Renormalization of second moment of GPD in lattice perturbation theory

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in collaboration with

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- Experimental access: DVCS (e.g. $ep \longrightarrow ep\gamma$), meson pair production (e.g. $ep \longrightarrow ep\pi^+\pi^-$), first data from e.g. HERMES show some evidence

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- Experimental access: DVCS (e.g. $ep \longrightarrow ep\gamma$), meson pair production (e.g. $ep \longrightarrow ep\pi^+\pi^-$), first data from e.g. HERMES show some evidence
- Moments of GPD: operators like for SF - sandwiched as non-forward matrix elements:

$$
\begin{aligned}
\langle p_2 | \mathcal{O}_q^{\mu_1 \cdots \mu_n} | p_1 \rangle \;=\;& \overline{u}(p_2)\gamma^{\mu_1} u(p_1) \\
& \times \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{qn,2i}(t) \Delta^{\mu_2} \ldots \Delta^{\mu_{2i+1}} \overline{p}^{\mu_{2i+2}} \ldots \overline{p}^{\mu_n} \\
& + \overline{u}(p_2) \frac{\sigma^{\mu_1 \alpha} i \Delta_\alpha}{2M} u(p_1) \\
& \times \sum_{i=0}^{\left[\frac{n-1}{2}\right]} B_{qn,2i}(t) \Delta^{\mu_2} \ldots \Delta^{\mu_{2i+1}} \overline{p}^{\mu_{2i+2}} \ldots \overline{p}^{\mu_n} \\
& + C_{qn}(t) \mathrm{Mod}(n+1, 2) \frac{1}{M} \overline{u}(p_2) u(p_1) \Delta^{\mu_1} \ldots \Delta^{\mu_n}
\end{aligned}
$$

with $\Delta = p_1 - p_2$, $\overline{p} = \frac{p_1 + p_2}{2}$ and $t = \Delta^2$.

There are four GPDs:

- Spin independent: $E_q(x,\xi,t), H_q(x,\xi,t)$
- Spin dependent: $\tilde{E}_q(x,\xi,t), \tilde{H}_q(x,\xi,t)$

Their moments are defined as

$$\begin{aligned}
\int_{-1}^{1} dx\, x^{n-1} E_q(x,\xi,t) &= E_{qn}(\xi,t) \\
\int_{-1}^{1} dx\, x^{n-1} H_q(x,\xi,t) &= H_{qn}(\xi,t)
\end{aligned}$$

with $\xi = -n\cdot\Delta$, $n\cdot\overline{p} = 1$

Connection to operator matrix elements:

$$
\begin{aligned}
H_{qn}(\xi,t) &= \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{qn,2i}(t)(-2\xi)^{2i} \\
&\quad + \mathrm{Mod}(n+1,2) C_{qn}(t)(-2\xi)^{2n} \\
E_{qn}(\xi,t) &= \sum_{i=0}^{\left[\frac{n-1}{2}\right]} B_{qn,2i}(t)(-2\xi)^{2i} \\
&\quad - \mathrm{Mod}(n+1,2) C_{qn}(t)(-2\xi)^{2n}
\end{aligned}
$$

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H_{qn}(\xi,t) &= \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{qn,2i}(t)(-2\xi)^{2i} \\
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&\quad - \mathrm{Mod}(n+1,2) C_{qn}(t)(-2\xi)^{2n}
\end{aligned}
$$

Sum rule (Ji) (total angular momentum carried by the quarks):

$$
\begin{aligned}
\langle J_q^3 \rangle &= \frac{1}{2}\big(A_q(0) + B_q(0)\big) \\
A_q(t) + B_q(t) &= \int_{-1}^{1} dx\, x\, \big(H_q(x,\xi,t) + E_q(x,\xi,t)\big) \\
&= H_{q2}(\xi,t) + E_{q2}(\xi,t)
\end{aligned}
$$

- As in the case of SF the question of relating lattice results to continuum data plays an important role $\longrightarrow$ renormalization factors (Z-factors)

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  - Computationally rather complicated (concerning clear signals)
  - Some aspects (explicit dependence on the lattice spacing a, mixing, ...) can be studied in lattice perturbation theory rather naturally

- As in the case of SF the question of relating lattice results to continuum data plays an important role $\longrightarrow$ renormalization factors (Z-factors)
- The consistent way would be a non-perturbative determination of the Z-factors. But
  - Computationally rather complicated (concerning clear signals)
  - Some aspects (explicit dependence on the lattice spacing a, mixing, ...) can be studied in lattice perturbation theory rather naturally
- In previous lattice calculations of GPDs perturbative Z-factors have been overtaken from SF. This is no problem for the first moment. However, for the second moment the correct mixing should be taken into account! It could (and will !!!) differ from the SF case.

- In this talk we present first results for the Z-factors of the second moment of GPDs for Wilson fermions in Feynman gauge

Second moment in lattice perturbation theory

We consider matrix elements of the following operators

$$\mathcal{O}^{DD}_{\mu\nu\omega} = \overline{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega \psi - \text{trace} \tag{1}$$

$$\mathcal{O}^{DD,5}_{\mu\nu\omega} = \overline{\psi}\gamma_\mu\gamma_5 \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega \psi - \text{trace} \tag{2}$$

$$\mathcal{O}^{\partial\partial}_{\mu\nu\omega} = \partial_\nu\partial_\omega \left(\overline{\psi}\gamma_\mu\psi\right) - \text{trace} \tag{3}$$

$$\mathcal{O}^{\partial\partial,5}_{\mu\nu\omega} = \partial_\nu\partial_\omega \left(\overline{\psi}\gamma_\mu\gamma_5\psi\right) - \text{trace} \tag{4}$$

$$\mathcal{O}^{\partial D}_{\mu\nu\omega} = \partial_\nu \left(\overline{\psi}\gamma_\mu \overleftrightarrow{D}_\omega \psi\right) - \text{trace} \tag{5}$$

$$\mathcal{O}^{\partial D,5}_{\mu\nu\omega} = \partial_\nu \left(\overline{\psi}\gamma_\mu\gamma_5 \overleftrightarrow{D}_\omega \psi\right) - \text{trace} \tag{6}$$

$$\overline{\mathcal{O}}_{\mu\nu\omega} = \partial_\nu \left(\overline{\psi}\left[\gamma_\mu, \gamma_\omega\right] \psi\right) - \text{trace} \tag{7}$$

$$\overline{\mathcal{O}}^{5}_{\mu\nu\omega} = \overline{\psi}\left[\gamma_\mu, \gamma_\nu\right] \overleftrightarrow{D}_\omega \psi - \text{trace} \tag{8}$$

The mixing problem for form factors has been studied by Shifman and Vysotsky in 1981 (NP B186 (1981)). They derived mixing matrices on the level of anomalous dimensions between operators (1) $\leftrightarrow$ (3) and (2) $\leftrightarrow$ (4).

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Computation:

Calculation is performed in symbolic terms completely.

We have enhanced our Mathematica package for SF (Göckeler et al. (NP B472 (1996)))

- **Feynman rules**

  – Standard realization of derivatives (I): For the case of two covariant derivatives we have in momentum space

$$\mathcal{O}^{DD}_{\mu\nu\omega}(q) \;=\; -\frac{1}{4}\sum_x \left(\bar\psi\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega \,\psi\right)(x)\,\exp(-iq\cdot x)$$

with

$$\begin{aligned}
\overrightarrow{D}_\mu\,\psi(x) &= \frac{1}{2a}\Big[U_{x,\mu}\,\psi(x+a\hat\mu) - U^\dagger_{x-a\hat\mu,\mu}\,\psi(x-a\hat\mu)\Big] \\
\bar\psi(x)\,\overleftarrow{D}_\mu &= \frac{1}{2a}\Big[\bar\psi(x+a\hat\mu)\,U^\dagger_{x,\mu} - \bar\psi(x-a\hat\mu)\,U_{x-a\hat\mu,\mu}\Big]
\end{aligned}$$

The total derivative is realized as

$$\partial_\mu\left[\bar\psi\cdots\psi\right](x) \;=\; \frac{1}{2a}\Big[[\bar\psi\cdots\psi](x+a\hat\mu) - [\bar\psi\cdots\psi](x-a\hat\mu)\Big]$$

– Compact realization of derivatives (II): Following a proposal of P. Rakow one could define

$$\left(\bar{\psi}\gamma_\mu D_\nu \psi\right)(q) = \frac{1}{4}\sum_x \Big\{ \bar{\psi}(x)\gamma_\mu U_{x,\nu}\psi(x+a\hat{\nu})\ \mathrm{exp}(-iq\cdot(x+\frac{a\hat{\nu}}{2})) - \bar{\psi}(x)\gamma_\mu U^\dagger_{x-a\hat{\nu},\nu}\psi(x-a\hat{\nu})\ \mathrm{exp}(-iq\cdot(x-\frac{a\hat{\nu}}{2}))\Big\}$$

and similar the operators with total derivatives!

$\longrightarrow$ eg. in order $O(g^0)$:

$$
\begin{aligned}
\mathcal{O}^{(DD,I)}_{\mu\nu\omega} &= \frac{1}{a^2}\gamma_\mu \cos\left(\frac{a}{2}(p_1-p_2)_\nu\right)\sin\left(\frac{a}{2}(p_1+p_2)_\nu\right) \\
&\quad \times \cos\left(\frac{a}{2}(p_1-p_2)_\omega\right)\sin\left(\frac{a}{2}(p_1+p_2)_\omega\right), \\
\mathcal{O}^{(DD,II)}_{\mu\nu\omega} &= \frac{1}{a^2}\gamma_\mu \sin\left(\frac{a}{2}(p_1+p_2)_\nu\right)\sin\left(\frac{a}{2}(p_1+p_2)_\omega\right), \\
\mathcal{O}^{(\partial\partial,I)}_{\mu\nu\omega} &= \frac{1}{4a^2}\gamma_\mu \sin(a\,(p_1-p_2)_\nu)\sin(a\,(p_1-p_2)_\omega), \\
\mathcal{O}^{(\partial\partial,II)}_{\mu\nu\omega} &= \frac{1}{a^2}\gamma_\mu \sin(\frac{a}{2}(p_1-p_2)_\nu)\sin(\frac{a}{2}(p_1-p_2)_\omega)
\end{aligned}
$$

$(q = p_1 - p_2 \neq 0)$

- **Two different external momenta**

In the Kawai scheme which we use the corresponding momentum integrals are computed as power series in the external momenta:

$$\mathcal{I}_{\mu_1\cdots\mu_n}(a,p_1,p_2) \;=\; \int \frac{d^d k}{(2\pi)^d}\,\mathcal{K}_{\mu_1\cdots\mu_n}(a,p_1,p_2,k)$$

$(d = 4 - 2\epsilon)$. It is calculated as

$$\mathcal{I} = \tilde{\mathcal{I}} + (\mathcal{I} - \tilde{\mathcal{I}}),$$

where

$$\begin{aligned}\tilde{\mathcal{I}}(a,p_1,p_2) \;=\;& \mathcal{I}(a,0) + \sum_{i=1}^{2}\sum_{\alpha} p_{i,\alpha}\,\frac{\partial}{\partial p_{i,\alpha}}\,\mathcal{I}(a,p_i)\Big|_{p_i=0}\\ &+\frac{1}{2!}\sum_{i,j=1}^{2}\sum_{\alpha,\beta} C_{ij}\,p_{i,\alpha}p_{j,\beta}\,\frac{\partial^2}{\partial p_{i,\alpha}\partial p_{j,\beta}}\,\mathcal{I}(a,p_i)\Big|_{p_i=0} + \cdots\end{aligned}$$

- **Continuum part**

The part $(\mathcal{I} - \tilde{\mathcal{I}})$ is actually computed in Euclidean continuum. In the case of $p_1 \neq p_2$ there are diagrams with three distinct propagators. Their finite parts are computed rather cumbersome.

In the literature some semi-analytic approaches can be found (Davydychev, Tarasov, Campbell, ...). The results can be expressed as polynomials of kinematic invariants and Spence functions.

We represent such types of integrals in the form

$$\begin{aligned} \mathcal{I}_{\mu_1 \cdots \mu_n}(p_1, p_2)^{ec} &= \frac{1}{\epsilon} A(p_1, p_2)_{\mu_1 \cdots \mu_n} \\ + \sum_{i,j,k,m} B(p_1, p_2, i, j, k, m)_{\mu_1 \cdots \mu_n} & \mathsf{FPI}(i, j, k, m, p_1, p_2). \end{aligned}$$

In A and B the general index structure is preserved.

FPI: integrals over the Feynman parameters to be carried out for specified external momenta numerically:

$$\mathrm{FPI}(i,j,k,m,p_1,p_2) = \int_0^1 dx \int_0^{1-x} dy\, x^i\, y^j (Q^2(x,y,p_1,p_2))^k \log^m Q^2(x,y,p_1,p_2)$$

with

$$Q^2(x,y,p_1,p_2) = p_1^2\, x(1-x) + p_2^2\, y(1-y) - 2p_1 \cdot p_2\, x\, y$$

In practice the integration over y has been carried out analytically.

For numeric evaluation the x integration remained to be done with appropriate values for p_1 and p_2 to be inserted.

Operators and the mixing problem

It is well known that operators of second and higher moments mix

In perturbation theory the one-loop result for a matrix element of a certain operator contains structures which differ from its own Born structure.

As a consequence the operators cannot be renormalized multiplicatively.

The set of possible operators is determined by the transformation properties under the hypercubic group and charge conjugation

Only operators can mix which belong to the same representation and with identical charge conjugation number!

A comprehensive derivation is given in Göckeler et al. (PR D54 (1996))

Mixing sets:

Let us define the following symmetrizations:

$$
\begin{aligned}
\mathcal{O}_{\{\nu_1\nu_2\nu_3\}} &= \frac{1}{6}\Big(\mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} \\
&\quad + \mathcal{O}_{\nu_2\nu_1\nu_3} + \mathcal{O}_{\nu_2\nu_3\nu_1} + \mathcal{O}_{\nu_3\nu_1\nu_2} + \mathcal{O}_{\nu_3\nu_2\nu_1}\Big) \\
\mathcal{O}_{|\nu_1\nu_2\nu_3|} &= \mathcal{O}_{\nu_1\nu_2\nu_3} - \mathcal{O}_{\nu_1\nu_3\nu_2} - \mathcal{O}_{\nu_3\nu_1\nu_2} + \mathcal{O}_{\nu_3\nu_2\nu_1} \\
\mathcal{O}_{\|\nu_1\nu_2\nu_3\|} &= \mathcal{O}_{\nu_1\nu_2\nu_3} - \mathcal{O}_{\nu_1\nu_3\nu_2} \\
&\quad + \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1} - 2\,\mathcal{O}_{\nu_2\nu_3\nu_1} + 2\,\mathcal{O}_{\nu_2\nu_1\nu_3} \\
\mathcal{O}_{\langle\nu_1\nu_2\nu_3\rangle} &= \mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} \\
&\quad + \mathcal{O}_{\nu_3\nu_1\nu_2} + \mathcal{O}_{\nu_3\nu_2\nu_1} - 2\,\mathcal{O}_{\nu_2\nu_3\nu_1} - 2\,\mathcal{O}_{\nu_2\nu_1\nu_3} \\
\mathcal{O}_{\langle\langle\nu_1\nu_2\nu_3\rangle\rangle} &= \mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} - \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1}
\end{aligned}
$$

We consider the following operators:

$\underline{\tau_2^{(4)},\ C=-1:}$ $\mathcal{O}^{DD}_{\{124\}}\,,\ \mathcal{O}^{\partial\partial}_{\{124\}}$

$\underline{\tau_3^{(4)},\ C=+1:}$ $\mathcal{O}^{DD,5}_{\{124\}}\,,\ \mathcal{O}^{\partial\partial,5}_{\{124\}}$

$\underline{\tau_1^{(8)},\ C=-1:}$

$$
\begin{aligned}
\mathcal{O}_1 &= \mathcal{O}^{DD}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\{224\}} + \mathcal{O}^{DD}_{\{334\}}\right) \\
\mathcal{O}_2 &= \mathcal{O}^{\partial\partial}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\{224\}} + \mathcal{O}^{\partial\partial}_{\{334\}}\right) \\
\mathcal{O}_3 &= \mathcal{O}^{DD}_{\langle\langle 411\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\langle\langle 422\rangle\rangle} + \mathcal{O}^{DD}_{\langle\langle 433\rangle\rangle}\right) \\
\mathcal{O}_4 &= \mathcal{O}^{\partial\partial}_{\langle\langle 411\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\langle\langle 422\rangle\rangle} + \mathcal{O}^{\partial\partial}_{\langle\langle 433\rangle\rangle}\right) \\
\mathcal{O}_5 &= \mathcal{O}^{\partial D,5}_{\|123\|} + 3\,\mathcal{O}^{\partial D,5}_{|123|} \\
\mathcal{O}_6 &= \mathcal{O}^{\partial D,5}_{\langle 123\rangle} - \mathcal{O}^{\partial D,5}_{\langle\langle 123\rangle\rangle}
\end{aligned}
$$

$\underline{\tau_2^{(8)},\ C=+1:}$

$$
\begin{aligned}
\mathcal{O}_1^5 &= \mathcal{O}^{DD,5}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{DD,5}_{\{224\}} + \mathcal{O}^{DD,5}_{\{334\}}\right)\\
\mathcal{O}_2^5 &= \mathcal{O}^{\partial\partial,5}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial,5}_{\{224\}} + \mathcal{O}^{\partial\partial,5}_{\{334\}}\right)\\
\mathcal{O}_3^5 &= \mathcal{O}^{DD,5}_{\langle\langle 411\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{DD,5}_{\langle\langle 422\rangle\rangle} + \mathcal{O}^{DD,5}_{\langle\langle 433\rangle\rangle}\right)\\
\mathcal{O}_4^5 &= \mathcal{O}^{\partial\partial,5}_{\langle\langle 411\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial,5}_{\langle\langle 422\rangle\rangle} + \mathcal{O}^{\partial\partial,5}_{\langle\langle 433\rangle\rangle}\right)\\
\mathcal{O}_5^5 &= \mathcal{O}^{\partial D}_{\|213\|}\\
\mathcal{O}_6^5 &= \mathcal{O}^{\partial D}_{213} + \mathcal{O}^{\partial D}_{231} - \mathcal{O}^{\partial D}_{321} - \mathcal{O}^{\partial D}_{312}
\end{aligned}
$$

These are rather non-trivial sets of operators!

Renormalization factor matrix

Let $\Gamma_j^D(p_1, p_2, \mu, g_R, \epsilon)$ the dimensionally regularized vertex function of operator $\mathcal{O}_j$.

One-loop perturbation theory yields

$$\begin{aligned}\Gamma_j^D(p_1,p_2,\mu,g_R,\epsilon) &= \Gamma_j^{\mathrm{Born}}(p_1,p_2)\\ &+g_R^2\left[\sum_{k=1}^{N}\gamma_{jk}\left(\frac{1}{\epsilon}-\gamma_E+\ln(4\pi)-\ln\frac{p_1^2+p_2^2}{4\mu^2}\right)\Gamma_k^{\mathrm{Born}}(p_1,p_2)+f_j(p_1,p_2)\right]+O(g_R^4)\end{aligned}$$

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In $\overline{MS}$ scheme the renormalized vertex is given as

$$\begin{aligned}\Gamma_j^R(p_1,p_2,\mu,g_R,\epsilon) \quad &= \quad \Gamma_j^{\mathrm{Born}}(p_1,p_2) \\ &+g_R^2\left[\sum_{k=1}^{N}\gamma_{jk}\left(-\ln\frac{p_1^2+p_2^2}{4\mu^2}\right)\Gamma_k^{\mathrm{Born}}(p_1,p_2)+f_j(p_1,p_2)\right]+O(g_R^4)\end{aligned}$$

On the lattice we find

$$\begin{aligned}\Gamma_j^L(p_1,p_2,\mu,g_R,\epsilon) \quad &= \quad \Gamma_j^{\mathrm{Born}}(p_1,p_2) \\ &+g_R^2\left[\sum_{k=1}^{N}\gamma_{jk}\left(-\ln\frac{a^2(p_1^2+p_2^2)}{4}\right)\Gamma_k^{\mathrm{Born}}(p_1,p_2)+f_j^L(p_1,p_2)\right]+O(g_R^4)\end{aligned}$$

On the lattice we find

$$\begin{aligned}
\Gamma_j^L(p_1,p_2,\mu,g_R,\epsilon) &= \Gamma_j^{\mathrm{Born}}(p_1,p_2) \\
&+g_R^2\left[\sum_{k=1}^{N}\gamma_{jk}\left(-\ln\frac{a^2(p_1^2+p_2^2)}{4}\right)\Gamma_k^{\mathrm{Born}}(p_1,p_2)+f_j^L(p_1,p_2)\right]+O(g_R^4)
\end{aligned}$$

There should a matrix ζ to relate lattice vertex functions to $\overline{MS}$ vertex functions as

$$\Gamma_j^R(p_1,p_2,\mu,g_R,\epsilon) = \sum_{k=1}^{N}\left(\delta_{jk}+g_R^2\zeta_{jk}+O(g_R^4)\right)\Gamma_k^L(p_1,p_2,a,g_R)$$

On the lattice we find

$$\Gamma_j^L(p_1,p_2,\mu,g_R,\epsilon) \quad = \quad \Gamma_j^{\text{Born}}(p_1,p_2)$$
$$+g_R^2\left[\sum_{k=1}^{N}\gamma_{jk}\left(-\ln\frac{a^2(p_1^2+p_2^2)}{4}\right)\Gamma_k^{\text{Born}}(p_1,p_2)+f_j^L(p_1,p_2)\right]+O(g_R^4)$$

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This implies

$$\Gamma_j^R(p_1,p_2,\mu,g_R,\epsilon) \quad = \quad \Gamma_j^{\text{Born}}(p_1,p_2)$$
$$+g_R^2\left[\sum_{k=1}^{N}\left(\zeta_{jk}-\gamma_{jk}\ln\frac{a^2(p_1^2+p_2^2)}{4}\right)\Gamma_k^{\text{Born}}(p_1,p_2)+f_j^L(p_1,p_2)\right]+O(g_R^4)$$

Comparing with the second relation we arrive at

$$\sum_{k=1}^{N} \left(\zeta_{jk} - \gamma_{jk} \ln(a^2\mu^2)\right) \Gamma_k^{\mathrm{Born}}(p_1, p_2) + f_j^L(p_1, p_2) - f_j(p_1, p_2) \quad = \quad 0$$

Comparing with the second relation we arrive at

$$\sum_{k=1}^{N}\left(\zeta_{jk}-\gamma_{jk}\ln(a^2\mu^2)\right)\Gamma_k^{\mathrm{Born}}(p_1,p_2)+f_j^L(p_1,p_2)-f_j(p_1,p_2) \;=\; 0$$

This equation must hold for arbitrary momenta p_1, p_2, therefore we must find constants c_{ij} sucht that

$$f_j^L(p_1,p_2)-f_j(p_1,p_2) \;=\; \sum_{k=1}^{N} c_{jk}\Gamma_k^{\mathrm{Born}}(p_1,p_2)$$

and we get

$$\zeta_{jk}=\gamma_{jk}\ln(a^2\mu^2)-c_{jk}$$

$f_j^L(p_1,p_2)-f_j(p_1,p_2)$ means additionally that for $\overline{MS}$ we have to compute the pure lattice part $\tilde{\mathcal{I}}$ of the momentum integrals only!

The connection between bare lattice vertex functions and $\overline{MS}$ renormalized vertex functions can generally be written as

$$\Gamma_j^R(p_1, p_2, \mu, g_R) \;\; = \;\; Z_\psi \sum_{k=1}^{N} Z_{jk} \Gamma_k^L(p_1, p_2, a, g_R).$$

Z_ψ has been calculated sometimes ago as (Feynman gauge)

$$Z_\psi \;\; = \;\; 1 + \frac{g_R^2}{16\pi^2} C_F \left(\ln(a^2\mu^2) + 1 - b_\psi \right).$$

The connection between bare lattice vertex functions and $\overline{MS}$ renormalized vertex functions can generally be written as

$$\Gamma_j^R(p_1, p_2, \mu, g_R) \;=\; Z_\psi \sum_{k=1}^{N} Z_{jk} \Gamma_k^L(p_1, p_2, a, g_R).$$

Z_ψ has been calculated sometimes ago as (Feynman gauge)

$$Z_\psi \;=\; 1 + \frac{g_R^2}{16\pi^2} C_F \left(\ln(a^2\mu^2) + 1 - b_\psi \right).$$

As result we obtain for the renormalization mixing matrix

$$\begin{aligned} Z_{jk}^{\overline{MS}} \;=\; & \delta_{jk} + g_R^2 \left[\left(\gamma_{jk} - \delta_{jk} \frac{C_F}{16\pi^2} \right) \ln(a^2\mu^2) - c_{jk} - \delta_{jk} \frac{C_F}{16\pi^2} (1 - b_\psi) \right] \\ & + O(g_R^4) \end{aligned}$$

Renormalization factors in $\overline{MS}$ - scheme

We give the results in a form

$$Z_{ij}^{\overline{MS},m} = \delta_{ij} - \frac{g_R^2 C_F}{16\pi^2} \left(\gamma_{ij}^{(m)} \ln(a^2\mu^2) + c_{ij}^{(m)} \right)$$

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1. $\mathcal{O}^{DD}_{\{124\}} \leftrightarrow \mathcal{O}^{\partial\partial}_{\{124\}}$

$$\gamma_{ij}^{(I,II)} = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} \\ 0 & 0 \end{pmatrix}$$

$$c_{ij}^{(I,II)} = \begin{pmatrix} -11.563 & 0.024 \\ 0 & 20.618 \end{pmatrix}$$

2. $\mathcal{O}^{DD,5}_{\{124\}} \leftrightarrow \mathcal{O}^{\partial\partial,5}_{\{124\}}$

$$\gamma_{ij}^{(I,II)} = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} \\ 0 & 0 \end{pmatrix}$$

$$c_{ij}^{(I,II)} = \begin{pmatrix} -12.117 & 0.167 \\ 0 & 15.796 \end{pmatrix}$$

3. $\{\mathcal{O}_1, ..., \mathcal{O}_6\}$, same dimension

$$\gamma_{ij}^{(I,II)} = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{6} & -\frac{5}{6} & -\frac{1}{2} & -\frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{pmatrix}$$

$$
c_{ij}^{(I)} = \begin{pmatrix}
-12.127 & -2.737 & -0.368 & -0.993 & 0.008 & -0.075 \\
0 & 20.618 & 0 & 0 & 0 & 0 \\
-3.306 & -18.184 & -14.852 & -4.302 & 0.464 & 0.369 \\
0 & 0 & 0 & 20.618 & 0 & 0 \\
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$$c_{ij}^{(II)} = \begin{pmatrix} -12.127 & 1.489 & -0.368 & 0.416 & 0.008 & -0.075 \\ 0 & 20.618 & 0 & 0 & 0 & 0 \\ -3.306 & 8.015 & -14.852 & 4.302 & 0.464 & 0.369 \\ 0 & 0 & 0 & 20.618 & 0 & 0 \\ 0 & 6.529 & 0 & 0 & 0.350 & -0.015 \\ 0 & 6.529 & 0 & 0 & 0.350 & 0.015 \end{pmatrix}$$

4. $\mathcal{O}_1$ - $\frac{1}{a}$ part

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$$\overline{\mathcal{O}}_{\mu\nu\omega} \;=\; \partial_\nu \left(\overline{\psi} \left[\gamma_\mu, \gamma_\omega \right] \psi \right)$$

The operator which is in the same representation as $\mathcal{O}_1$ is

$$\mathcal{O}_7 \;=\; \overline{\mathcal{O}}_{114} - \frac{1}{2} \left(\overline{\mathcal{O}}_{224} + \overline{\mathcal{O}}_{334} \right)$$

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We find a multiplicative mixing

$$\mathcal{O}_1 \Big|_{1/a-part} = \frac{g_R^2 C_F}{16\pi^2} (-0.518) \frac{1}{a} \mathcal{O}_7^{\mathrm{Born}}$$

5. $\{\mathcal{O}_1^5, ..., \mathcal{O}_6^5\}$, same dimension

$$\gamma_{ij}^{(I,II)} = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{6} & -\frac{5}{6} & -1 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 2 & -2 \end{pmatrix}$$

$$c_{ij}^{(I)} = \begin{pmatrix} -12.861 & -2.095 & -0.349 & -0.854 & 0.051 & 0.030 \\ 0 & 15.796 & 0 & 0 & 0 & 0 \\ -3.422 & -15.821 & -15.359 & -5.164 & -0.170 & 0.472 \\ 0 & 0 & 0 & 15.796 & 0 & 0 \\ 0 & -8.912 & 0 & 0 & 0.960 & -0.480 \\ 0 & -8.912 & 0 & 0 & 0.060 & -0.960 \end{pmatrix}$$

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c_{ij}^{(II)} = \begin{pmatrix} -12.861 & 1.489 & -0.349 & 0.331 & 0.051 & 0.030 \\ 0 & 15.796 & 0 & 0 & 0 & 0 \\ -3.422 & 7.796 & -15.359 & 2.543 & -0.170 & 0.472 \\ 0 & 0 & 0 & 15.796 & 0 & 0 \\ 0 & -8.912 & 0 & 0 & 0.960 & -0.480 \\ 0 & -8.912 & 0 & 0 & 0.060 & -0.960 \end{pmatrix}
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We find a multiplicative mixing

$$\mathcal{O}_1^5\Big|_{1/a-part} = \frac{g_R^2 C_F}{16\pi^2}(-0.252)\frac{1}{a}\mathcal{O}_7^{5,\mathrm{Born}}$$

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- Next tasks:

  $$\mathcal{O}_{\mu\nu\omega\sigma} = \overline{\psi}\,\sigma_{\mu\nu}\,\overleftrightarrow{D}_\omega \overleftrightarrow{D}_\sigma\,\psi \qquad \text{for Wilson fermions}$$

  The whole set of operators for clover and overlap fermions.