High-Temperature Series Analysis of the p-State Potts Glass Model on d-Dimensional Hypercubic Lattices

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Abstract

We analyze recently extended high-temperature series expansions for the "Edwards-Anderson" spin glass susceptibility of the p-state Potts glass model on d-dimensional hypercubic lattices for the case of a symmetric bimodal distribution of ferro- and antiferromagnetic nearest-neighbor couplings $J_{ij} = \pm J$. In these star-graph expansions up to order 22 in the inverse temperature $K \equiv J\beta \equiv J/k_BT$, the number of Potts states p and the dimension d are kept as free parameters which can take any value. By applying several series analysis techniques to the new series expansions, this enabled us to determine the critical coupling K_c and the critical exponent γ of the spin glass susceptibility in a large region of the two-dimensional (p, d)-parameter space. We discuss the thus obtained information with emphasis on the lower and upper critical dimensions of the model and present a careful comparison with previous estimates for special values of p and d.

The p-State Potts Glass Model

- Hamiltonian [1]:

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij}\delta_{S_i S_j}$$
$$S_i = 1, 2, \ldots, p$$

- Bimodal distribution:

$$P_b(J_{ij}) = x\delta(J_{ij} - J) + (1 - x)\delta(J_{ij} + J)$$
$$J > 0; x = 1/2$$

- Free energy:

$$\beta F = -\left[\ln\left(\sum_{\{S_i\}} \exp(-\beta\mathcal{H})\right)\right]_{\rm av}$$

$$\beta = 1/k_B T$$

$[\ldots]_{\rm av}$: Average over **quenched**, random disorder.

- "Edwards-Anderson" (EA) susceptibility:

$$\chi = \lim_{V\to\infty} \frac{1}{V} \sum_{i,j=1}^{V} \left[\left\langle \frac{1}{(p-1)} \left(p\delta_{S_i S_j} - 1\right)\right\rangle_T^2\right]_{\rm av}$$

- **Goal:** Compute and analyze high-temperature series expansions in $K = J\beta$ for the free energy and susceptibility for **general** p **and** d,

$$\beta F = 1 + a_8 K^8 + a_{10} K^{10} + a_{12} K^{12} + \ldots + a_{22} K^{22} + \ldots$$

$$\chi = 1 + b_2 K^2 + b_4 K^4 + b_6 K^6 + \ldots + b_{22} K^{22}$$

where $a_i = a_i(p,d)$, $b_i = b_i(p,d)$.

High-Temperature Series Expansion

- Star graph expansion: Star graphs are multiply connected graphs without articulation points (Fig. 1). For Potts models only star graphs yield non-vanishing contributions to the expansion of the free energy and (inverse) susceptibility [2].

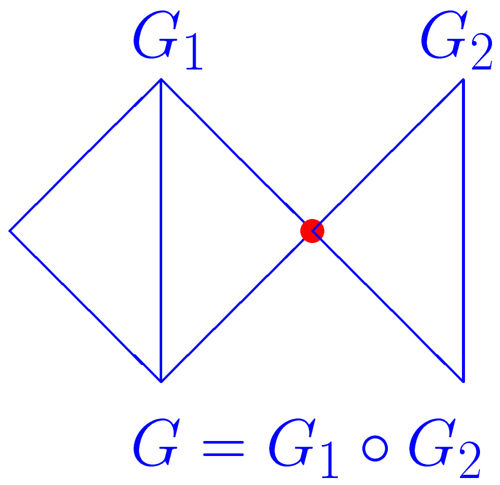

Figure 1: A graph G with articulation point (•). By cutting at the articulation point, G can be decomposed into the two star graphs G_1 and G_2.

- Inversion test [3]: Solve

$$1/\chi(K_c) = 0$$

recursively with the ansatz

$$K^2 = K_c^2 = (p^2/2d)\left(1 + \sum_{i=1} c_i \left(\frac{1}{2d}\right)^i\right)$$

$\longrightarrow$ $(1/d)$-expansion for K_c^2 (Fig. 2) – if χ expansion is correct.

Series Analysis Techniques

- Dlog-Padé: If

$$F(z) \sim A(z_c - z)^{-\lambda} \doteq \sum a_n z^n$$

then

$$\frac{d}{dz} \ln F(z) = \sum a_n^* z^n \sim \frac{\lambda}{z_c - z}$$

Compute $[L/M]$ Padé-Approximant [4]

$$[L/M] \equiv \frac{P_L(z)}{Q_M(z)} \equiv \frac{p_0 + p_1 z + p_2 z^2 + \ldots + p_L z^L}{q_0 + q_1 z + q_2 z^2 + \ldots + q_M z^M}$$

where the coefficients are chosen such that the Taylor expansions agree in the first $L + M + 1$ terms.

- Correction terms: If

$$F(z) = A(z_c - z)^{-\lambda} \left[1 + A_1(z_c - z)^{\Delta_1} + A_2(z_c - z) + \ldots\right]$$

use methods

  - M1/M2 [5]
  - Differential Padé approximants (DPA) [6]

$\longrightarrow$ Comparison in Fig. 3.

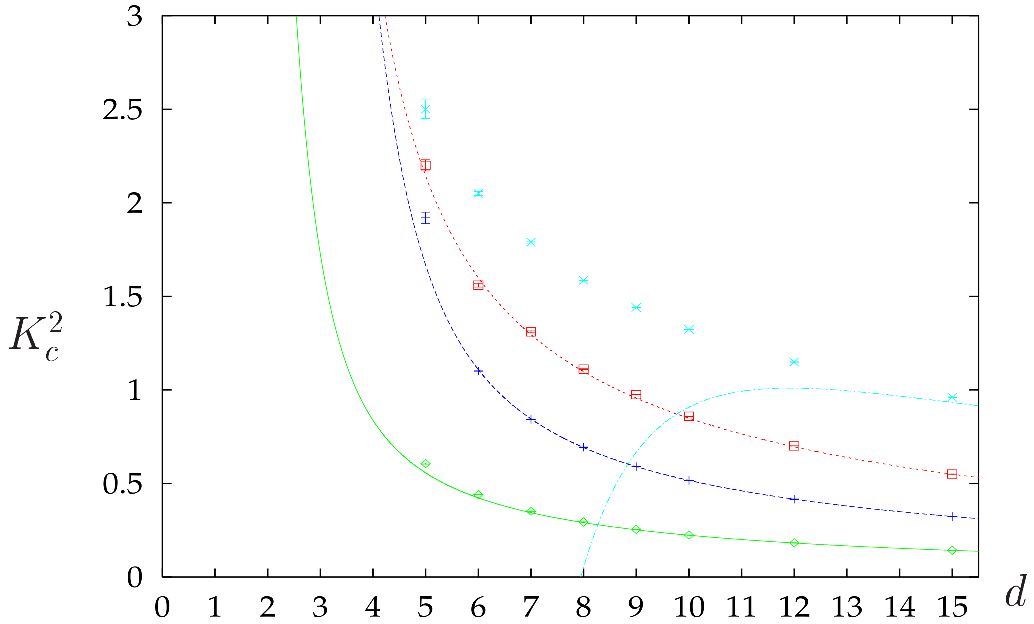

Figure 2: Comparison of the large-d expansions for K_c^2 (dashed lines) with directly obtained estimates: green: $p = 2$; blue: $p = 3$; red: $p = 4$; turquois: $p = 6$.

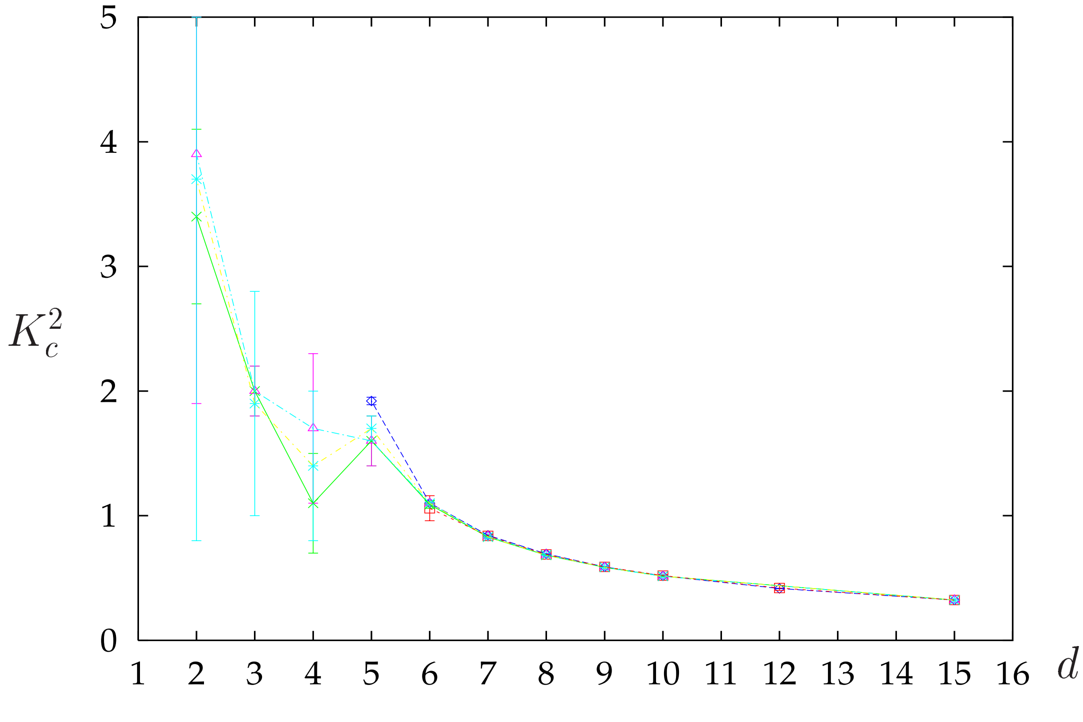

Figure 3: Comparison of the analysis methods for $p = 3$: blue: M1/M2; red: Dlog-Padé; green: DPA1; turquoise: DPA2; yellow: DPA3.

Results and Conclusions [7]

- Consistent $1/d$-expansion for K_c^2 (Fig. 2).
- Susceptibility exponent $\gamma \geq 1$ for $p = 2$, 3, and 4, and $\gamma < 1$ for $p = 5$, 6, and 8 (Fig. 4). From a comparative study of the pure ferromagnetic model we conclude that $\gamma < 1$ should be interpreted as an *effective* exponent, signalizing the metastability boundary of a *first-order* phase transition.
- Rough estimates of the lower critical dimension d_l from fits to $1/K_c^2 = a_0 + a_1 d + a_2 d^2 + a_3 d^3$ (Fig. 5): $d_l \approx 2.28$ ($p = 2$); $d_l \approx 3.20$ ($p = 3$); $d_l \approx 1.81$ ($p = 4$). But series are still too short to warrant a more detailed investigation.
- The only safe statement about the upper critical dimension d_u (where $\gamma = \gamma_{\mathrm{MF}} = 1$) is that $d_u \leq 15$. In Fig. 4 we do not see any decisive difference between the γ-value at $d = 6$ (the commonly accepted value of d_u), or $d = 8$ (the value of d_u tentatively supported in Ref. [8]), or even $d = 10$.
- With the present data for $p = 3$, the claim that $d = 8$ is a special dimension is not justified at all (Fig. 6).

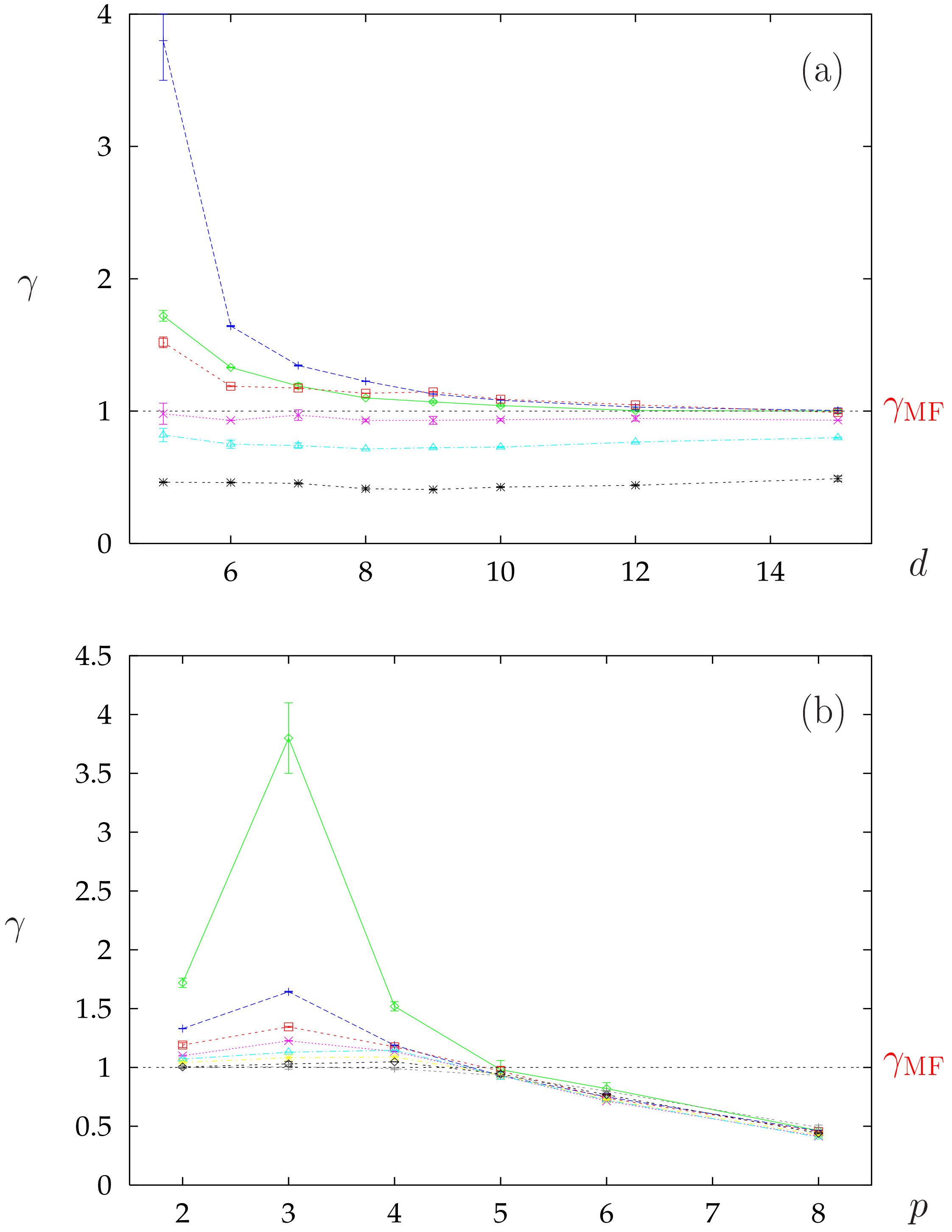

Figure 4: The critical exponent γ of the susceptibility χ (a) as a function of the dimension d: green: $p = 2$; blue: $p = 3$; red: $p = 4$; rosé: $p = 5$; turquois: $p = 6$; black: $p = 8$;
(b) as a function of the number of Potts states p: green: $d = 5$; blue: $d = 6$; red: $d = 7$; rosé: $d = 8$; turquois: $d = 9$; yellow: $d = 10$; black: $d = 12$; grey: $d = 15$.

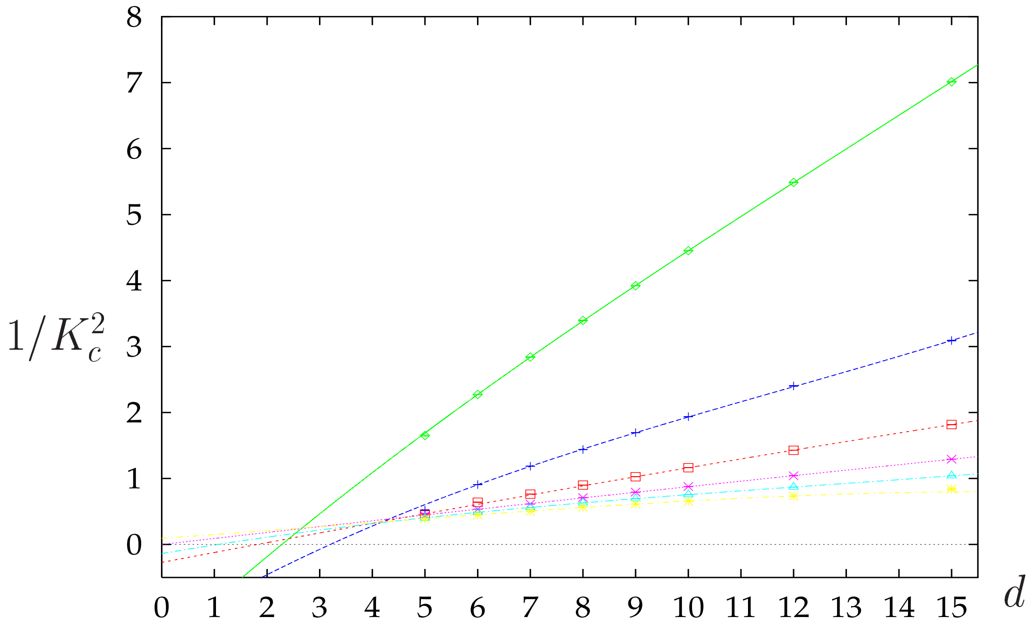

Figure 5: Least-squares fits of the inverse critical couplings to the ansatz $1/K_c^2 = a_0+a_1d+a_2d^2+a_3d^3$: green: $p = 2$; blue: $p = 3$; red: $p = 4$; rosé: $p = 5$; turquois: $p = 6$; yellow: $p = 8$.

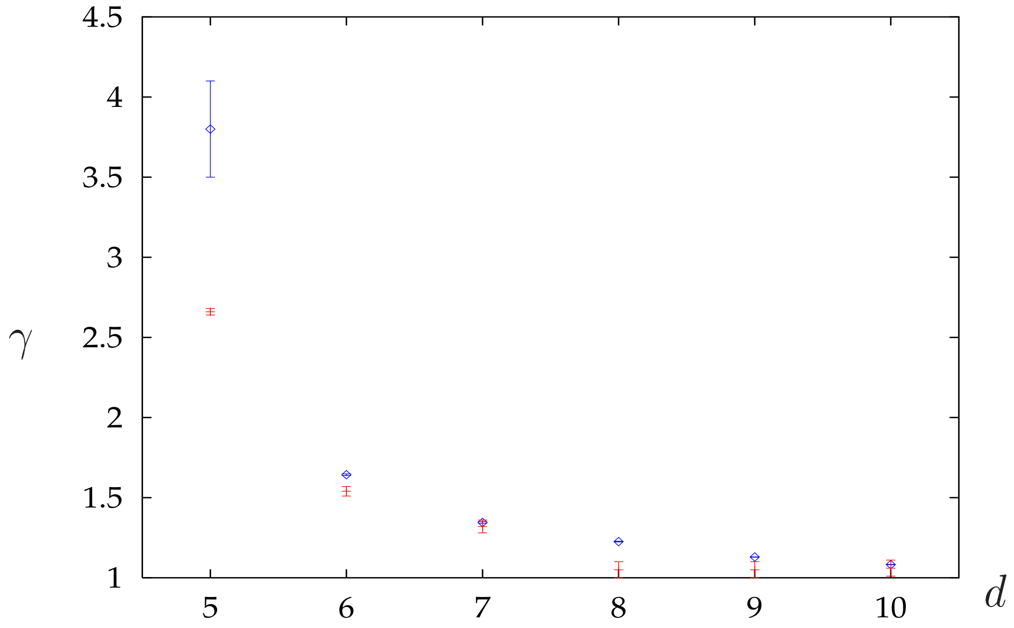

Figure 6: Critical exponents γ for $p = 3$. Comparison of the present results (blue) with previous estimates of Ref. [8] (red).

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