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Monte Carlo study of 8-state Potts model on 2D random lattices *

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We study the effect of quenched coordination-number disorder of random lattices on the nature of the phase transition in the two-dimensional eight-state Potts model, which is of first order on regular lattices. We consider Poissonian random lattices of toroidal topology constructed according to the Voronoi/Delaunay prescription. Monte Carlo simulations yield strong evidence that the phase transition remains first order.

1. INTRODUCTION

Pure systems exhibiting a continuous phase transition are very susceptible to the addition of random disorder. The critical behaviour can be driven to new universality classes or the phase transition can be eliminated altogether [1]. Also for first-order phase transitions phenomenological renormalization-group arguments suggest strong effects caused by random disorder [2]. In particular the order of the transition can change from first to second.

The well-known paradigm to investigate such effects is the two-dimensional q-state Potts model which undergoes on regular lattices for $q \geq 5$ a temperature driven first-order phase transition [3]. Monte Carlo (MC) simulations for $q = 8$ with a certain type of quenched bond-disorder provided clear evidence for a continuous phase transition of the Ising type [4]. Also in two-dimensional quantum gravity studies of Potts "matter" coupled to dynamically triangulated random surfaces (DTRS), a similar softening effect was observed [5]. From a statistical mechanics viewpoint, in this case the Potts model is subject to annealed disorder in the local coordination numbers of the dynamical triangulation.

Here we report on a study [6] of the same model on static Poissonian random lattices constructed according to the Voronoi/Delaunay prescription [7]. The locally varying coordination numbers cause the disorder similar to Ref. [5], but in our case the disorder is assumed to be frozen in, i.e. "quenched", as in Ref. [4].

2. MODEL AND SIMULATION

The 8-state Potts model is defined by the partition function

$$Z = \sum_{\{\sigma_i\}} e^{-\beta E}; E = -\sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}; \sigma_i = 1, \ldots, q, \quad (1)$$

with $q = 8$. The σ_i are integer valued spins at the lattice sites i, $\delta_{\sigma_i \sigma_j}$ denotes the usual Kronecker delta symbol, and the nearest-neighbor bonds $\langle ij \rangle$ are determined by the Voronoi/Delaunay construction of the random lattices. We always used periodic boundary conditions, i.e., toroidal topology as depicted in Fig. 1.

Using a standard algorithm [8] we generated 20 independent replica of random lattices with $V = 250$, 500, 750, 1000, 2000, and 3000 sites and performed long single-cluster simulations near the transition point at $\hat{\beta} = 0.826$, 0.830, 0.830, 0.830, 0.832, and 0.833, respectively. After equilibration we recorded 1 000 000 measurements (taken after 1, 1, 1, 1, 2, 4 clusters had been flipped) of the energy E and the magnetization $M = (q \max\{n_i\} - V)/(q-1)$ in a time-series file, where

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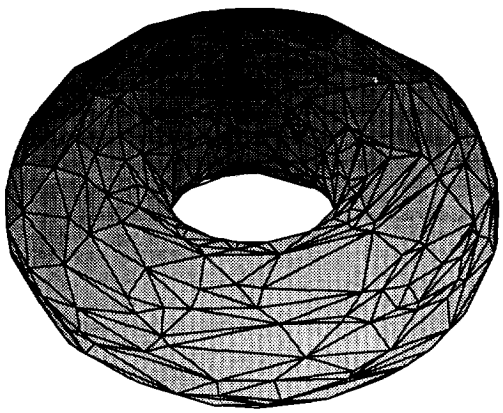

Figure 1. Random lattice with toroidal topology.

$n_i \leq V$ denotes the number of spins of "orientation" $i = 1, \ldots, q$ in one lattice configuration. The corresponding quantities per site will be denoted by $e = E/V$ and $m = M/V$.

We then applied the reweighting method to compute, e.g., the specific heat $C^{(i)}(\beta) = \beta^2 V \left(\langle e^2 \rangle - \langle e \rangle^2\right)$ for each replica labeled by the superindex (i), performed the replica average $C(\beta) = [C^{(i)}(\beta)] \equiv (1/20) \sum_i^{20} C^{(i)}(\beta)$, and finally determined the maximum, $C_{\max} = C(\beta_{C_{\max}})$. For the magnetic susceptibility, $\chi(\beta) = \beta V \left([\langle m^2 \rangle - \langle m \rangle^2]\right)$ we followed exactly the same lines.

The proper replica average for the specific heat and susceptibility follows from the general rule that in the quenched case the free energy (and its derivatives) should be averaged [9]. For the (energetic) Binder parameter, usually defined for pure systems as $B(\beta) = 1 - \langle e^4 \rangle / 3 \langle e^2 \rangle^2$, the proper replica average is less clear to us. We have therefore studied three different definitions: $B_1(\beta) = 1 - \left[\langle e^4 \rangle / 3 \langle e^2 \rangle^2\right]$, $B_2(\beta) = 1 - \left[\langle e^4 \rangle\right] / 3 \left[\langle e^2 \rangle^2\right]$, and $B_3(\beta) = 1 - \left[\langle e^4 \rangle\right] / 3 \left[\langle e^2 \rangle\right]^2$. While in spin glass simulations [10] usually the analog of B_3 (with e replaced by the overlap) is used, for a random bond Ising chain [11] a better scaling behaviour was observed for the analog of B_1 (with e replaced by m).

3. RESULTS

Our estimates of the extrema of C, χ, and B_1 for the various lattice sizes are collected in Table 1. The error bars are estimated by jackkniving over the 20 replica. This takes into account both the statistical errors on each $C^{(i)}(\beta)$ and the fluctuations among the different replica. Already a first qualitative inspection of the data indicates that the first-order nature of the phase transition persists on quenched random lattices.

To make this statement more precise we performed a finite-size scaling (FSS) analysis. Assuming a first-order phase transition, we expect for large system sizes an asymptotic FSS behaviour of the form [12–14]

$$C_{\max} = a_C + b_C V + \ldots, \tag{2}$$

$$\chi_{\max} = a_\chi + b_\chi V + \ldots, \tag{3}$$

$$B_{i,\min} = a_{B_i} + b_{B_i}/V + \ldots, \tag{4}$$

and

$$\beta_{C_{\max}} = \beta_t + c_C/V + \ldots, \tag{5}$$

etc., where β_t is the infinite volume transition point. The data for $C_{\max}$ and $\chi_{\max}$ shown in Fig. 2 are clearly consistent with this assumption. From least-square fits we obtained $a_C = 23.3(2.0), b_C = 0.0659(30)$, with a goodness-of-fit parameter $Q = 0.16$ (corresponding to a chi-square per degree of freedom of 1.7), and $a_\chi = -0.70(43), b_\chi = 0.0629(13)$, with $Q = 0.45$.

Also the data for the Binder parameter minima confirms the hypothesis of a first-order phase transition. Here the least-square fits gave $a_{B_1} = 0.6240(20)$, $b_{B_1} = -18.8(1.4)$, $Q = 0.17$, $a_{B_2} = 0.6236(22)$, $b_{B_2} = -18.5(1.4)$, $Q = 0.47$, and $a_{B_3} = 0.61125(68)$, $b_{B_3} = -16.45(71)$, $Q = 0.55$. Notice the much higher accuracy of B_3.

Our data for the pseudo-transition points and the corresponding fits through all data points are shown in Fig. 3. The resulting estimates for β_t are 0.83360(14) from $C_{\max}$ ($Q = 0.51$), 0.83365(14) from $\chi_{\max}$ ($Q = 0.47$), and 0.83371(14) from $B_{1,\min}$ ($Q = 0.40$). On the scale of Fig. 3 the data points for $B_{2,\min}$ and $B_{3,\min}$ could hardly be disentangled from $B_{1,\min}$ and are therefore

Table 1
Extrema of the specific heat ($C_{\max}$), the susceptibility ($\chi_{\max}$), and the Binder parameter ($B_{1,\min}$), together with the respective pseudo-critical couplings.

V	$\beta_{C_{\max}}$	$C_{\max}$	$\beta_{\chi_{\max}}$	$\chi_{\max}$	$\beta_{B_{1,\min}}$	$B_{1,\min}$
250	0.82500(44)	33.15(45)	0.82404(46)	14.96(20)	0.81872(48)	0.5662(11)
500	0.82946(35)	55.51(93)	0.82907(34)	31.09(56)	0.82655(34)	0.5875(13)
750	0.83087(23)	76.1(2.0)	0.83065(24)	47.7(1.3)	0.82901(24)	0.5960(18)
1000	0.83112(31)	90.4(2.6)	0.83095(31)	61.0(1.8)	0.82972(32)	0.6044(17)
2000	0.83232(22)	144.8(9.0)	0.83225(21)	114.8(7.7)	0.83164(21)	0.6180(31)
3000	0.83300(16)	216(11)	0.83297(16)	185.1(9.9)	0.83257(16)	0.6190(25)

omitted. The results for β_t are 0.83350(13) from $B_{2,\min}$ ($Q = 0.25$), and 0.83362(13) from $B_{3,\min}$ ($Q = 0.23$). By taking the average of these estimates we finally obtain

$$\beta_t = 0.83362 \pm 0.00013. \qquad (6)$$

Notice that this value is very close to the exactly known transition point of the 8-state Potts model on a triangular lattice ($\beta_t^{\text{triang.}} = 0.85666\ldots$) [3].

Finally we show in Fig. 3 the "ratio-of-weights" definition of pseudo-transition points, β_W, which are expected to approach β_t exponentially fast with increasing lattice size [15]. Basically the idea is to reweight the energy histograms to a point β_W where the weights of the ordered and disordered phase are in a ratio $q : 1$. As in earlier studies for regular square lattices [14,15], we find also here that the β_W are quite accurate estimates of β_t already for very small system sizes.

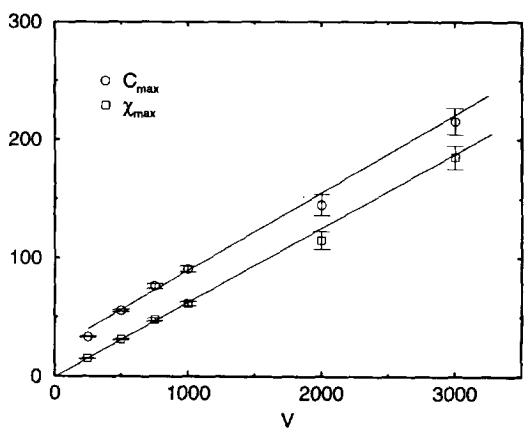

Figure 2. FSS of specific-heat and susceptibility maxima.

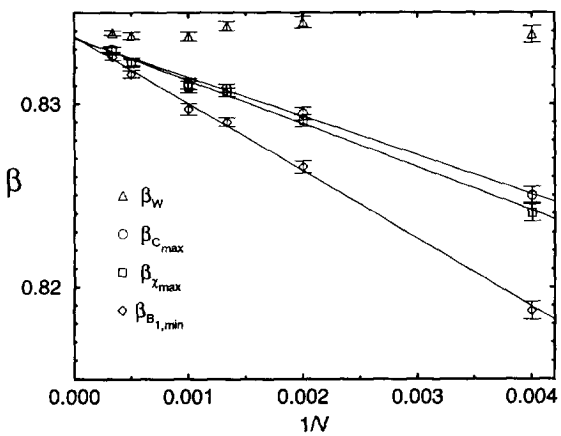

Figure 3. FSS of pseudo-transition points.

4. CONCLUSIONS

Summarizing, we have obtained clear numerical evidence for a first-order phase transition in the 8-state Potts model on quenched random lattices of Voronoi/Delaunay type. We can savely exclude a cross-over to a continuous transition as was observed for a certain type of quenched bond disorder on square lattices [4] and for the annealed disorder of dynamically triangulated surfaces [5].

This conclusion is based on the FSS behaviour of standard thermodynamic observables. We are currently extending the analysis to quantities that are directly related to the probability distributions of the energy or magnetization, such as

the interface tension and the briefly mentioned "ratio-of-weights" definition of pseudo-transition points. Details of this study, which is based on a much larger set of 128 replica, will be presented elsewhere [16].

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