Introduction to Computer Simulation II

Homework 11

Due: Monday, 07 July 2025

21. **Reweighting of magnetisation data**

    Simulate the two-dimensional Ising model at the exactly known transition point $\beta_c = \ln(1+\sqrt{2})/2 \approx 0.440\,686\ldots$ with the single-cluster algorithm for linear lattice sizes $L = 16$ and 32 (and periodic boundary conditions). Store the time series for the energy and magnetisation and use this data to determine by reweighting $\langle |m| \rangle(\beta)$, $\langle m^2 \rangle(\beta)$, and $\langle m^4 \rangle(\beta)$ in a "small" region around β_c. Compare these curves with the results of direct simulations at two $\beta > \beta_c$ and two $\beta < \beta_c$. The "small" region, in which the reweighting method yields dependable results, scales proportional to $L^{-1/\nu}$ (that is $1/L$ in the special case of the two-dimensional Ising model with $\nu = 1$).

22. **Autocorrelation times of the single-cluster algorithm for the 2D 3-state Potts model**

    Repeat for the 2D 3-state Potts model (cf. problem 6) problem 7 for the 2D Ising model with linear lattice sizes $L = 16$, 32, and 64 (and periodic boundary conditions), i.e., estimate the autocorrelation times τ_{int} of the single-cluster algorithm of problem 6 at the critical point $\beta_c = \ln(1+\sqrt{3})$ of the infinite system (e.g., by means of the binning method). Use this data to investigate the scaling behavior of the autocorrelation times as function of the lattice size, that is $\tau_{\text{int}} \propto L^z$.

    Optional: An additional simulation for $L = 128$ should not take too long and reveal the scaling behavior more pronounced.