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Introduction to Computer Simulation II

Homework 10

Due: Monday, 30 June 2025

19. **Derivative of the Binder parameter U_4 and of $\ln\langle m^2\rangle$**

    a) Show that the derivative of the Binder parameter $U_4 = 1 - \langle m^4\rangle/3\langle m^2\rangle^2$ (cp. problems 1 and 5) with respect to inverse temperature β can be expressed by the following combination of expectation values:

    $$\frac{dU_4(\beta)}{d\beta} = V(1-U_4)\left(\langle e\rangle - 2\frac{\langle m^2 e\rangle}{\langle m^2\rangle} + \frac{\langle m^4 e\rangle}{\langle m^4\rangle}\right) .$$

    How does the analogous equation read for $dU_2/d\beta$ with $U_2 = 1 - \langle m^2\rangle/3\langle |m|\rangle^2$?

    b) Show similarly that

    $$\frac{d\ln\langle m^2\rangle(\beta)}{d\beta} = V\left(\langle e\rangle - \frac{\langle m^2 e\rangle}{\langle m^2\rangle}\right) .$$

    How does the analogus equation read for $d\ln\langle |m|\rangle/d\beta$?

20. **Finite-size scaling in the two-dimensional Ising model at the critical point**

    Simulate the two-dimensional Ising model at the exactly known transition point $\beta_c = \ln(1+\sqrt{2})/2 \approx 0.440\,686\ldots$ with the single-cluster algorithm for linear lattice sizes L = 8, 16, 32, 64, and 128 (and periodic boundary conditions). Determine from the time-series data for the energy and magnetisation the specific heat, magnetic susceptibility, and the Binder parameter as well as the derivatives of the Binder parameter and magnetisation with respect to inverse temperature as discussed in the lecture (see also problem 19).

Check the finite-size scaling behavior of the two-dimensional Ising model at the critical point β_c ($C(\beta_c) = a_1 \ln(L) + b_1 + \ldots$, $\chi(\beta_c) = a_2 L^{\gamma/\nu} + b_2 + \ldots$, $\frac{dU_4}{d\beta}(\beta_c) = a_3 L^{1/\nu} + \ldots$, $\frac{d \ln\langle m^2 \rangle}{d\beta}(\beta_c) = a_4 L^{1/\nu} + \ldots$) and determine the critical exponents by least-squares fits.