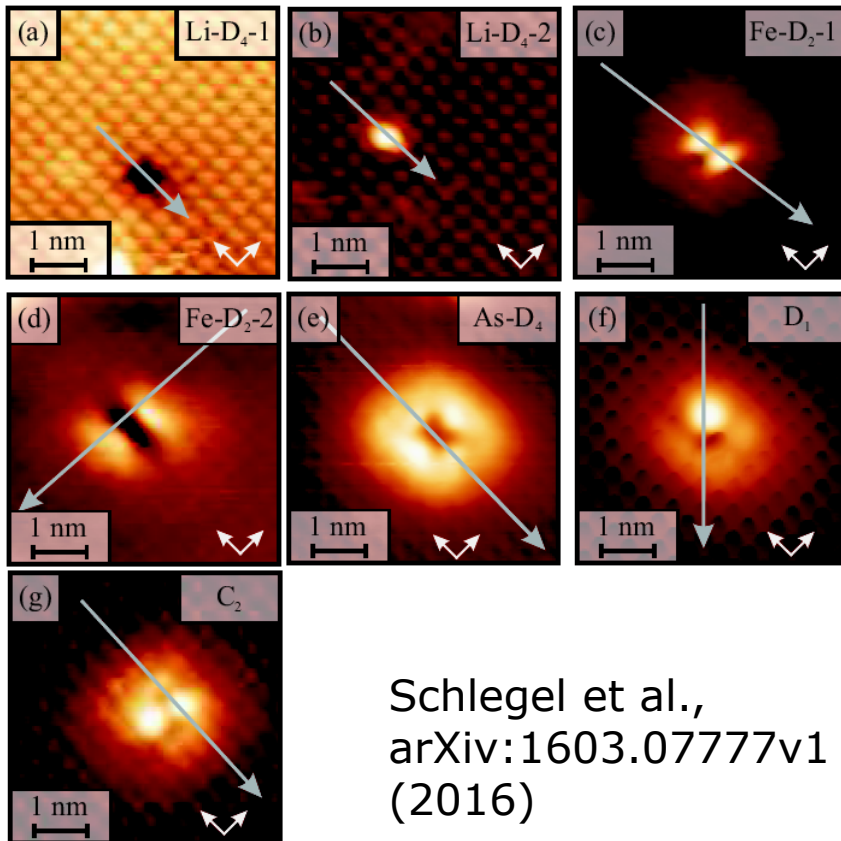
relative spectra  
Ni impurity



# LiFeAs: other native impurities

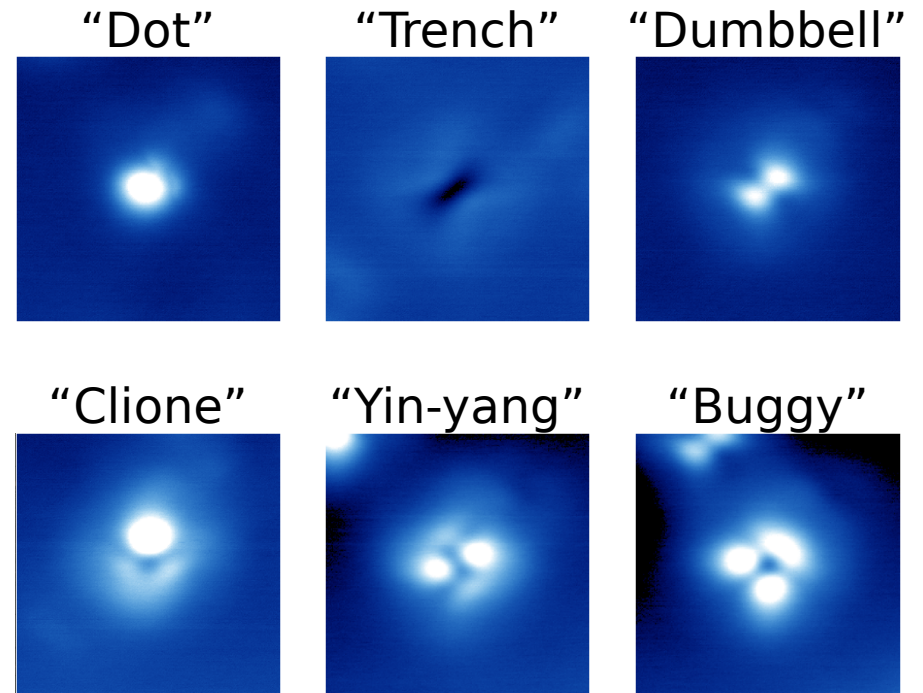


Schönflies classification of impurities



Schlegel et al.,  
arXiv:1603.07777v1  
(2016)

Hanaguri, unpublished (KITP 2011)

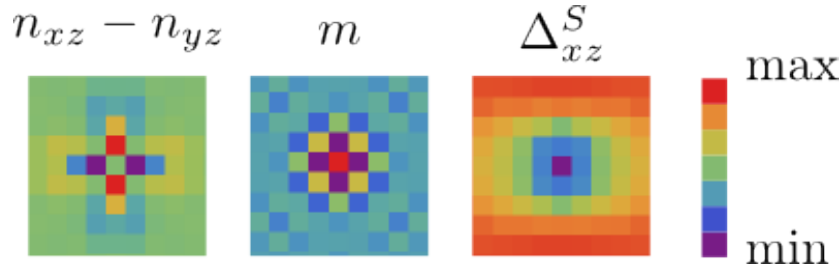




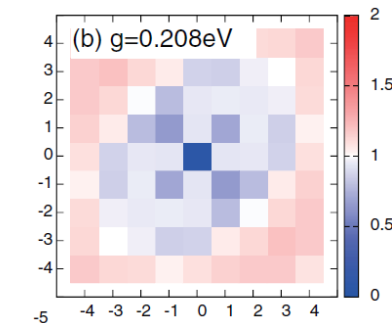
# Chiral defects

- from a symmetry perspective not compatible to impurities on any single site in LiFeAs
- multiple impurities?
- local order?

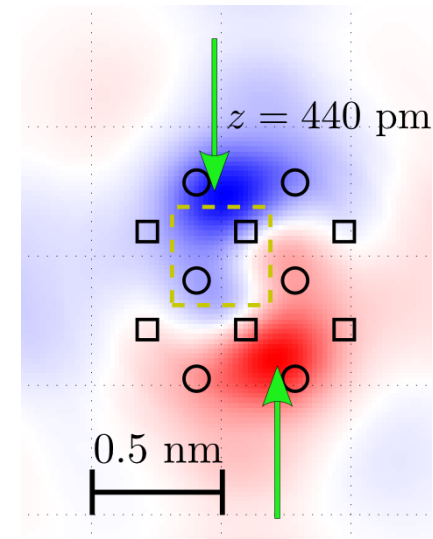
local orbital order  
 + Wannier function  
 → chiral defect structure



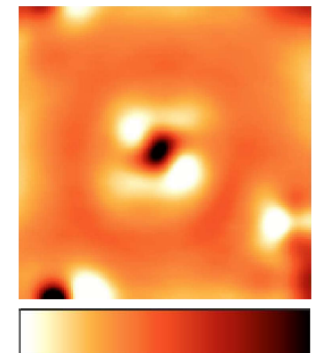
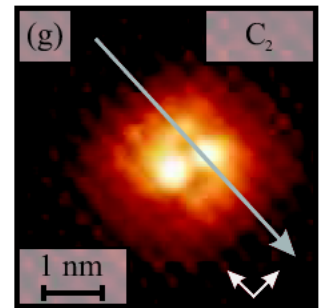
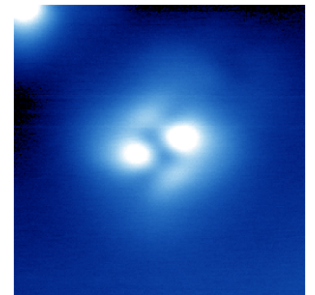
Gastiasoro, Andersen, J. Supercond Nov. Magn., **26**, 2651 (2013)



Inoue, Yamakawa, Kontani PRB 85, 224506 (2012)



$d_{xz}$  Wannier function on Fe(2)

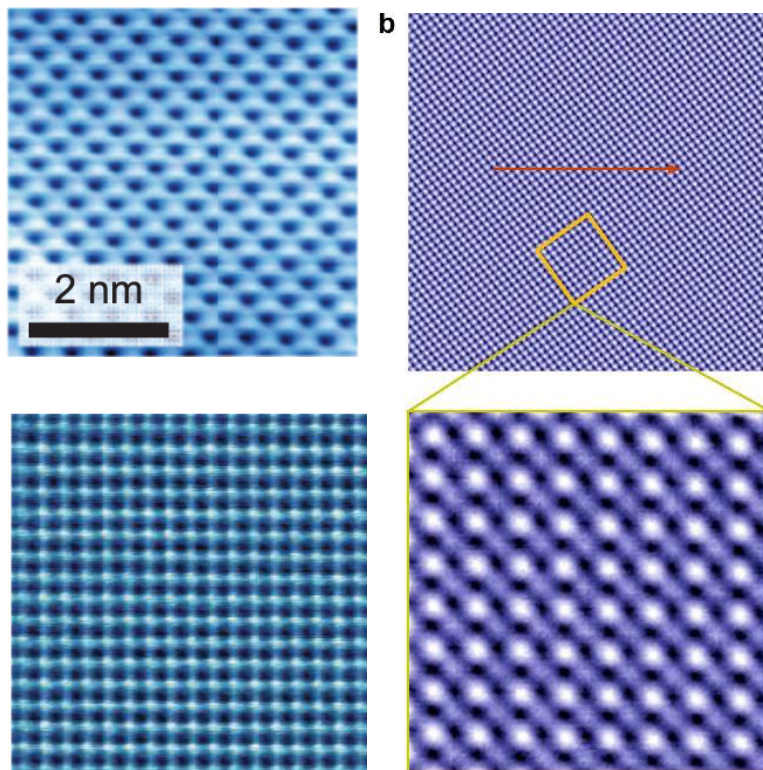


high low

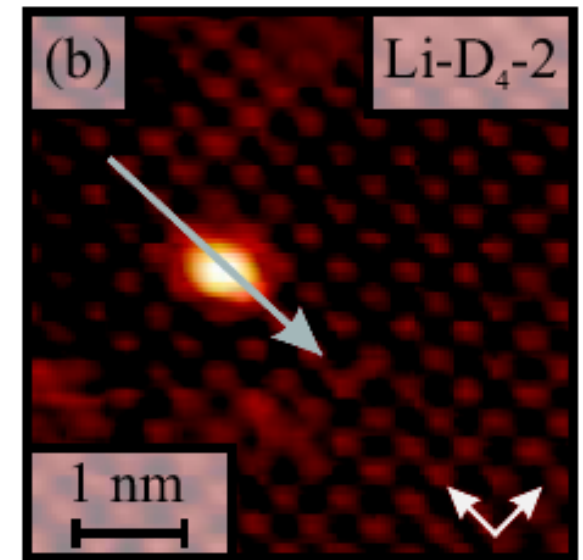


# Height and current dependence of topographs

- experiment: Li or As lattice?



height maxima at Li positions!?  
counter-intuitive from chemistry point of view

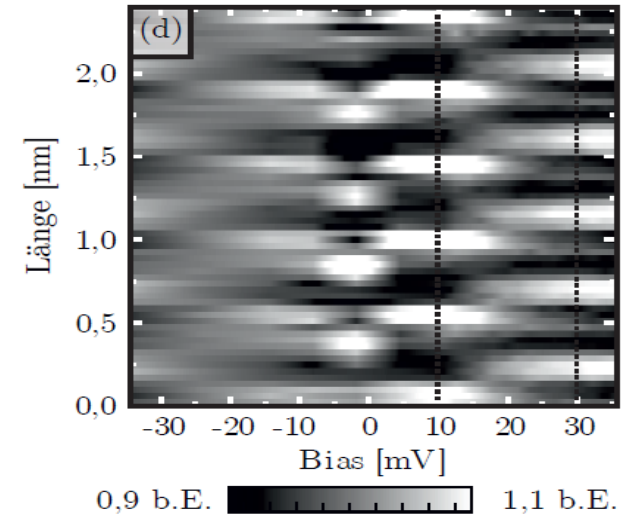
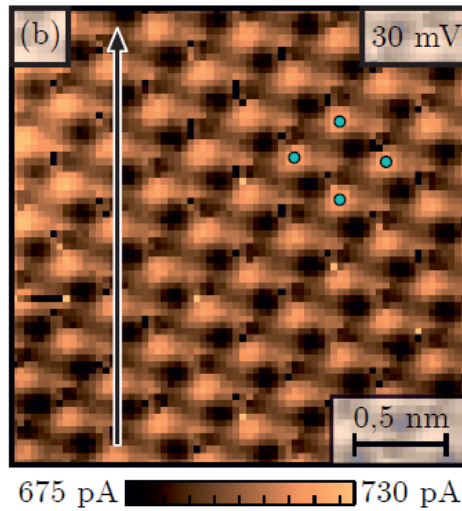
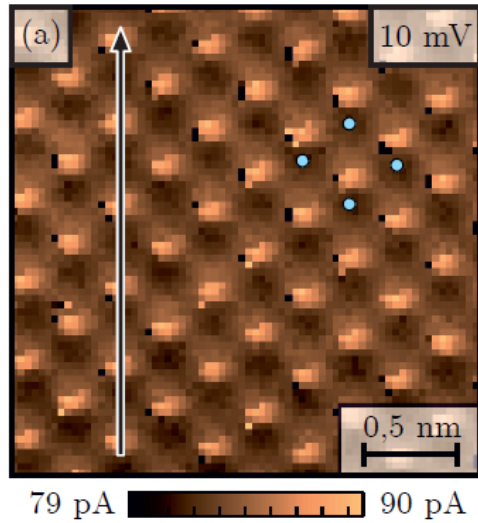


Shun Chi, et al., PRL 109, 087002 (2012)  
T. Hanaguri, et al. PRB 85, 214505 (2012)  
S. Grothe, et al., PRB 86, 174503 (2012)  
J. -X. Yin, et al., arXiv, 1602.04949 (2016)

Schlegel et al.,  
arXiv:1603.07777v1  
(2016)

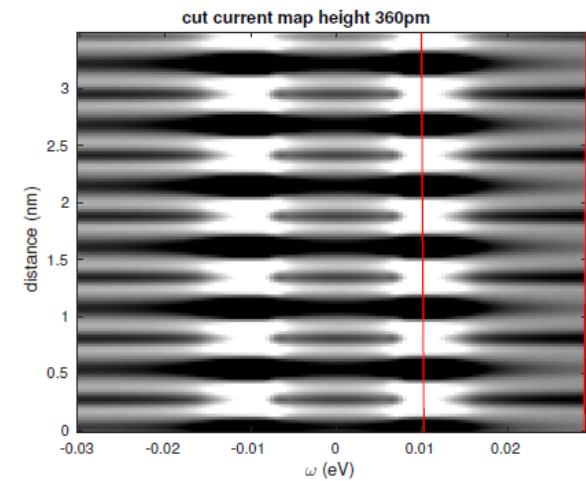
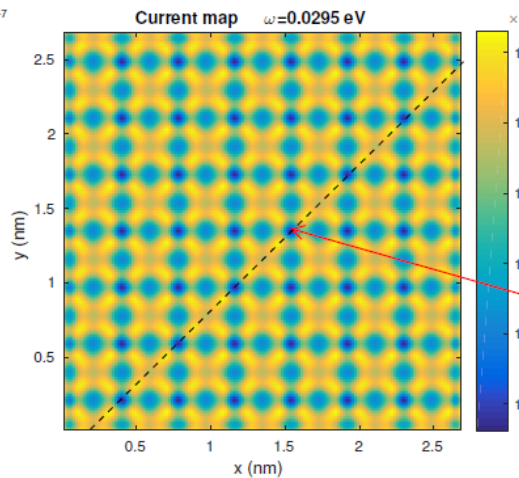
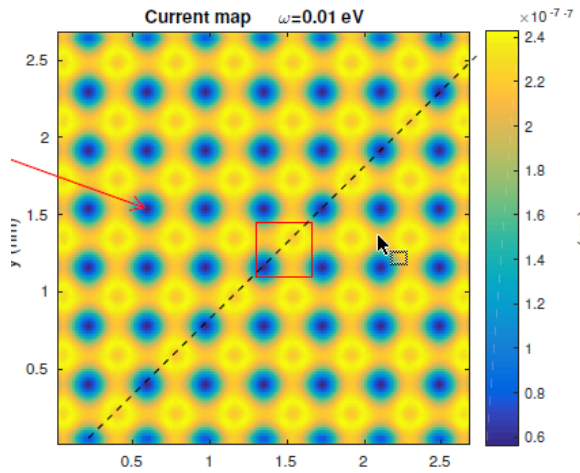
# Further experimental evidences?

experiment (current maps)



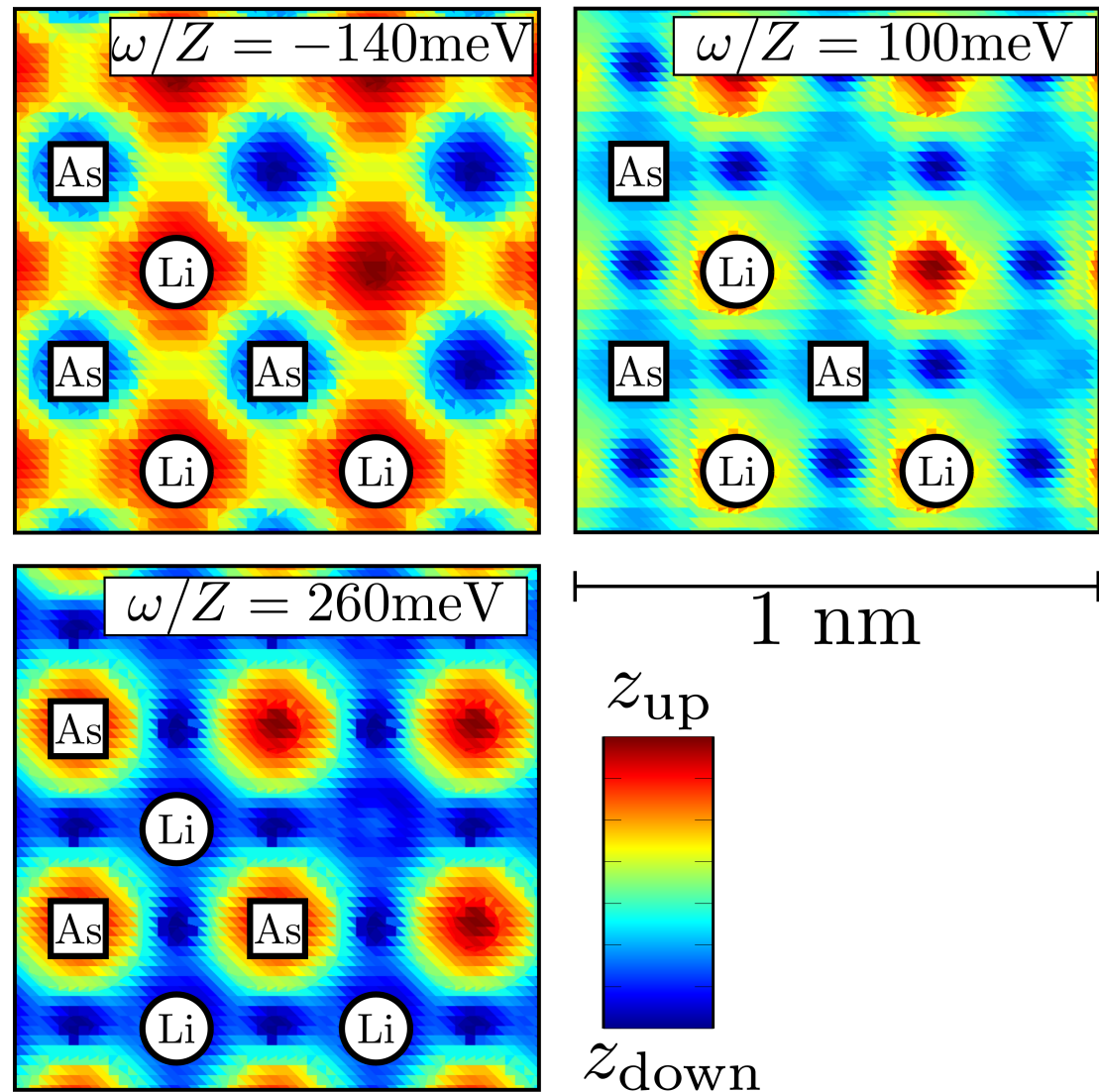
Ronny Schlegel, Dissertation, TU Dresden  
(thanks to C. Hess)

theory



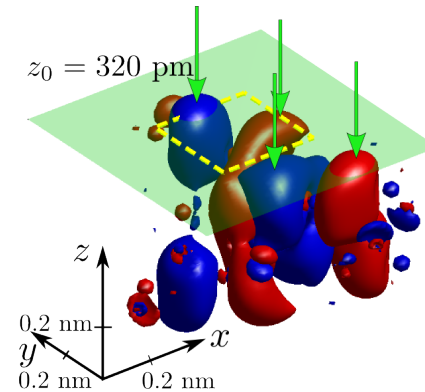
# Simulation of topographs

- switching of height maxima as a function of bias voltage

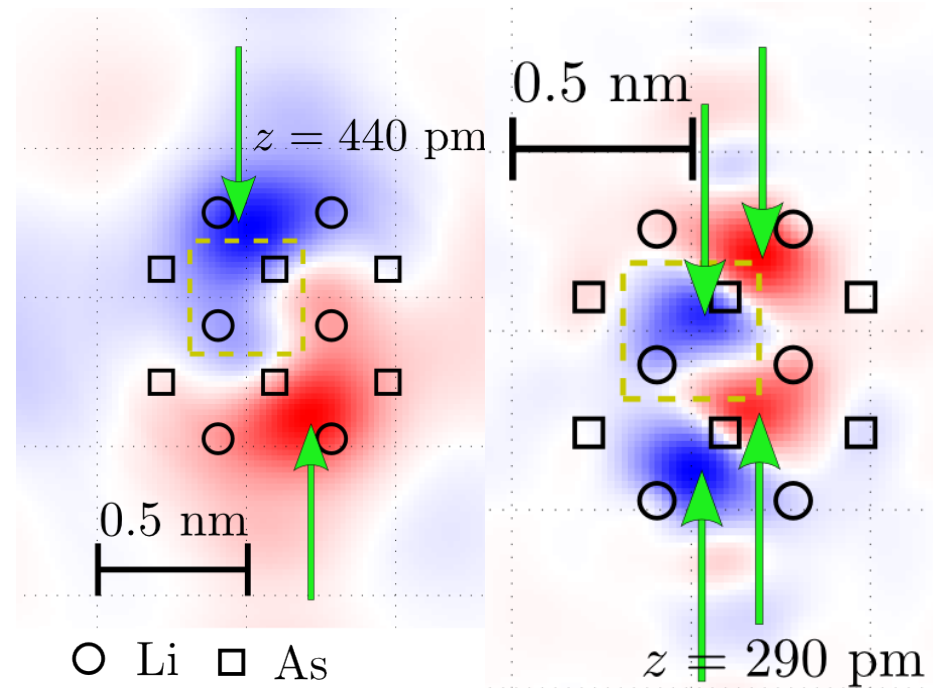
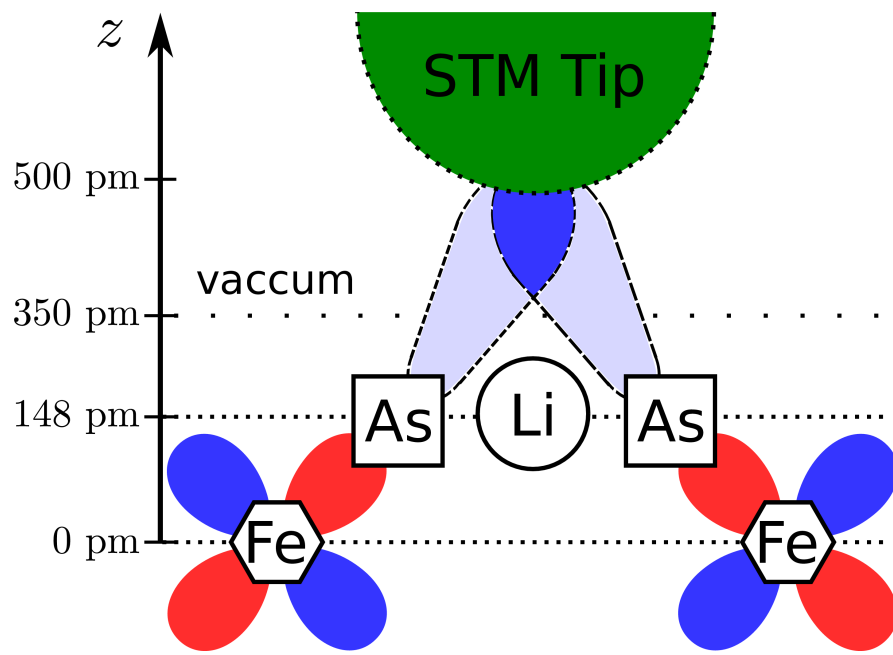


# Simulation of topographs

- interplay of interference of wavefunctions and their spatial structure



$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mu, \nu, \mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mu, \mathbf{R}', \nu; \omega) w_{\mathbf{R}, \mu}(\mathbf{r}) w_{\mathbf{R}', \nu}^*(\mathbf{r}')$$

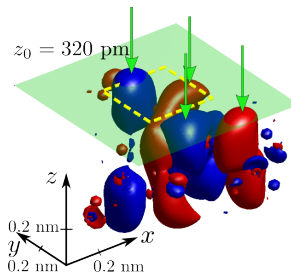


# Recapitulation: BdG/T-matrix+W

- **simple**: just a basis transformation of the Green's function

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{\mathbf{R}, \mathbf{R}'} G(\mathbf{R}, \mathbf{R}'; \omega) w_{\mathbf{R}}(\mathbf{r}) w_{\mathbf{R}'}^*(\mathbf{r}')$$

- **powerful** tool for calculation of local density of states at the surface (STM tip position) of superconductors



- takes into account atomic scale information and symmetries of the elementary cell and the contained atoms

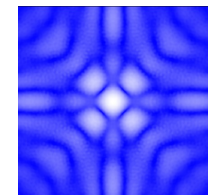
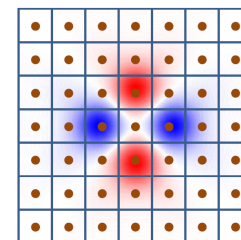
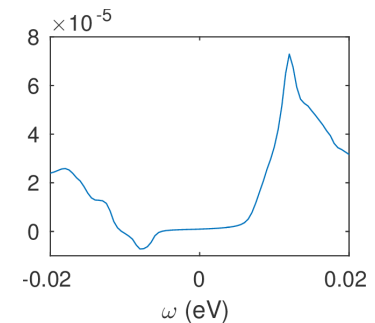
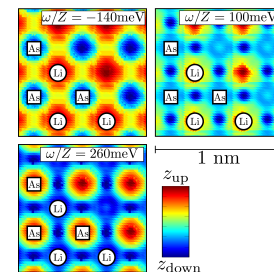
- **shown to work**

- LiFeAs: metal ion impurities

Shun Chi, (...), A. Kreisel, et al. arXiv:1607.03192

- BiSrCaCuO: Zn impurity, QPI

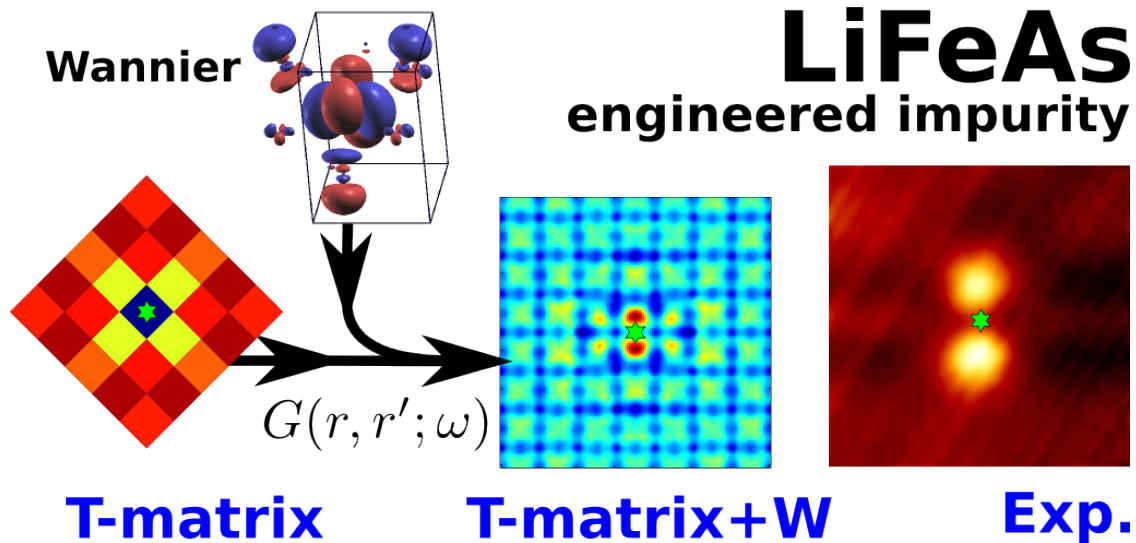
Kreisel *et al.* PRL **114**, 217002 (2015)





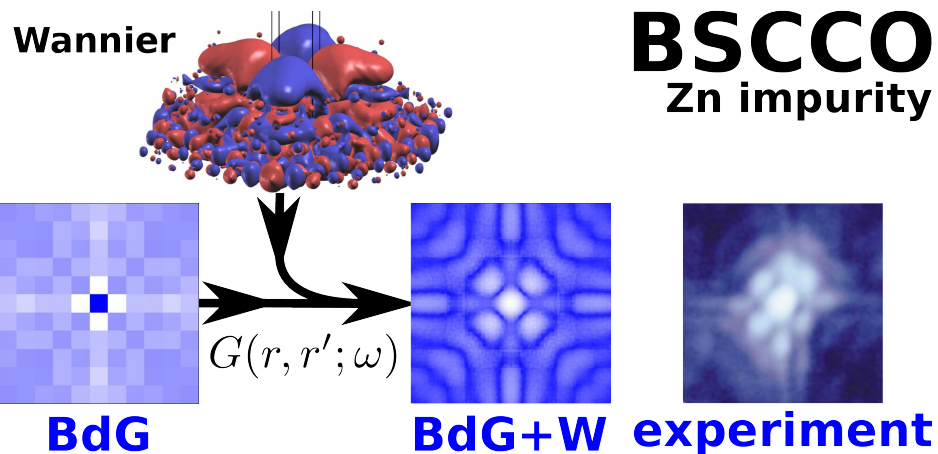
# Summary

Kreisel *et al.*  
PRL 114, 217002 (2015)



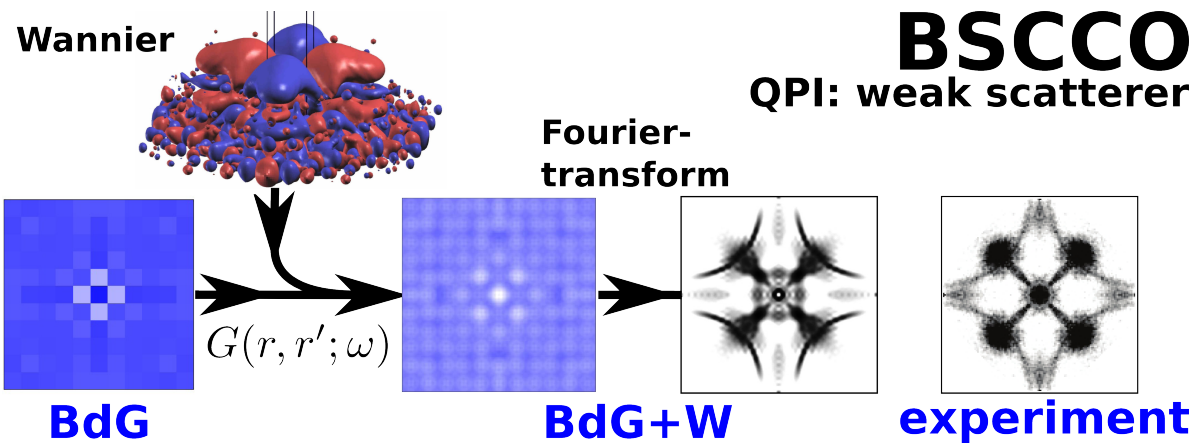
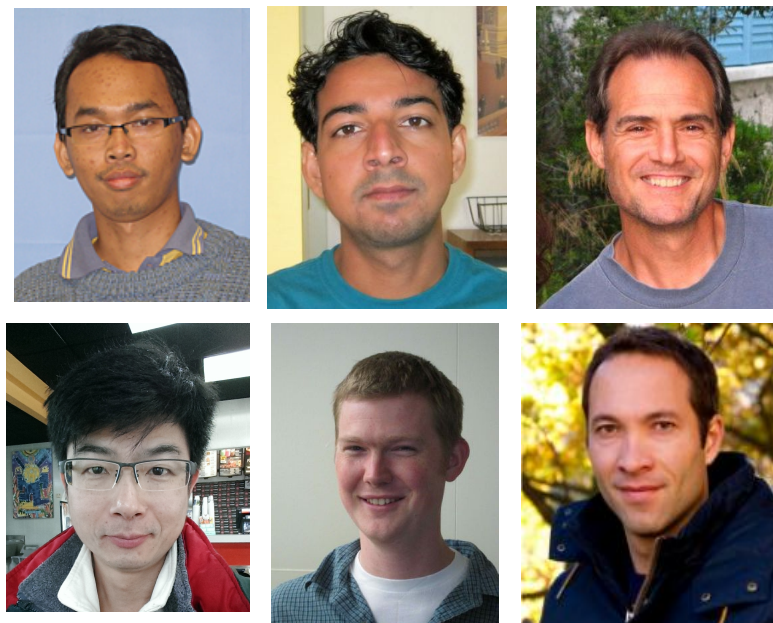
# LiFeAs

engineered impurity



arXiv:1607.03192

Acknowledgements



University of  
St Andrews



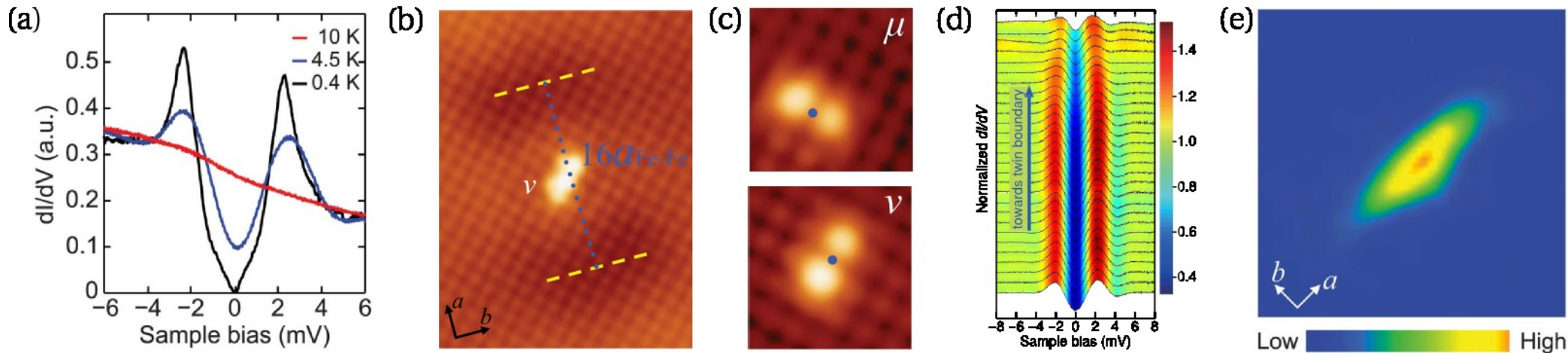
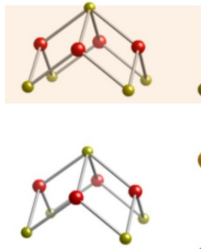
THE UNIVERSITY OF BRITISH COLUMBIA



# FeSe: simplest crystal structure

- Tc 8K, under pressure  $\sim 40$ K Medvedev, et al. Nat. Mater. 8, 630 (2009)
- Tc 100K (single layer) Ge et al. Nat. Mater. 14, 285 (2015)
- nematic phase  
no magnetism Baek, et al. Nat. Mat. 14, 210 (2015)
- consequences: nodal gapstructure, anisotropy

11  
FeSe



Song et al. PRL 109, 137004 (2012)

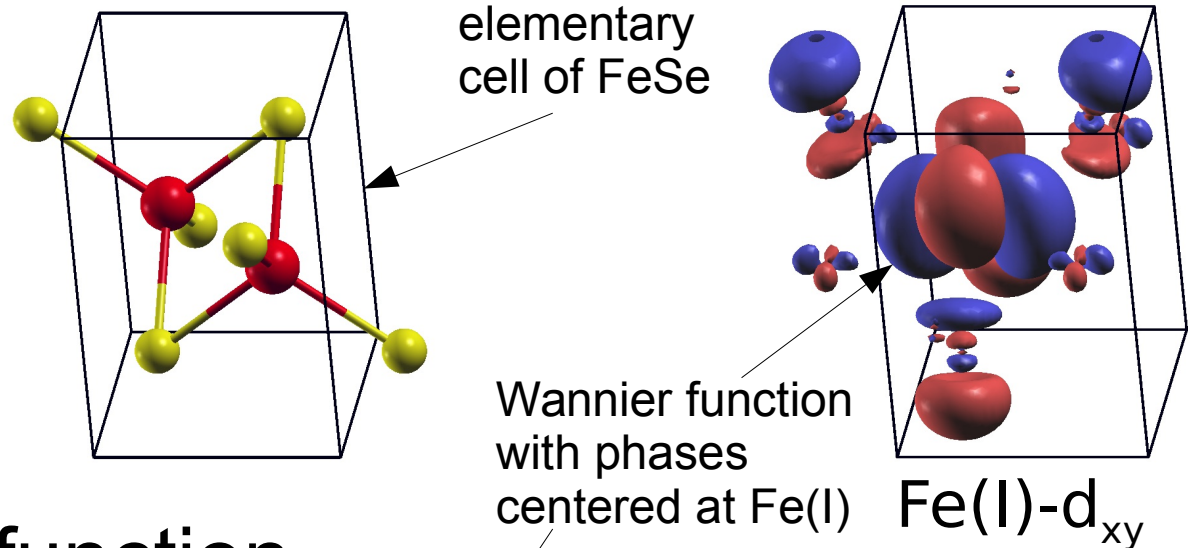
Song et al. Science 332, 1410 (2011)

# BdG+Wannier method

- first principles calculation

- band structure
- Wannier functions wavefunctions in real space

$$H_0 = \sum_{R R', \sigma} t_{R R'} c_{R \sigma}^\dagger c_{R' \sigma} - \mu_0 \sum_{R, \sigma} c_{R \sigma}^\dagger c_{R \sigma}$$



- continuum Green function

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \sum_{R, R'} G(R, R'; \omega) w_R(\mathbf{r}) w_{R'}^*(\mathbf{r}')$$

continuum position

nonlocal contributions

lattice Green function

local density of states (LDOS)

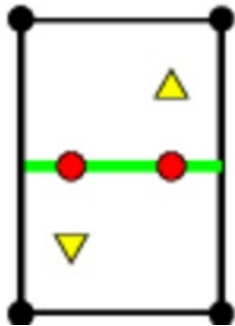
$$\rho(\mathbf{r}, \omega) \equiv -\frac{1}{\pi} \text{Im} G(\mathbf{r}, \mathbf{r}; \omega)$$

# FeSe: BdG+W Results

$$I(V, x, y, z) = -\frac{4\pi e}{\hbar} \rho_t(0) |M|^2 \int_0^{eV} \rho(x, y, z, \epsilon) d\epsilon$$

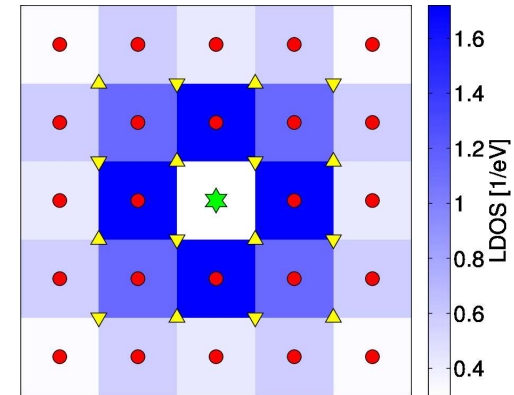
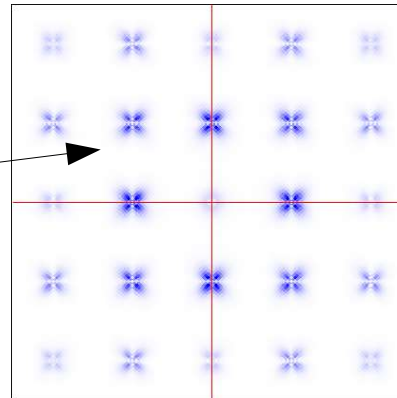
- continuum density of states

- at Fe plane

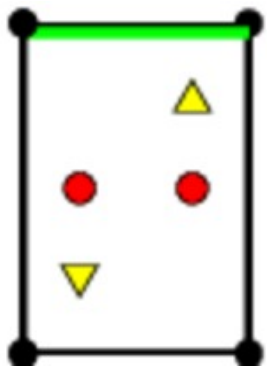


C4 symmetry!

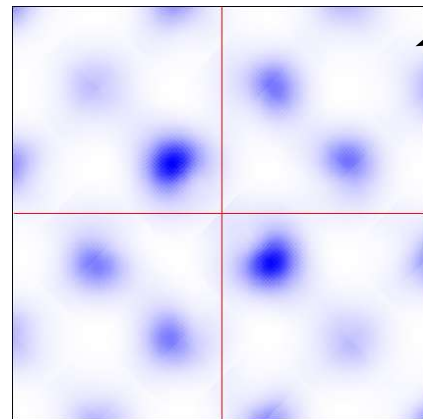
2 meV



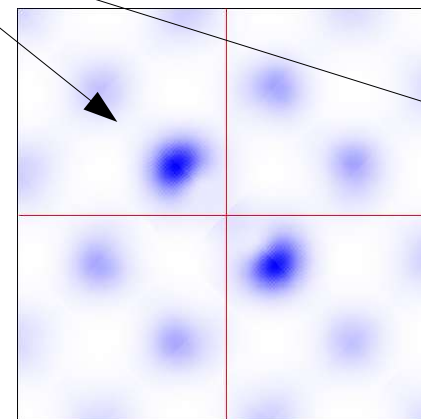
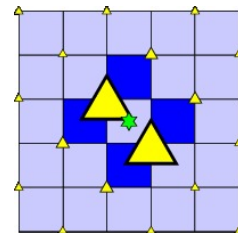
- at STM tip position



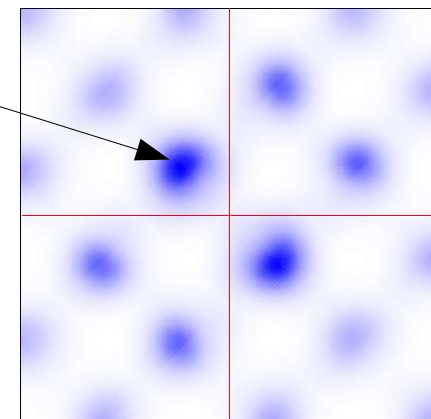
C2 symmetry!



-2 meV



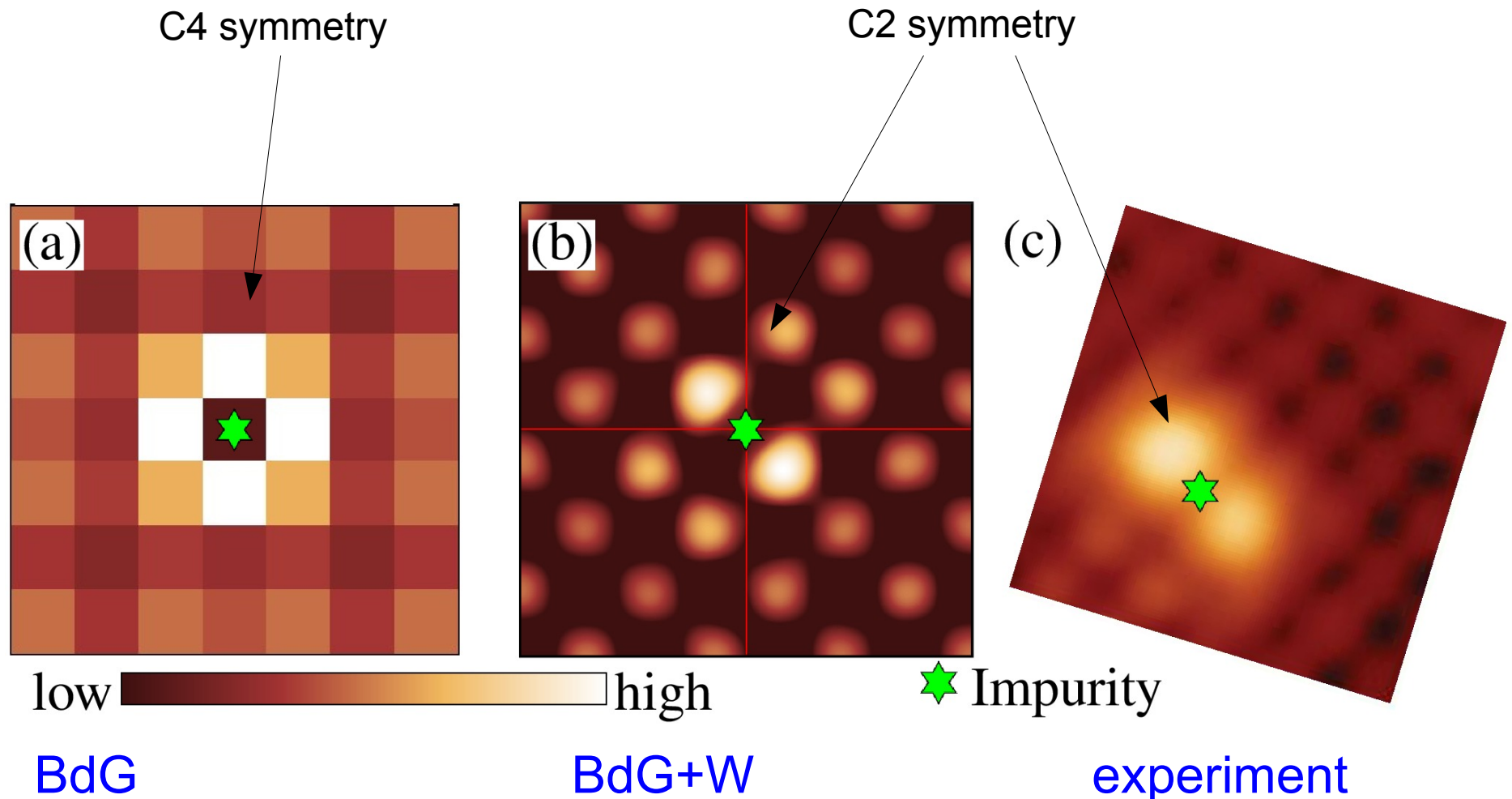
+2 meV



+30 meV

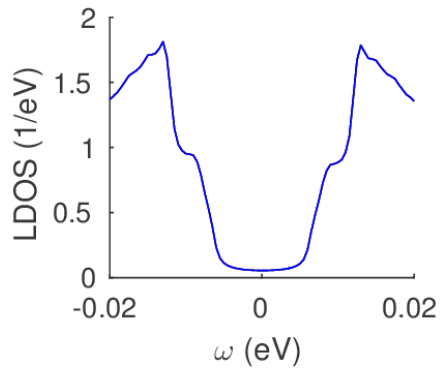
# FeSe: Comparison to experiment

STM topography on (bulk) FeSe with Fe-centered impurity

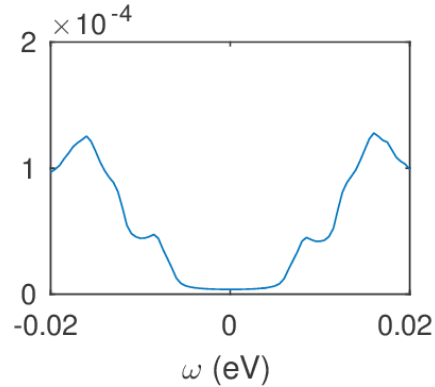


# LiFeAs: spectra

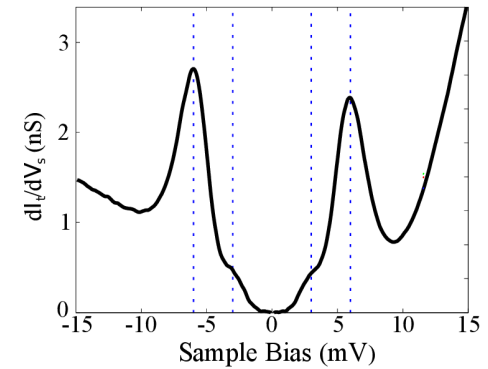
T-matrix



T-matrix+Wannier (T+W)

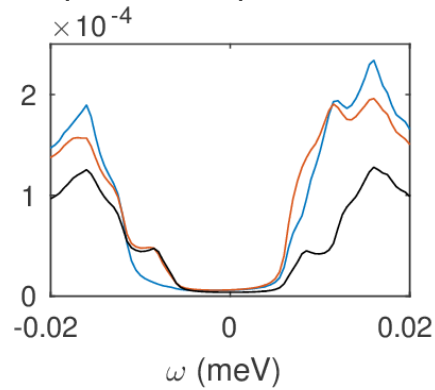
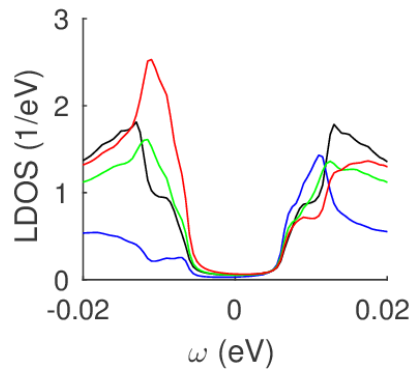


experiment

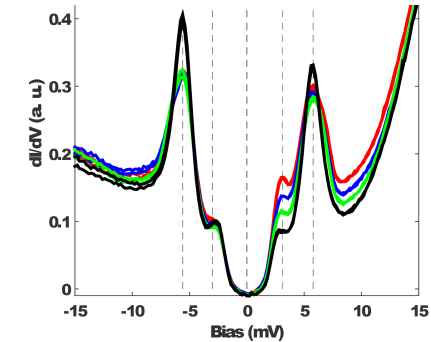


Chun Chi  
PhD thesis

Ni impurity potential (*ab initio*)

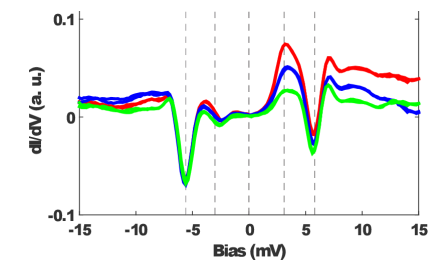
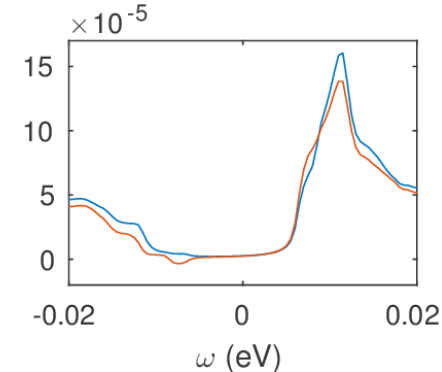
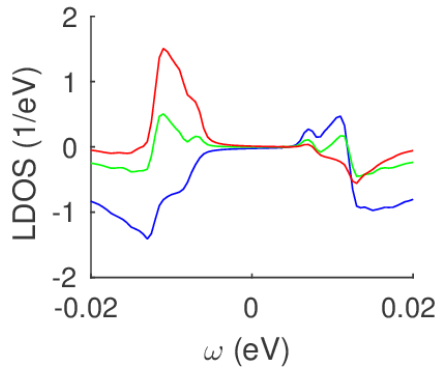


engineered Ni impurity



Chun Chi  
*et al.* (in  
preparation)

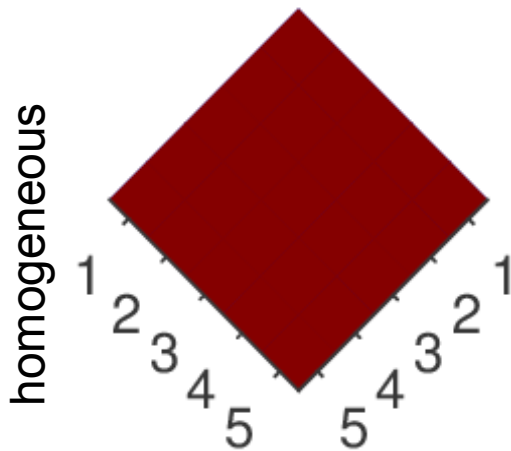
relative spectra



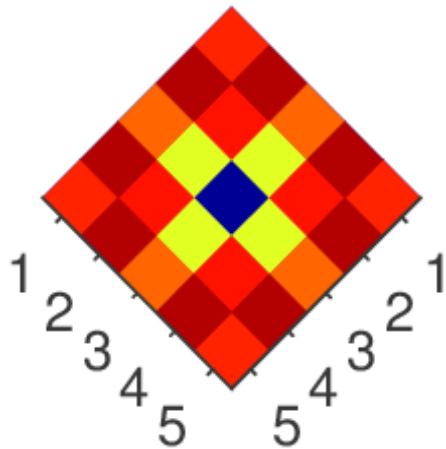


# LiFeAs: spatial conductance

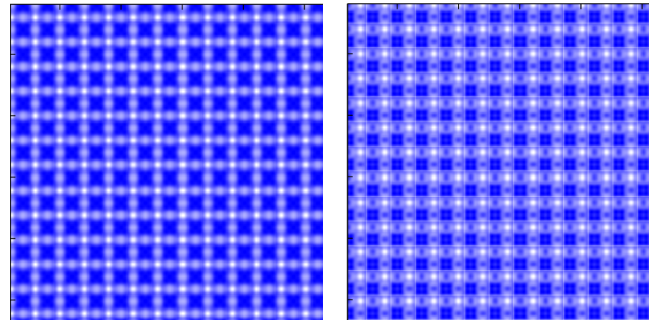
T-matrix



one pixel per Fe atom!



T-matrix+Wannier (T+W)

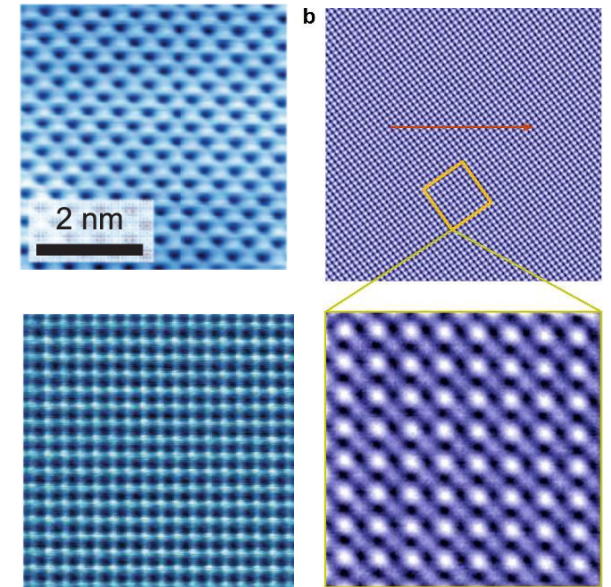


variation of tip height:

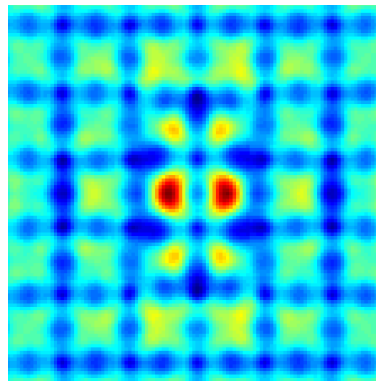
As lattice

Li lattice

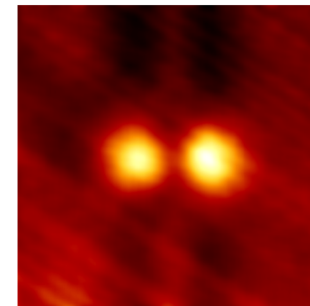
experiment (Li  $\leftrightarrow$  As lattice ?)



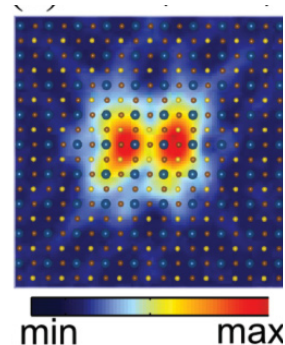
Ni impurity:  
C2 symmetric impurity state



engineered  
Ni defect



Fe centered  
defect



Shun Chi, et al., PRL 109, 087002 (2012)  
T. Hanaguri, et al. PRB 85, 214505 (2012)  
S. Grothe, et al., PRB 86, 174503 (2012)  
J. -X. Yin, et al., arXiv, 1602.04949 (2016)

min max