Prof. Dr. Wolfhard Janke SS 2010

# TP4 – Statistical Physics I
# Homework 11

Due: Monday, 28 June 2010

31. **Volume and surface of a sphere in $d$ dimensions**

    A $d$-dimensional sphere of radius $r$ is defined by

    $$x_1^2 + x_2^2 + \ldots + x_d^2 = r^2 \,,$$

    where $x_i$, $i = 1, \ldots, d$, are the Cartesian coordinates.

    a) Calculate the volume and surface of this sphere.
    b) Determine the layer thickness $\Delta r$, for which the volume of the surface layer equals the volume of the (inner) sphere. Consider in particular the limiting case $d \to \infty$.

    *Hint:* Consider the integral

    $$I_d = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} dx_1 dx_2 \ldots dx_d \, e^{-(x_1^2 + x_2^2 + \ldots + x_d^2)} \,.$$

32. **Linear harmonic oscillator in the microcanonical ensemble**

    Consider the classical linear harmonic oscillator

    $$H(q,p) = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2 \,.$$

    a) Determine the normalized density distribution function $\rho(q,p)$ of the microcanonical ensemble.
    b) Calculate the mean values of the potential and the kinetic energy.

33. **Random walks in the computer**

Write a computer program for the numerical generation of random walks. The start position is $x = 0$, the probability for a unit step to the right is $p$ (and to the left thus $1 - p$) and the total number of steps is $N$. The number of steps to the right is denoted by $N_R$ and the number of steps to the left by $N_L$; the final position of the "drunken walker" is thus given by $x = N_R - N_L$.

a) Plot 20 random walks with $N = 10$, 20, and 100 (the current position as function of the steps). In principle, a single plot would be sufficient. Why?

b) Check for $N = 10$, 20, and $p = 0.5$, 0.7, that the values of $N_R$ are distributed according to the binomial distribution. To this end, about 100 000 random walks should be generated (which takes a few seconds of computing time on a typical PC).

c) Study the limiting cases $p \ll 1$ and $N_R \ll N$, and compare with the approximation by a Poisson distribution of problem 22b); see also problem 23.

d) Measure the histogram $\hat{P}(x)$ of the final positions $x$ after $N$ steps and compare with a (suitably normalized) Gaussian approximation $G(x)$. Generate again about 100 000 random walks. Besides the direct comparison of $\hat{P}(x)$ with $G(x)$, to assess the accuracy of this approximation, it is useful to plot the ratio $\hat{P}(x)/G(x)$. Consider the cases $N = 10$, 20, and 100, and $p = 0.3$, 0.5, 0.7, and 0.9.

e) Measure the probability $\tilde{P}(N, s)$ to arrive after $N$ steps for the first time at $x = s$. Compare your numerical data with the analytical result of the lecture.

f) "Play" with your computer program and think about further questions and comparisons with analytical results, i.e., start your own original research into this problem ...