Ghost and gluon propagators from SU(3) lattice gluodynamics

— progress report —

Berlin-Leipzig-Dubna collaboration

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Outline:

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2. Lattice gauge fixing, the (lattice) Gribov problem
3. Gribov copies and the ghost propagator: SU(2) case
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Introduction, motivation

- Non-perturbative study of gluon and ghost propagators is of interest for understanding the confinement phenomenon.

- Singular behavior of the ghost propagator for the Landau gauge

$$G^{ab} = \delta^{ab} \frac{Z_{gh}(q^2)}{q^2} \quad \text{for} \quad q \to 0$$

$\Longleftrightarrow$ Kugo-Ojima confinement criterion,

$\Longleftrightarrow$ absence of colored states in physical spectrum.

- Suppression of the gluon propagator:

$$D^{ab}_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \frac{Z_{gl}(q^2)}{q^2} \quad \text{for} \quad q \to 0$$

$\Longleftrightarrow$ confinement of gluons.

- Behavior of both intimately connected.

- Running coupling in MOM scheme

$$\alpha_s(q) \sim Z_{gh}(q^2)^2 \;\; Z_{gl}(q^2)\; \alpha_s(\mu)$$

- Gribov, Zwanziger: Infrared behavior related to the restriction of gauge fields to the Gribov region:

$$\Omega = \{A : \partial_\mu A_\mu = 0, \quad M \geq 0\}$$

where $M^{ab} \; = \; -\partial_\mu D_\mu^{ab}(A)$ Faddeev-Popov operator.

Expect gauge fields to dominate belonging to the Gribov horizon:

$\partial\Omega = \{A : A \in \Omega, \;\; M(A)$ gets nontrivial zero-mode$\}$.

Gribov problem: non-uniqueness of gauge fixing.

Continuum approach: Dyson-Schwinger equations for the Euclidean Greens functions

$$\left(\frac{\delta S}{\delta \phi_i}\left[\frac{\delta}{\delta J}\right] + J\right) Z_E(J) = 0$$

- need gauge fixing: Landau gauge,
- provides infinite set of equations, which has to be truncated

Alkofer et al. [1997]: Critical behavior as $q \to 0$:

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z_{gl}(q^2)}{q^2} \qquad Z_{gl}(q^2) \propto (q^2)^{2\kappa}$$

$$G^{ab} = \delta^{ab}\frac{Z_{gh}(q^2)}{q^2} \qquad Z_{gh}(q^2) \propto (q^2)^{-\kappa}$$

critical exponent: $\kappa \approx 0.595$

$\Rightarrow$ IR fixed point $\alpha_s(q) \to$ const. for $q \to 0$?

Lattice gauge fixing, the Gribov problem

- vector potential: $A_\mu(x) \equiv \frac{1}{2i}(U_{x\mu} - U^\dagger_{x\mu})\Big|_{\text{traceless}}$
- Landau gauge: $\max_x \text{Tr}\left(\partial_\mu A_\mu(x) \cdot [\partial_\mu A_\mu(x)]^\dagger\right) < \epsilon$

  $\Longleftrightarrow$

  maximize the gauge functional

$$F_U(g) = \frac{1}{4V} \sum_{x,\mu} \Re\mathfrak{e}\, \text{Tr}\, U^g_{x\mu}$$

  with resp. to $\quad U^g_{x\mu} = g_x \cdot U_{x\mu} \cdot g^\dagger_{x+\mu}$

  $\Longrightarrow$ nonuniqueness of the solution, i.e. Gribov copies!

- Way out: fundamental modular region ?

$$\Lambda \equiv \{U : F_U(1) \geq F_U(g)\,, \quad \forall g\}$$

Method: Take thermalized U fixed,

generate initial random gauge $g_x^{(0)}$,

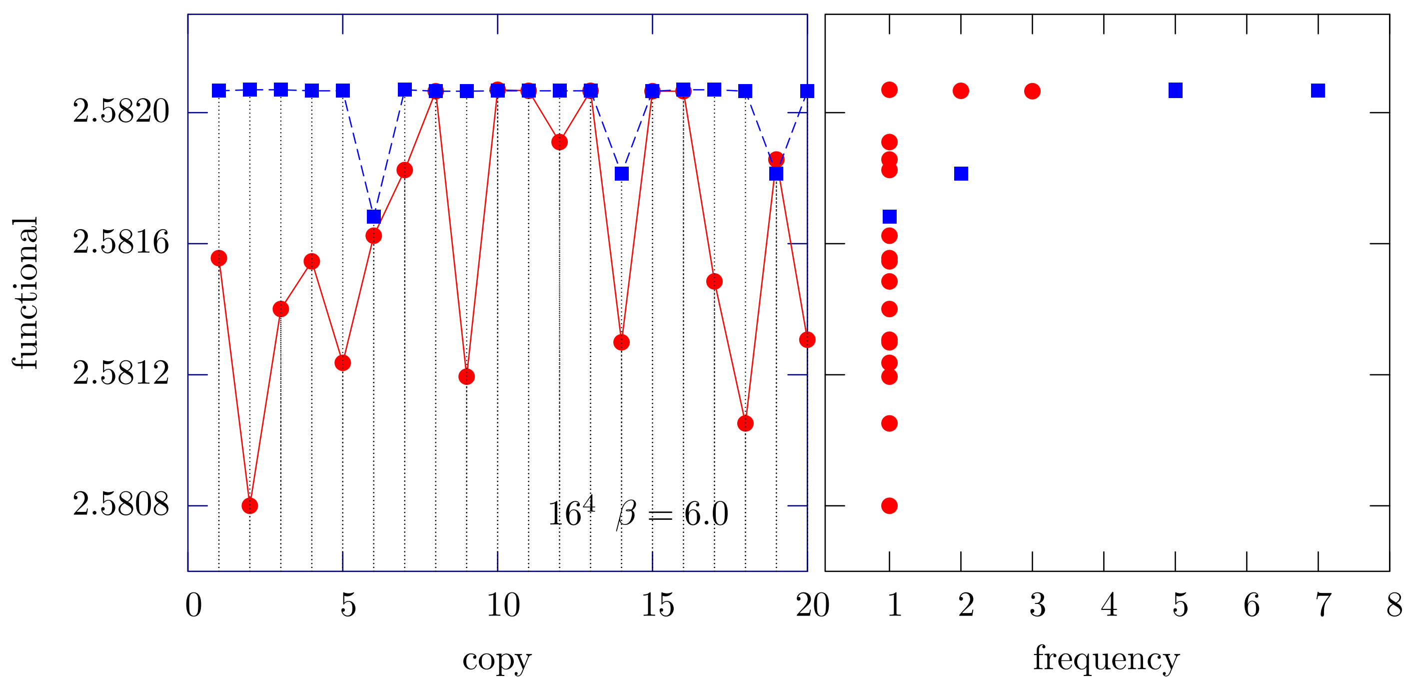
(over) relax g_x until $\max_x \mathrm{Tr}\left(\partial_\mu A_\mu^g(x) \cdot [\partial_\mu A_\mu^g(x)]^\dagger\right) < \epsilon$

w.r. to local gauge transformation: $U_{x\mu}^g = g_x \cdot U_{x\mu} \cdot g_{x+\mu}^\dagger$

Functional: $F_U(g) = \frac{1}{4V} \sum \mathfrak{Re}\,\mathrm{Tr}\, U_{x\mu}^g = \mathrm{Max.}$

Example: 1 gauge field configuration $\{U_{x\mu}\}$, 20 $\times$ random gauges

red: stand. overrelaxation, blue: simul. annealing



Remark on gauge fixing algorithms:

Goal: Find those gauge transformations which maximize (globally)

(Over-)relaxation:
$(g_x)^\omega$ where g_x is local maximum

Simulated annealing
ground state of spin sytem
$\exp\left\{ - F_U(g)/T \right\}$

$$F_U(g) = \frac{1}{4V} \sum \Re \, \mathrm{Tr}\, U^g_{x\mu}$$

$\cdots\cdots$

Fourier acceleration

Smeared gauge fixing
start with g_x which gauge fix the smeared configuration $\tilde{U}_{x,\mu}$

Applied numerical techniques:

- Parallelized (MPI) $SU(3)$ code running on IBM 690p (HLRN)
  $\longrightarrow$ very large lattices, i.e. small momenta.
- Update: standard Wilson action and hybrid overrelaxation (Cabibbo-Marinari heatbath, mod. à la Fabricius-Haan + overrelaxation steps)
- Gauge fixing: start with a number of random gauge copies, apply standard overrelaxation (vs. simulated annealing)
- Ghost propagator:

$$G(p) = \frac{1}{3V} \sum_{xy} e^{-2\pi i\, k\cdot(x-y)} \left\langle \left(M^{-1} \right)^{aa}_{xy} [U] \right\rangle$$

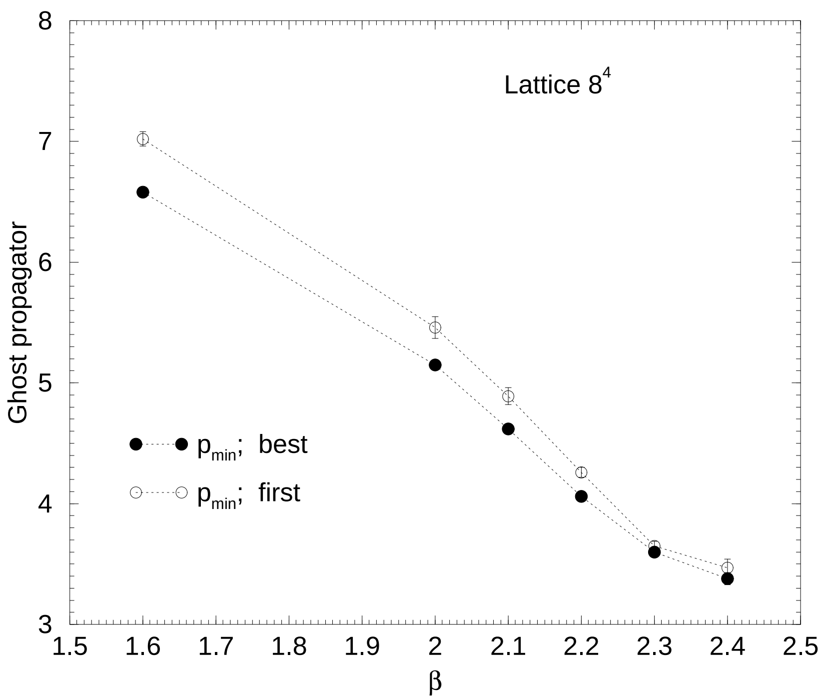
  using conjugate gradient algorithm with source:
  $\psi^{ac}(y) = \delta^{ac}\, e^{2\pi i\, k\cdot y}$ $\qquad k \neq (0,0,0,0)$ (Cucchieri)
- Gluon propagator: fast Fourier transform

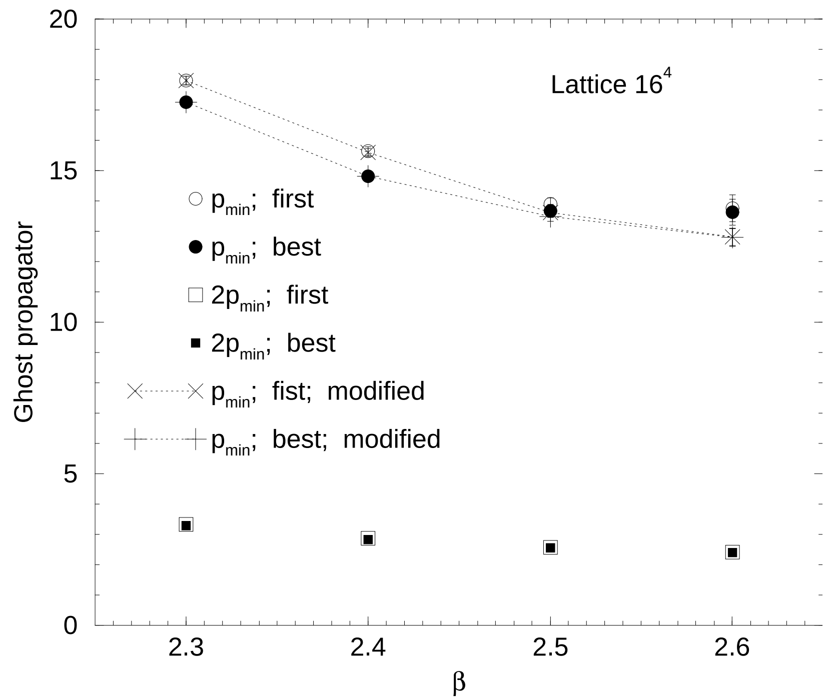
Gribov copies and the ghost propagator: the SU(2) case

Dependence on Gribov copies clearly visible for $0 \leq \beta < 2.5$, weakens with increasing β.

A. Cucchieri (Nucl. Phys. B508, 353)

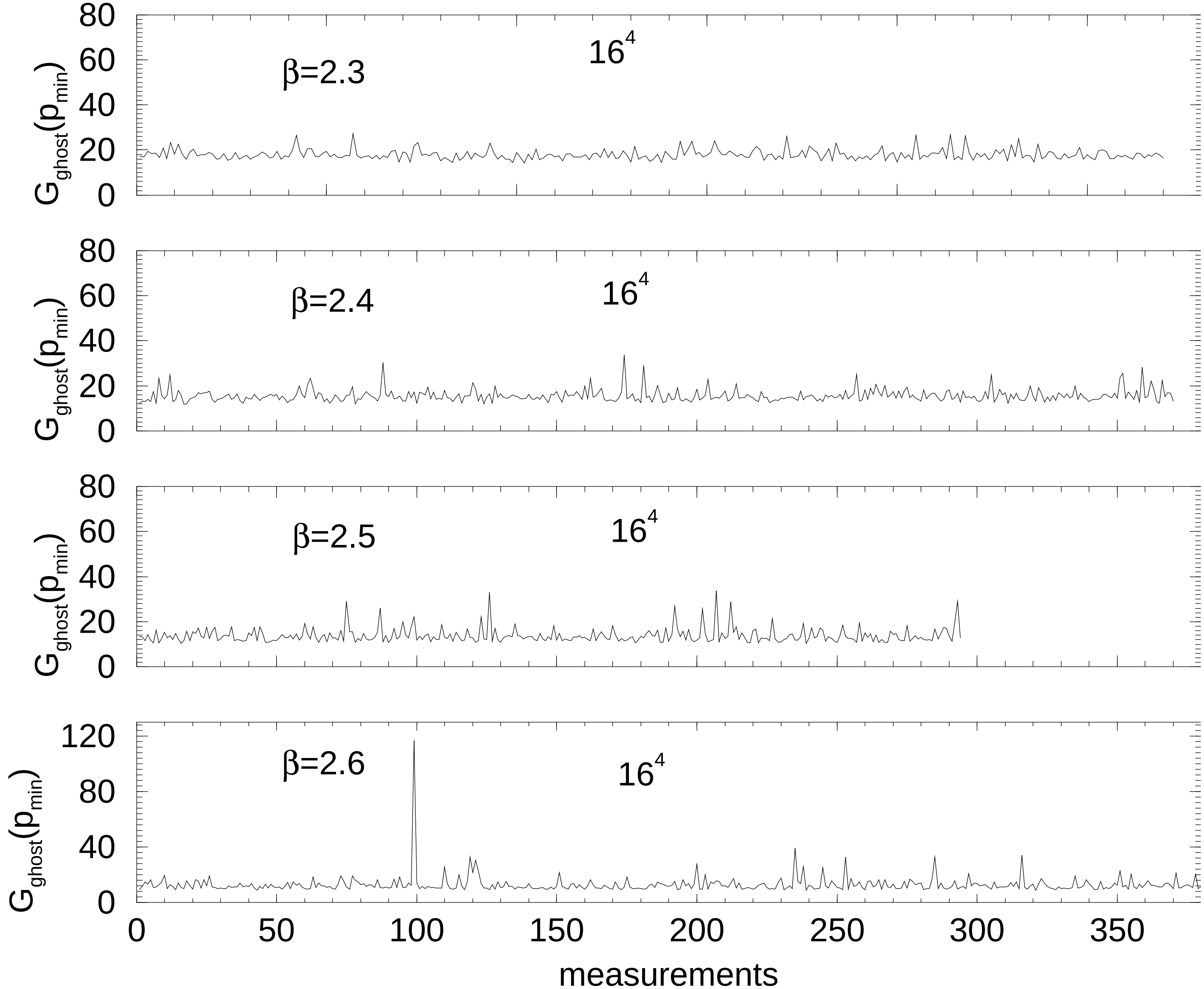
T.D. Bakeev, E.M. Ilgenfritz, V.K. Mitrjushkin, M. M.-P., hep-lat/0311041, PRD to appear.





For increasing β a new problem arises:

exceptional configurations, origin still unclear.



The SU(3) case: ghost and gluon propagators

- Ghost propagator at lowest momenta:

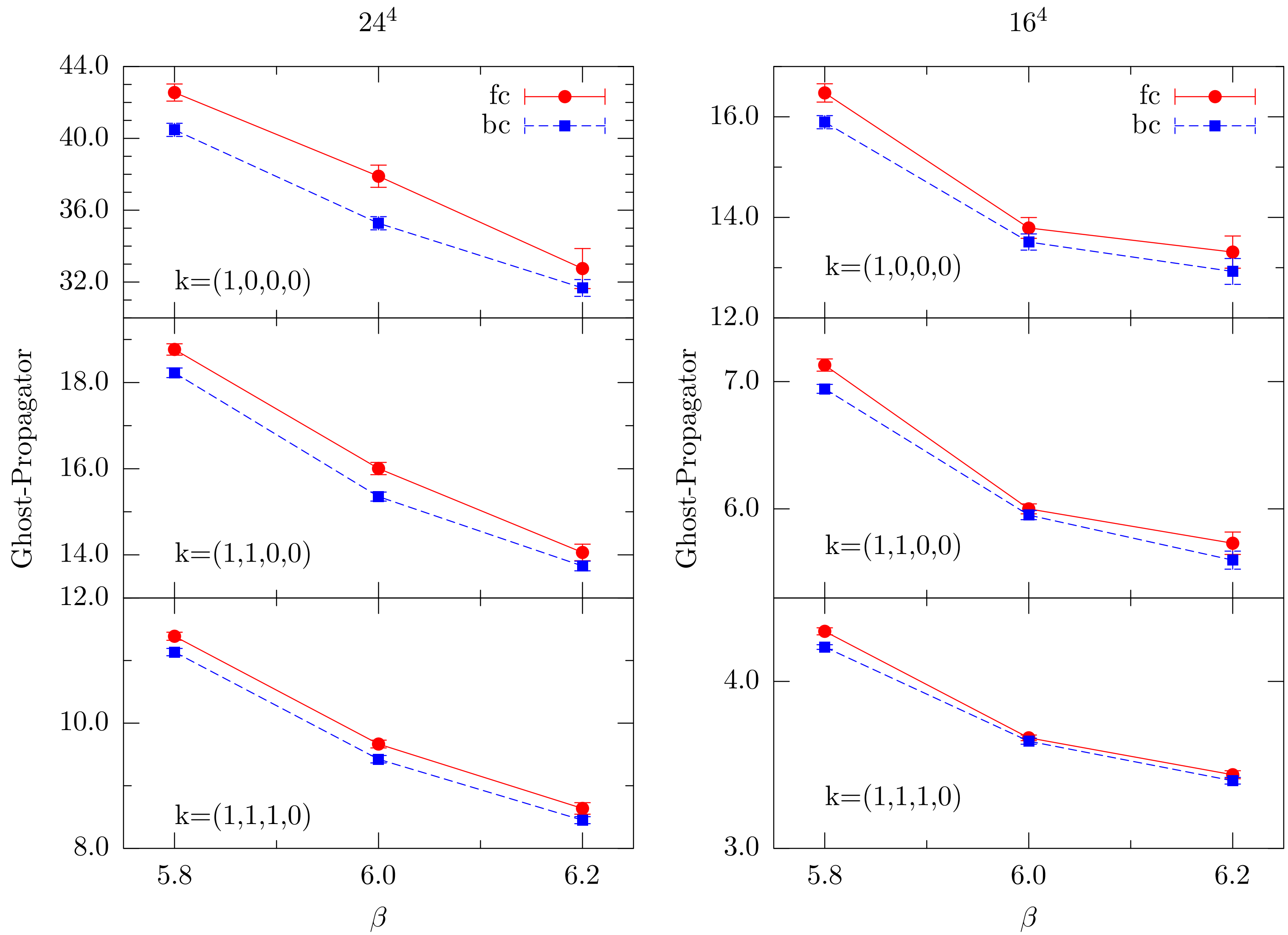
  $\Longrightarrow$ Gribov copy dependence visible again.

  $\Longrightarrow$ Exceptional configurations appear at $\beta \simeq 6.2$.

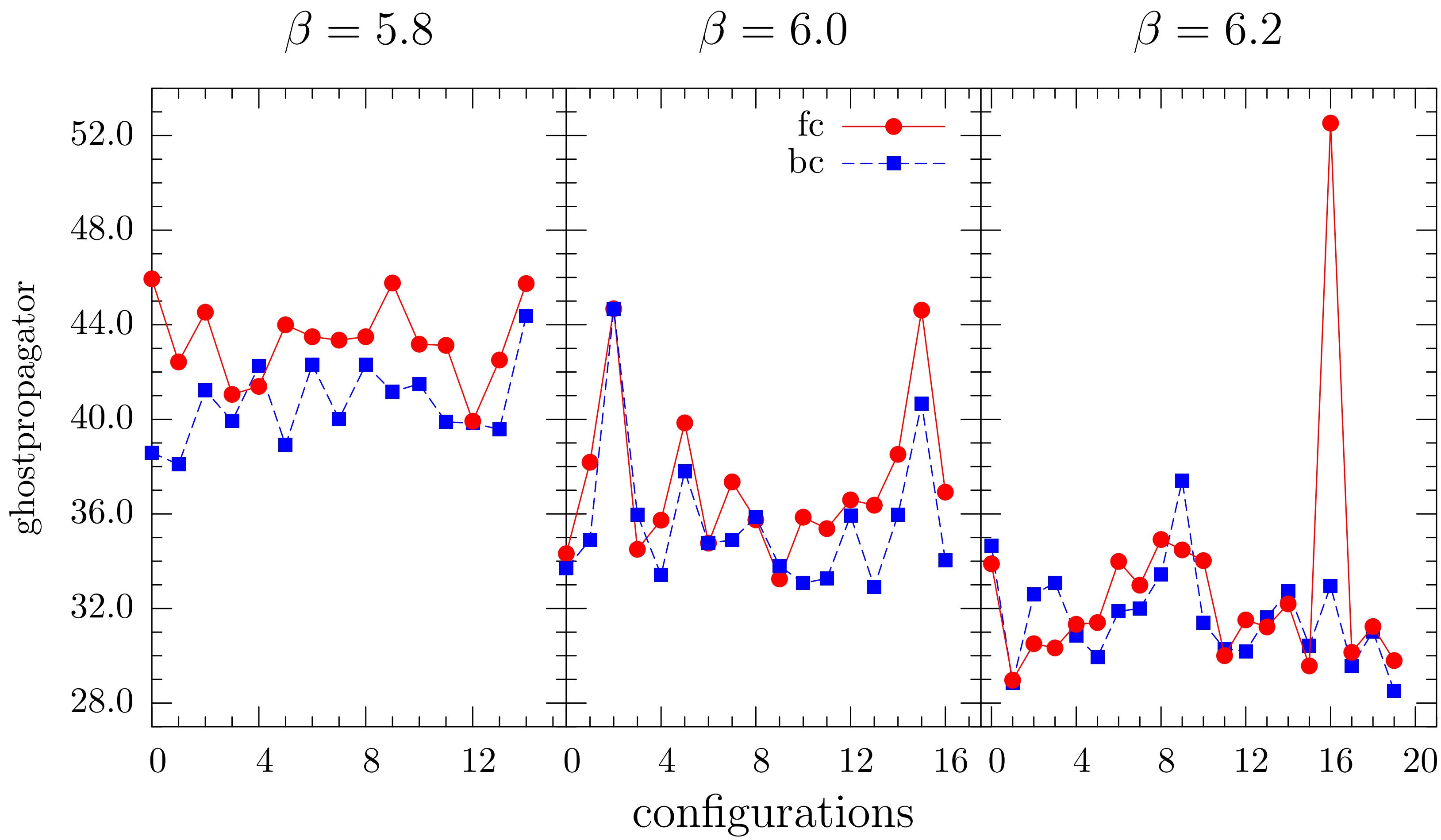
- Gluon propagator at lowest momenta:

  $\Longrightarrow$ Gribov noise within the statistical noise.

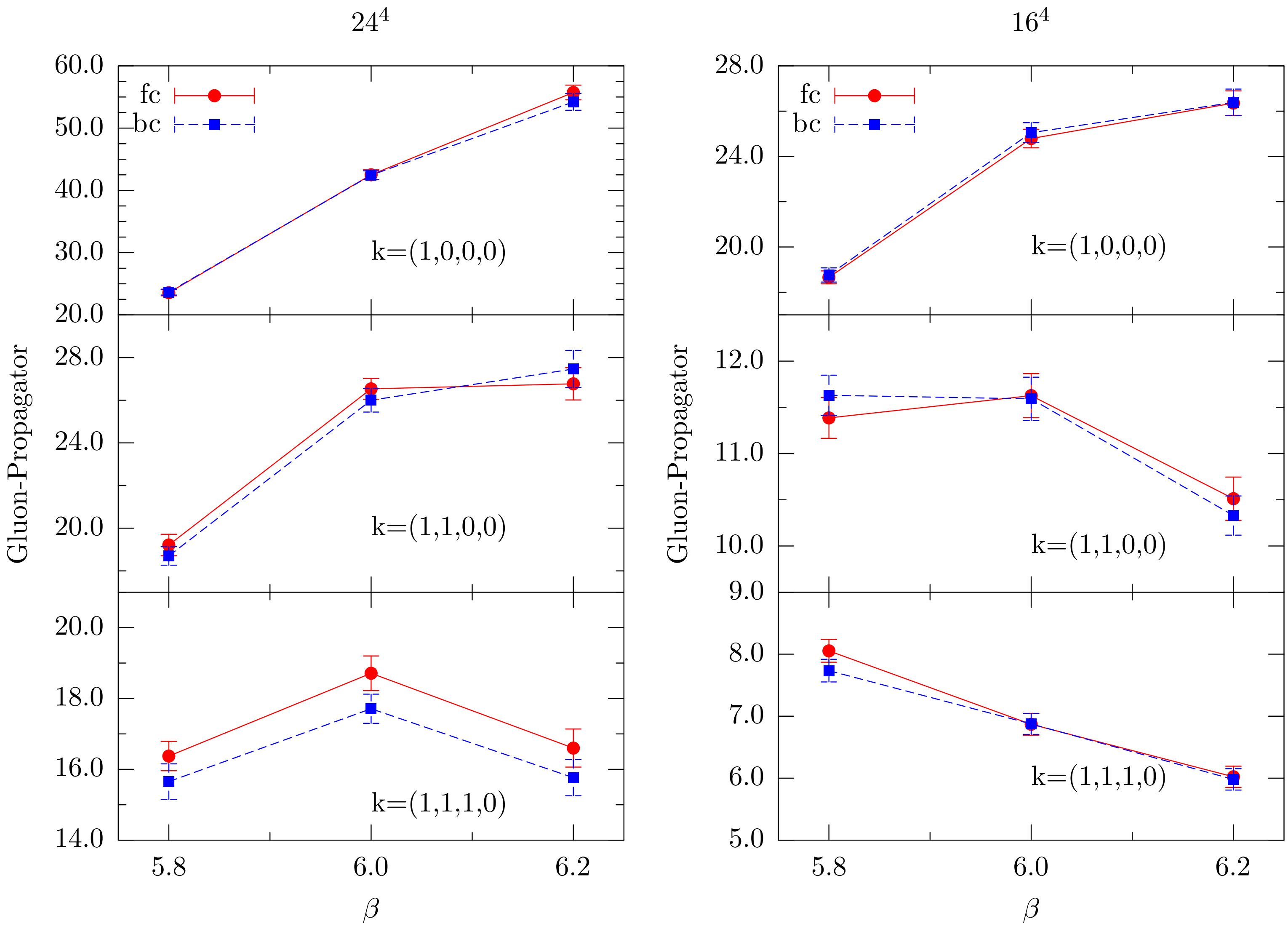
Ghost propagator at lowest momenta:



Appearance of exceptional configurations:



Gluon propagator at lowest momenta:

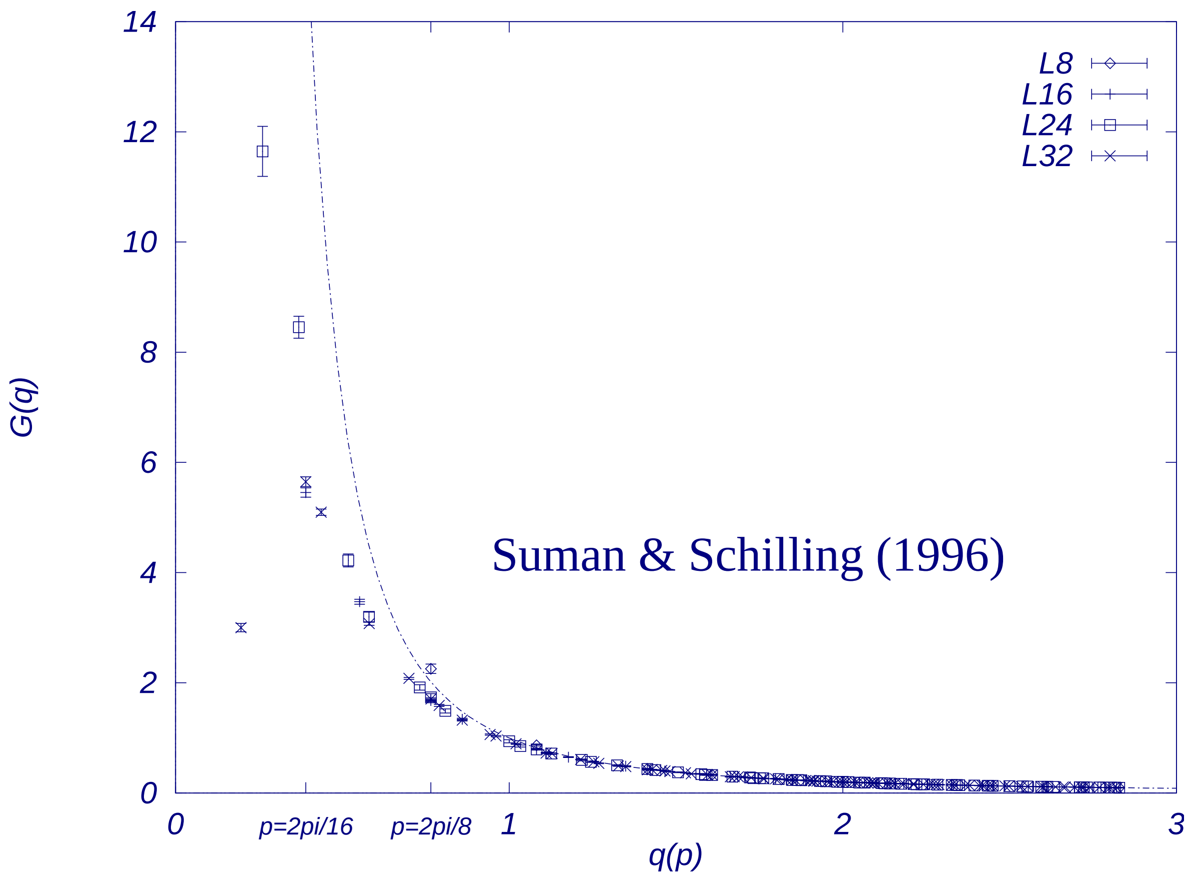


Ghost propagator in the infrared:

First SU(3) lattice study by Suman & Schilling ('96) at $\beta = 6.0$

1) turnover at very small q ?

2) $G(q) \propto \frac{1}{q^2}$ for $p \to \frac{\pi}{L}$ $\Rightarrow$ no $\frac{1}{q^4}$ behavior



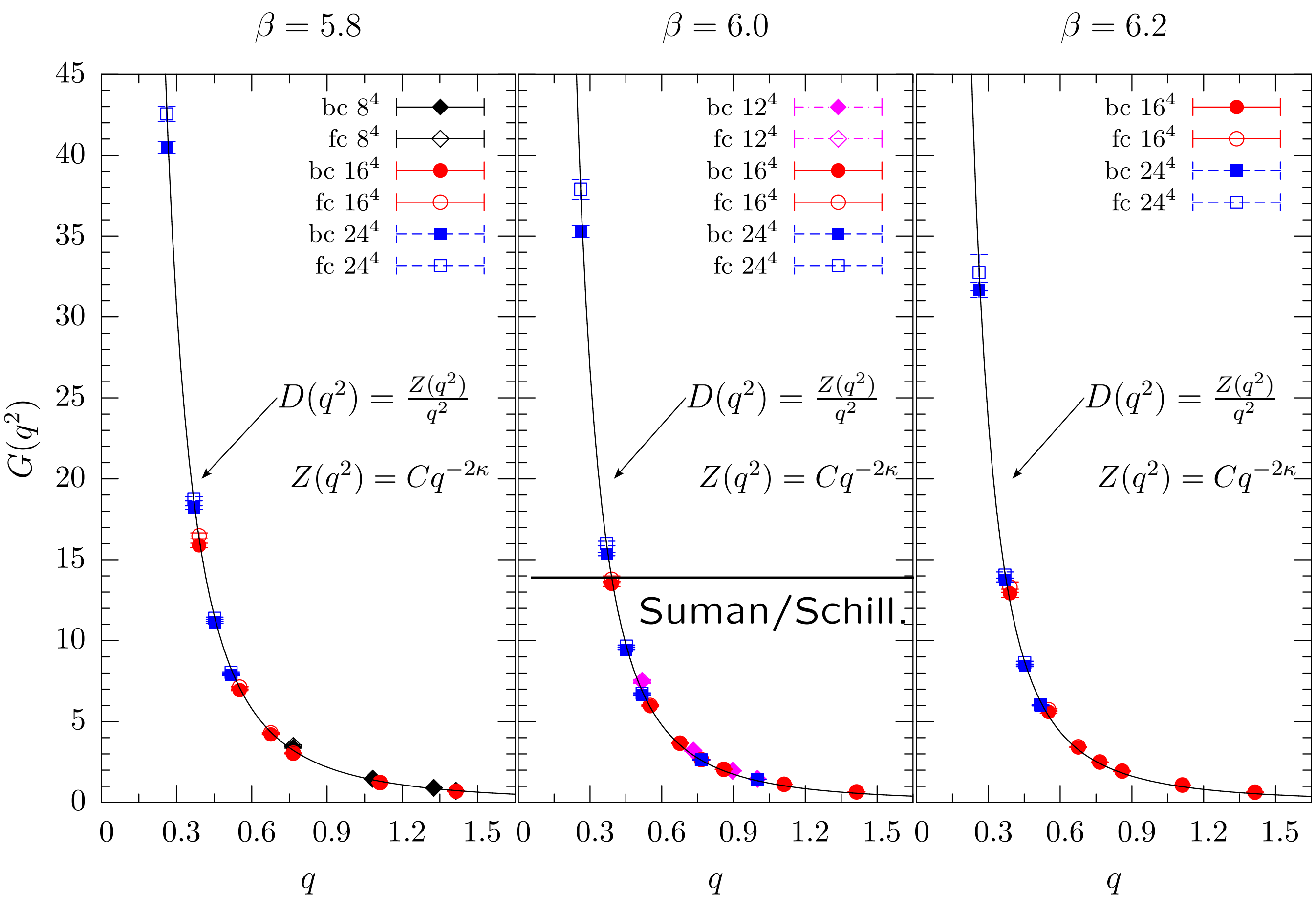
Alkofer et.al. ('97)

$$Z_{gh}(q^2) \propto (q^2)^{-\kappa}$$

unknown at this time

Fit: $G(q) = \frac{A}{q^2} + \frac{B}{q^4} + \ldots$

Ghost propagator in the infrared:

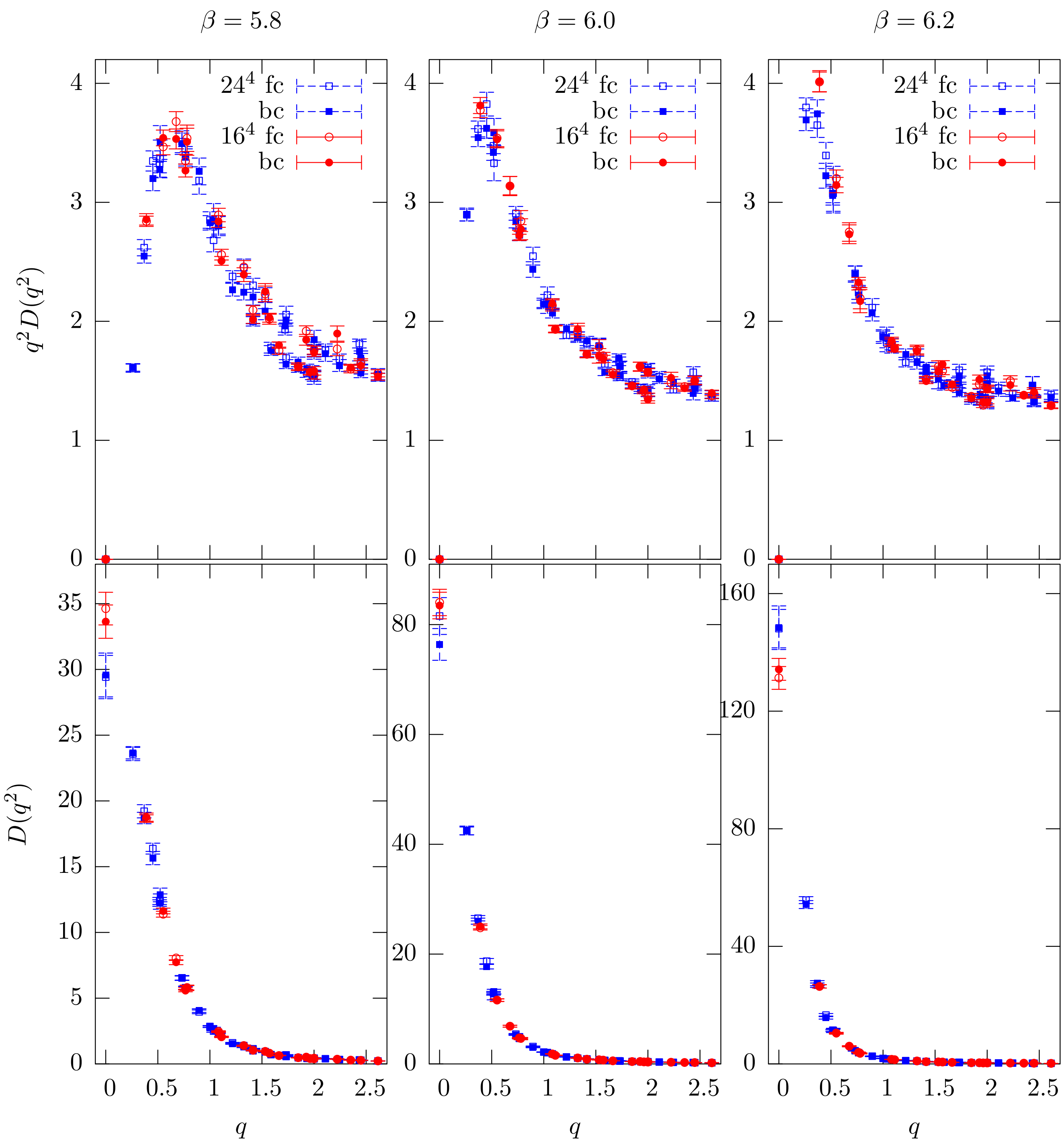


Fit parameter:

	bc	fc
β	5.8	
κ	0.21(2)	0.23(2)
$\chi^2/$	13	6
β	6.0	
κ	0.23(2)	0.27(2)
$\chi^2/$	4.9	2.9
β	6.2	
κ	0.23(1)	0.25(2)
$\chi^2/$	2.3	1.8

Again: the Gribov copies affect the infrared behavior

Gluon propagator in the infrared



Conclusions and future plans

- Gribov effects have to be carefully taken into account at smaller β

  $\Longrightarrow$

  better algorithms for finding the global maxima are desired.

- With increasing β exceptional configurations can spoil the ghost propagator estimate ?

- Larger lattices in order to explore the low momentum limit.

- Finiteness of the infrared limit of α_s ?

- Investigate the spectrum of the Faddeev-Popov operator.