

Introduction to Computer Simulation II

Homework 5

Due: Friday, 16 May 2025

9. Mean cluster size as improved estimator for the susceptibility of the 2D Ising model

The mean cluster size of the single-cluster algorithm is an estimator for the susceptibility in the high-temperature phase of the Ising model. Not too close to the critical point the cluster estimator yields much more accurate averages than the usual definition in terms of spins, i.e., it is an “improved estimator”. Verify this claim by comparison of the statistical errors of the two estimators for square lattices of linear extent $L = 8, 16, 32$, and 64 with periodic boundary conditions. Compare also with problems 2 and 8.

10. Embedded cluster algorithm for the 2D XY model

Generalize your computer code for the Wolff single-cluster algorithm of problem 4 for the 2D Ising model to the 2D XY model.

Validate your program by comparison with the results in the publication of U. Wolff, Nucl. Phys. B **322** (1989) 759, especially in Table 1 and Table 3a. At least for the smallest lattice sizes and maybe with a little bit reduced statistics ($2 - 3$ times larger error bars are sufficient for this comparison) the simulation times should be in the minute range.

When comparing the susceptibilities pay attention to the definition in the original publication, $\chi = V \langle m^2 \rangle$ (that is *without* β factor), where $m^2 = (1/V^2)(M_x^2 + M_y^2)$ is the normalized and squared value of the magnetization vector $\vec{M} = \sum_i ((\sigma_i)_x, (\sigma_i)_y)$.