

On the continuum limit of the Discrete Regge model in 4d

Elmar Bittner^{a,b,*}, Wolfhard Janke^{b,†} and Harald Markum^a

^aAtominstitut der Österreichischen Universitäten, A-1040 Vienna, Austria

^bInstitut für Theoretische Physik, Universität Leipzig, D-04109 Leipzig, Germany

The Regge Calculus approximates a continuous manifold by a simplicial lattice, keeping the connectivities of the underlying lattice fixed and taking the edge lengths as degrees of freedom. The Discrete Regge model employed in this work limits the choice of the link lengths to a finite number. This makes the computational evaluation of the path integral much faster. A main concern in lattice field theories is the existence of a continuum limit which requires the existence of a continuous phase transition. The recently conjectured second-order transition of the four-dimensional Regge skeleton at negative gravity coupling could be such a candidate. We examine this regime with Monte Carlo simulations and critically discuss its behavior.

1. REGGE QUANTUM GRAVITY

One promising method to quantize the theory of gravitation employs the Euclidean path integral

$$Z = \int D[q] e^{-I(q)} , \quad (1)$$

where the partition function describes a fluctuating space-time manifold. In the Regge approach the (quadratic) link lengths q_l represent the dynamical degrees of freedom [1], deforming continuously a simplicial lattice with fixed incidence matrix, whereas in the somehow complementary approach of dynamical triangulation the incidence matrix is fluctuating with constant link lengths. Any of these two approaches is plagued with various problems; for the Regge approach see [2,3].

In “conventional” Regge theory the Regge-Einstein action including a cosmological term [4],

$$I_r = -\beta \sum_t A_t \delta_t + \lambda \sum_s V_s , \quad (2)$$

is used. The first sum runs over all products of triangle area A_t times corresponding deficit angle δ_t weighted by the bare gravitational coupling β .

*Poster presented by E. B. and supported by Hochschuljubiläumsstiftung der Stadt Wien.

†W. J. acknowledges partial support by the EC IHP Network grant HPRN-CT-1999-00161: “EUROGRID”.

The second sum extends over the volumes V_s of the 4-simplices of the lattice and allows together with the cosmological constant λ to set an overall scale in the action.

The Discrete Regge model was invented in an attempt to reformulate (1) as the partition function of a spin system [5,6]. It is defined by restricting the squared link lengths to take on only two values

$$q_l = b_l(1 + \epsilon \sigma_l) , \quad \sigma_l \in Z_2 , \quad (3)$$

where $b_l = 1, 2, 3$, and 4 for edges, face diagonals, body diagonals and the hyperbody diagonal of a hypercube, respectively, is chosen to allow for fluctuations around flat space. The Euclidean triangle inequalities are fulfilled automatically as long as ϵ is smaller than a maximum value ϵ_{\max} . The measure $D[q]$ in the quantum gravity path integral is taken to unity for all possible link configurations.

Numerical simulations of the Z_2 system become extremely efficient by implementing look-up tables and a heat-bath algorithm. In this work typically 200 000 – 500 000 iterations have been generated. Calculations have been performed with the parameter $\epsilon = 0.0875$ and the cosmological constant $\lambda = 0$ because (3) already fixes the average lattice volume.

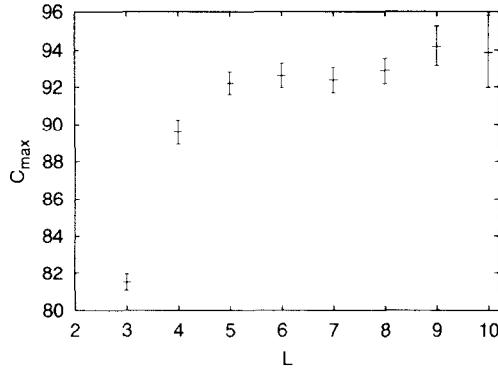


Figure 1. FSS of the specific-heat maxima C_{\max} as a function of the lattice size L .

2. RESULTS

The Discrete Regge model – like full Regge theory – exhibits two phase transitions [7]. One is located at a negative value and the other one at a positive value of the bare gravitational coupling. Although earlier work concentrated on the latter transition, there is no reason for favoring a positive value of β over a negative one. Since a Wick rotation from the Lorentzian to the Euclidean sector of quantum gravity is not feasible in general, the sign in the exponential of the path integral is not fixed a priori.

The transition for positive gravitational coupling has been observed before the one at negative β [8]. There, histograms, e.g. of $A_t \delta_t$ [7], clearly show a two-peak structure. The two phases coexist and tunneling from one phase to the other and back takes place. The system also exhibits a hysteresis; the transition occurs at a larger value of β if the simulation is started from a configuration in the well-defined phase than it does if the calculation is started from a “frozen” configuration. Given all these pieces of evidence, we conjecture that the transition at $\beta > 0$ is of first order.

To determine the order of the transition at negative β we employed in Ref. [7] histogram techniques and used the Binder–Challa–Landau (BCL) cumulant criterion [9]. The BCL cumulant is defined as $B_L := 1 - \frac{\langle E^4 \rangle}{3 \langle E^2 \rangle^2}$, with E being the action of the system under consideration. It was evaluated for $A_t \delta_t$ on different lattice sizes

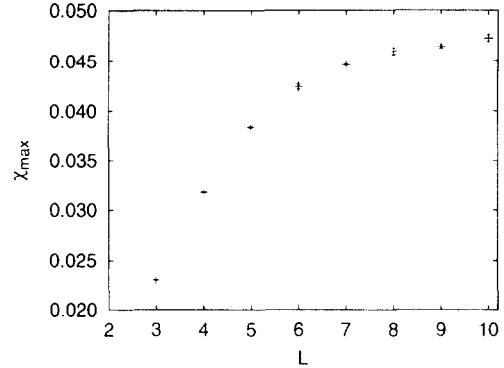


Figure 2. FSS of the susceptibility maxima χ_{\max} , indicative for a cross-over.

with $L = 3$ to 10 vertices per direction, simulating the system at several values of the bare coupling β with high statistics. For the BCL cumulant a trend towards $2/3$ was observed and all histograms showed a clear one-peak structure, cf. [7].

In the present Monte Carlo simulations we recorded for every run the time series of the energy density $e = E/N_0$ and the magnetization density $m = \sum_l \sigma_l/N_0$, with the lattice size $N_0 = L^4$. To obtain results for the various observables \mathcal{O} at values of the bare gravitational coupling β in an interval around the simulation point β_0 , we applied the reweighting method [10].

With the help of the time series we can compute the specific heat,

$$C(\beta) = \beta^2 N_0 (\langle e^2 \rangle - \langle e \rangle^2), \quad (4)$$

and the (finite lattice) susceptibility,

$$\chi(\beta) = N_0 (\langle m^2 \rangle - \langle |m| \rangle^2). \quad (5)$$

Figures 1 and 2 show the finite-size scaling (FSS) of the maxima of the specific heat C_{\max} and the susceptibility χ_{\max} , respectively. While the behavior of C_{\max} could still be explained as critical scaling with a negative exponent α , the flattening of χ_{\max} is rather unusual for a second-order transition and should be taken as an indication for a cross-over regime.

Another feature of the system can be seen in Figs. 3 and 4 depicting the critical gravitational coupling, as determined from the specific-heat maxima C_{\max} and the susceptibility maxima

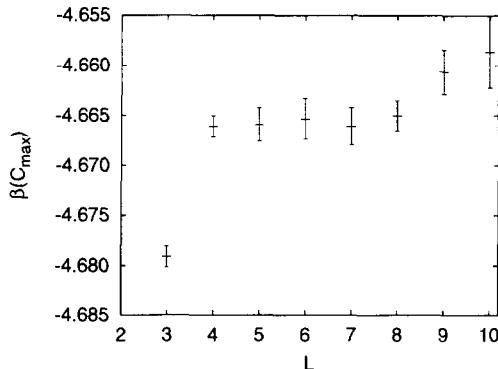


Figure 3. Volume dependence of the critical gravitational coupling $\beta(C_{\max})$, as determined from the specific-heat maxima C_{\max} in Fig. 1.

χ_{\max} . In the case of a second-order transition the extrapolations of all pseudo-transition points lead to one infinite-volume critical value. This seems to be violated in the four-dimensional Discrete Regge model and thus favors the interpretation as a cross-over phenomenon over a true, thermodynamically defined phase transition.

3. CONCLUSION

In the present analysis of the scaling of the maxima of the specific heat and susceptibility we found evidence for a cross-over regime in the four-dimensional Discrete Regge model of quantum gravity in the negative coupling region. If this can be substantiated by further investigations and also for the Regge theory with continuous link lengths, the existence of a continuum limit at negative bare gravitational coupling is questionable. This is of major concern for a continuum theory of quantum gravity with matter fields [11].

REFERENCES

1. J. Hartle, J. Math. Phys. 26 (1985) 804; 27 (1986) 287; 30 (1989) 452.
2. H.W. Hamber, Phys. Rev. D45 (1992) 507; Nucl. Phys. B400 (1993) 347.
3. W. Beirl, E. Gerstenmayer, H. Markum and J. Riedler, Phys. Rev. D49 (1994) 5231.

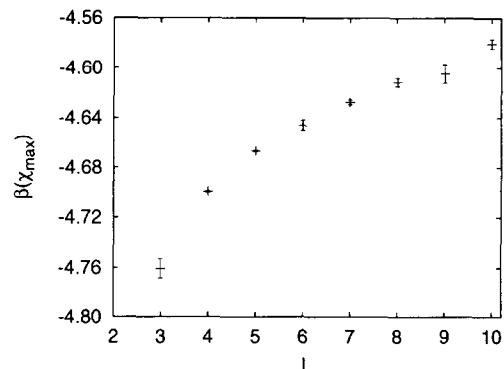


Figure 4. Volume dependence of the critical gravitational coupling $\beta(\chi_{\max})$, as determined from the susceptibility maxima χ_{\max} in Fig. 2.

4. T. Regge, Nuovo Cimento 19 (1961) 558.
5. W. Beirl, H. Markum and J. Riedler, Int. J. Mod. Phys. C5 (1994) 359; W. Beirl, P. Homolka, B. Krishnan, H. Markum and J. Riedler, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 710.
6. T. Fleming, M. Gross and R. Renken, Phys. Rev. D50 (1994) 7363.
7. W. Beirl, A. Hauke, P. Homolka, B. Krishnan, H. Markum and J. Riedler, Nucl. Phys. B (Proc. Suppl.) 47 (1996) 625; J. Riedler, W. Beirl, E. Bittner, A. Hauke, P. Homolka and H. Markum, Class. Quant. Grav. 16 (1999) 1163.
8. H.W. Hamber in: *Proceedings of the 1984 Les Houches Summer School, Session XLIII*, edited by K. Osterwalder and R. Stora (North Holland, Amsterdam, 1986); H.W. Hamber and R.M. Williams, Phys. Lett. B157 (1985) 368; H.W. Hamber, Phys. Rev. D61 (2000) 124008.
9. K. Binder and D.P. Landau, Phys. Rev. B30 (1984) 1477; M.S.S. Challa, D.P. Landau and K. Binder, Phys. Rev. B34 (1986) 1841.
10. A.M. Ferrenberg and R.H. Swendsen, Phys. Rev. Lett. 61 (1988) 2635; 63 (1989) 1195; 1658.
11. E. Bittner, W. Janke and H. Markum, Phys. Rev. D66 (2002) 024008; Acta Physica Slovaca 52 (2002) 241.