## Introduction to Computer Simulation II

## Homework 09

Due: Monday, 23 June 2025

## 17. Multiple-histogram reweighting

Write a computer program for multiple-histogram reweighting, the socalled "Weighted Histogram Analysis Method (WHAM)", and test it for the 2D 16<sup>2</sup> Ising model with the three simulation points  $\beta_1 = 0.375$ ,  $\beta_0 = \beta_c = \ln(1 + \sqrt{2})/2 \approx 0.440\,686\,79\ldots$ ,  $\beta_2 = 0.475$  (cp. problem 22 of Computer Simulation I).

## 18. Master plot for the susceptibility

Determine the susceptibility  $\chi$  of the 2D Ising model on lattices of size  $16 \times 16$ ,  $32 \times 32$ ,  $64 \times 64$ , and  $128 \times 128$  for 31 inverse temperatures  $\beta$  in the high-temperature phase in the range  $0.85\beta_c, \ldots, \beta_c$  close to the critical point at  $\beta_c$  analogously to problem 16. Plot these data

- i) as function of  $\beta$  and then
- ii) in a log-log plot as function of  $b \equiv 1 \beta/\beta_c$ . Here it does make sense to compare with  $b^{-7/4}$ . Finally
- iii) generate the Master plot of  $\chi L^{-\gamma/\nu}$  as function of the scaling variable  $x = (1 \beta/\beta_c)L^{1/\nu}$  in a log-log representation. Use for this illustration of the scaling property the exact critical exponents of the 2D Ising model  $\nu = 1$  and  $\gamma = 7/4$  and compare with  $x^{-7/4}$ .