QCD Thermodynamics from an Imaginary μ_B

LEILAT04

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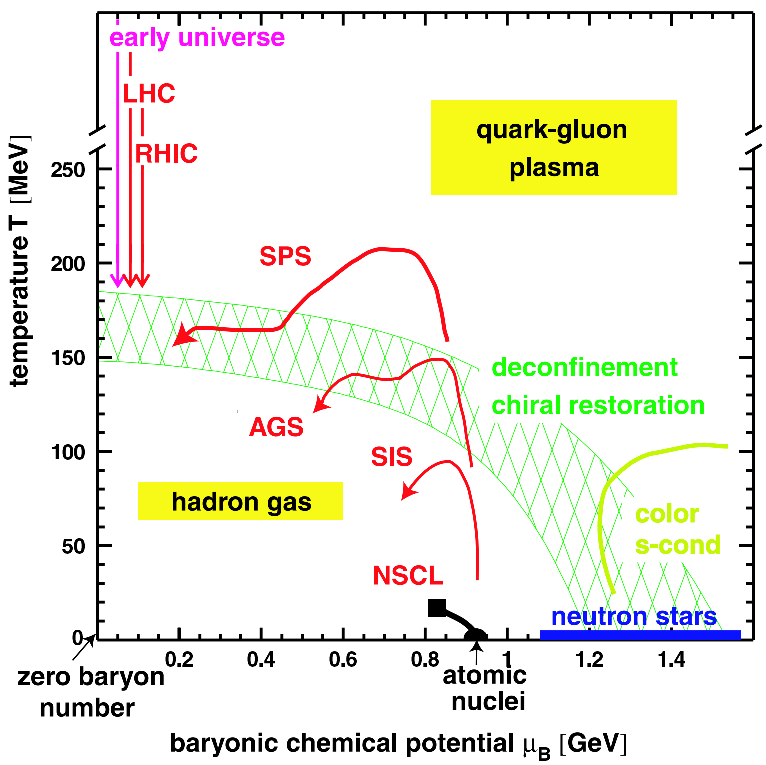
QCD at nonzero baryon density is a twenty years old, and difficult problem. In the last four years a few lattice techniques proven successful for

$$\mu_B/T < 1.$$



Importance Sampling and The Positivity Issue

$$\mathcal{Z}(T,\mu,U) = \int dU e^{-(S_{YM}(U) - log(\det M))}$$

$\det M > 0 \rightarrow$ Importance Sampling

$$M^\dagger(\mu_B) = -M(-\mu_B)$$

$\mu = 0 \rightarrow \det M$ is real

Particles-antiparticles symmetry

Imaginary $\mu \neq 0 \rightarrow \det M$ is real

(Real) Particles-antiparticles symmetry

Real $\mu \neq 0$ Particles-antiparticles asymmetry

$\rightarrow \det M$ is complex in QCD

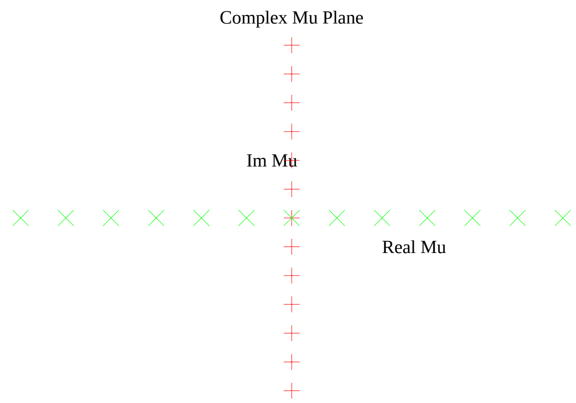
QCD with a real baryon chemical potential:

use information from the accessible region

$$Real \mu = 0, Im \mu \leq 0$$

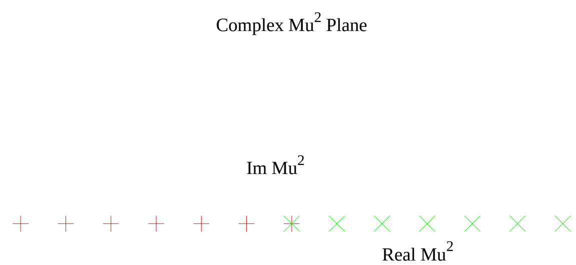
QCD and a Complex μ_B

Analytic continuation *along one arc* in the complex μ plane:



$$Z(\mu) = Z(-\mu)$$

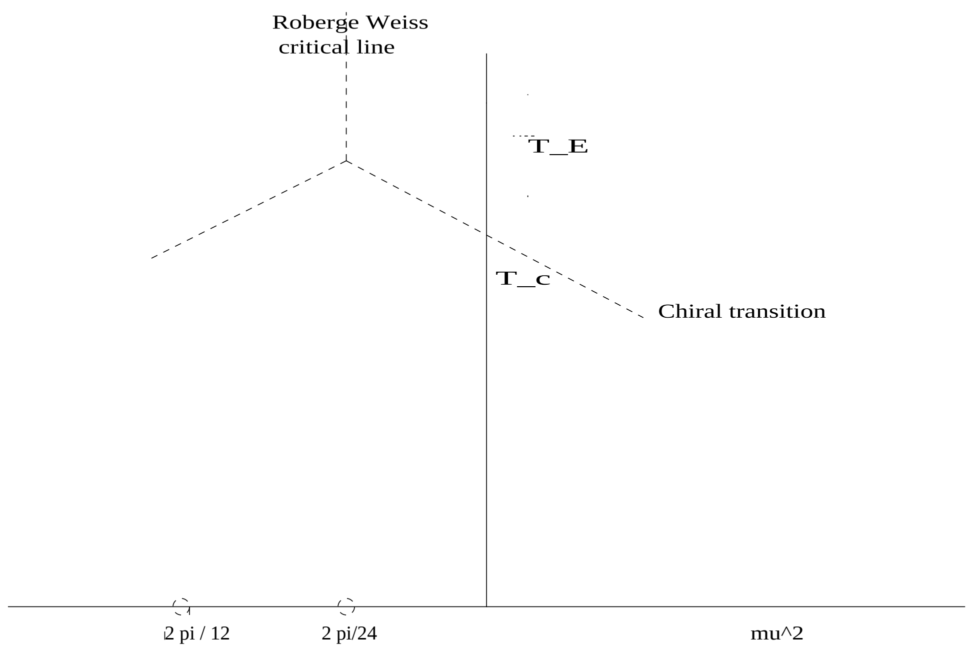
Can be mapped onto the complex μ^2 plane



To define $\mathcal{Z}(\mu^2)$ which is
REAL VALUED FOR REAL μ^2

Analogy with statistical models in external fields

The Phase Diagram in the T, μ_B^2 Plane



Region accessible to simulations: μ^2 real ≤ 0.

Multiparameter Reweighting:
Fodor, Katz, Csikor, Egri, Szabo, Toth

Derivatives: Gupta, Gavai; MILC; QCD-Taro

Expanded Reweighting: Bielefeld-Swansea

Im μ: de Forcrand, Philipsen; D'Elia, MpL

Analytic continuation from an imaginary μ

The method

Strong Coupling MpL
Dimensionally reduced model Hart, Laine, Philipsen
Check: Two colors P. Giudice, A. Papa

QCD Results for

The critical line
2,3,2+1 flavor : Ph. de Forcrand O. Philipsen
4 Flavor: M. D'Elia, MpL *

Order parameter, Pressure, Baryon Density
4 Flavor: M. d'Elia, MpL *

$*$ This Talk

Outline

I-QCD in the T, μ^2 plane

II-The critical line

III-The hadronic phase

IV-The QGP phase

The Critical Line

Analytic continuation of the critical line from an imaginary μ:
Ph. de Forcrand and O. Philpsen

Consideration of the T, μ^2 plane helps the analysis:

Model analysis suggests this parametrization confirmed by numerical results:

$$(T + aT_c)(T - T_c) + k\mu^2 = 0,\, k > 0$$

Gross Neveu Model

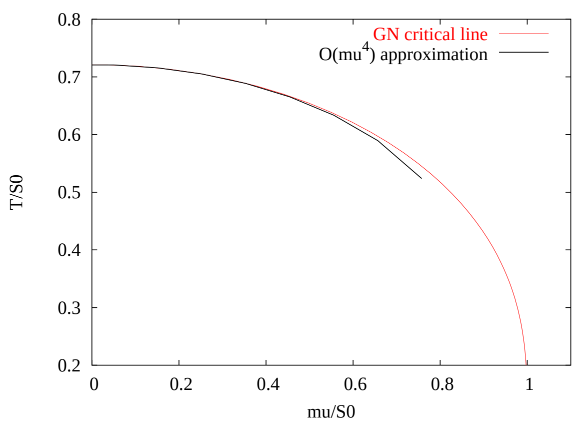
The critical line:

$$1 - \mu/\Sigma_0 = 2T/\Sigma_0 ln(1 + e^{-\mu/T})$$

Reduces to:

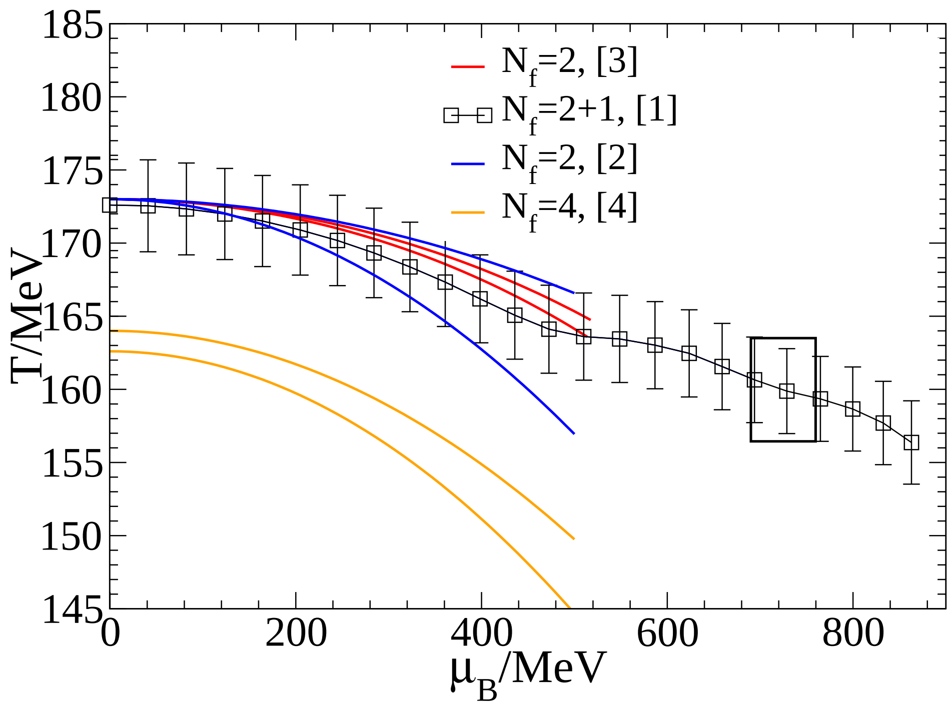
$$T(T - T_c) + \mu^2/(8ln2) = 0$$

Second order approximation good up to $\mu \simeq T_c$



From O. Philipsen and E. Laermann

Ann. Rev. Nucl. Part. Phys. 2003

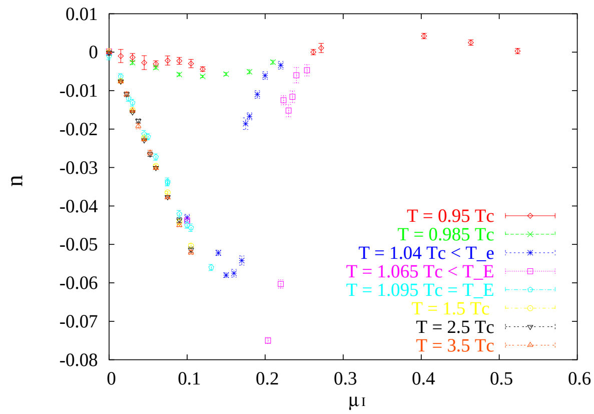


Method	N_f	m_q
reweighting [1]	2+1	$am = 0.025, m_s = 8m_u$, ($m_\pi \approx 300 MeV$)
rew. +Taylor [2]	2	$am = 0.1$, ($m_\pi \approx 600$) MeV
imag. μ [3]	2	$am = 0.025$, ($m_\pi \approx 300$) MeV
imag. μ [4]	4	$am = 0.05$

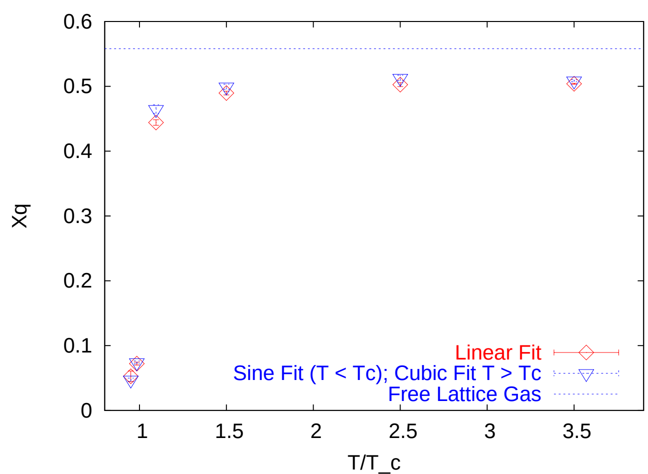
1. Fodor Z and Katz SD, *JHEP* 0203:014 (2002).
2. Allton CR et al., *Phys. Rev. D* 66:074507 (2002).
3. de Forcrand P and Philipsen O, *Nucl. Phys.* B642:290 (2002).
4. D'Elia M and Lombardo MP,*Phys. Rev. D* 1:074507 (2003).

Baryon Density

$$n(T, i\mu) = \frac{\partial log(\mathcal{Z}(T, i\mu)}{V\partial\mu}$$

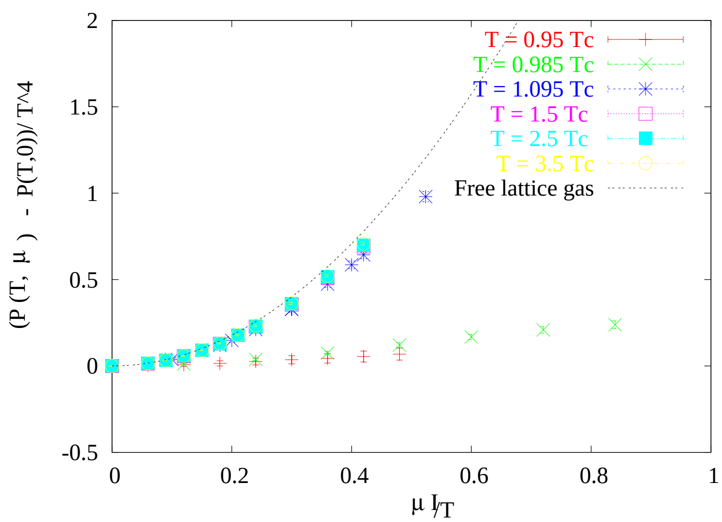


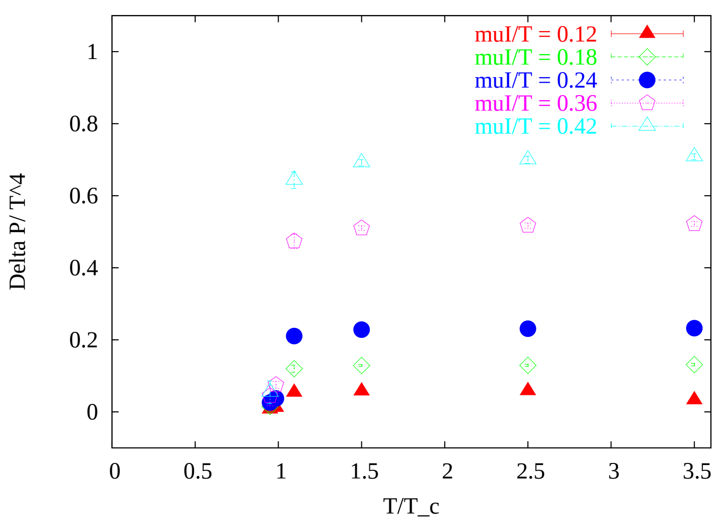
$$\chi_q(\mu = 0) = \frac{i\partial n(i\mu)}{\partial\mu}|_{\mu=0}$$



Pressure: integral method

$$\frac{P(T,i\mu)-P(T,\mu=0)}{T^4} = N_t^4 \int d\mu n(i\mu)$$





Hadronic Phase: $T < T_c$

Observables are smooth , analytic continuation in the $\mu^2 > 0$ half plane possible, but interesting only when $\chi_q(\mu = 0, T) > 0$

Analytic continuation is valid till $\mu < \mu_c(T)$

Even and odd observables

For observables which are even/odd in the chemical potential O_e/O_o we consider two different parametrizations

- A Fourier serie

$$O_e = ae_F + \sum be_F cos(N_c N_t \mu)$$

$$O_o = ao_F + \sum bo_F sin(N_c N_t \mu)$$

- A Taylor serie - useful to compare with $\mu = 0$ results: MILC; Bielefeld-Swansea; R. Gavai and S. Gupta.

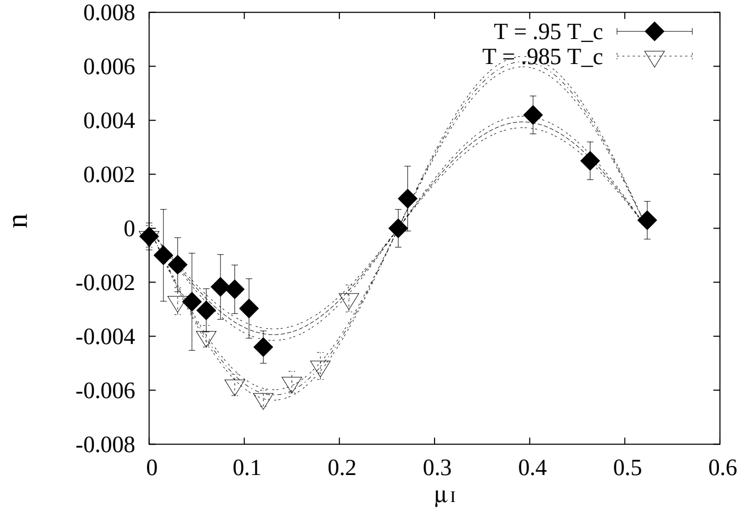
$$O_e = ae_T + be_T \mu^2 + ce_T \mu^4$$

$$O_o = ao_T + bo_T \mu + co_T \mu^3$$

Even/odd observables purely real/imaginary at imaginary chemical potential

Number density

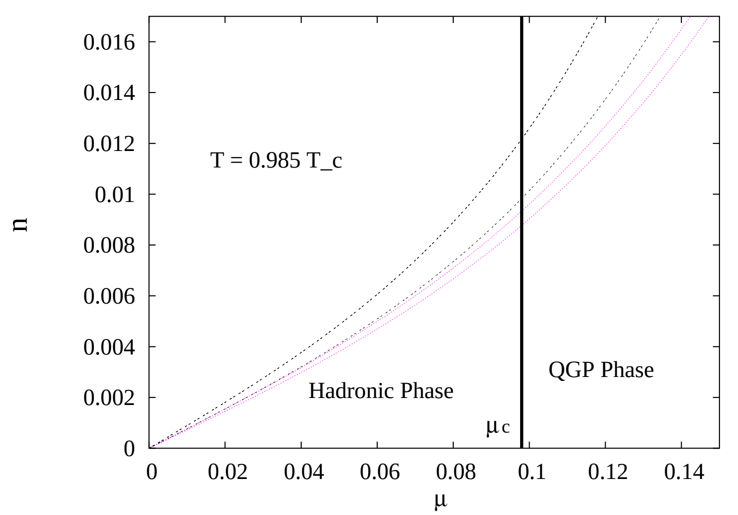
$$n(i\mu) = a_1 sin(i\mu N_c N_T) + b_1 sin(i2\mu N_c N_T)\ {}_{b_1}$$



Analytic continuation up to $\mu = \mu_c(T)$:

$$a_1 sin(i\mu N_c N_T) + a_2 sin(i2\mu N_c N_T)$$

$$\rightarrow a_1 sinh(\mu N_c N_t) + a_2 sinh(i2\mu N_c N_T)$$



critical density at T = .985 $T_c n_c(\mu_c)/T^3 \simeq 0.5$

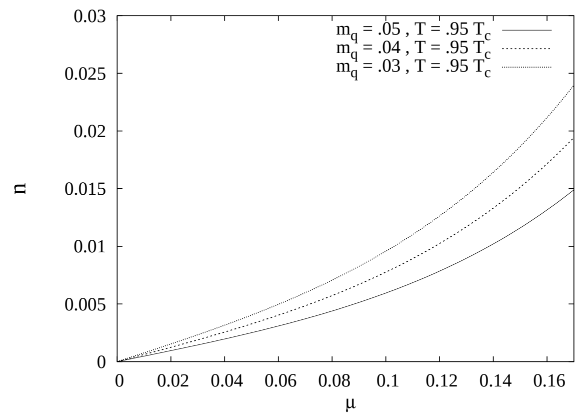
The errors from one and two Fourier coefficient fits are shown.

Mass dependence

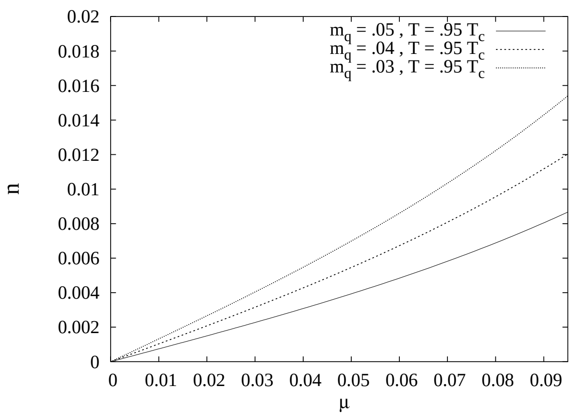
From derivatives: $\partial < \bar{\psi}\psi > / \partial\mu = \partial n(\mu)/\partial m$

When: $\bar{\psi}\psi(\mu, m_q)) = a_C cosh(3\mu N_T) + b_C$ and
$n(\mu, m_q) = a_n sinh(3\mu N_T)$

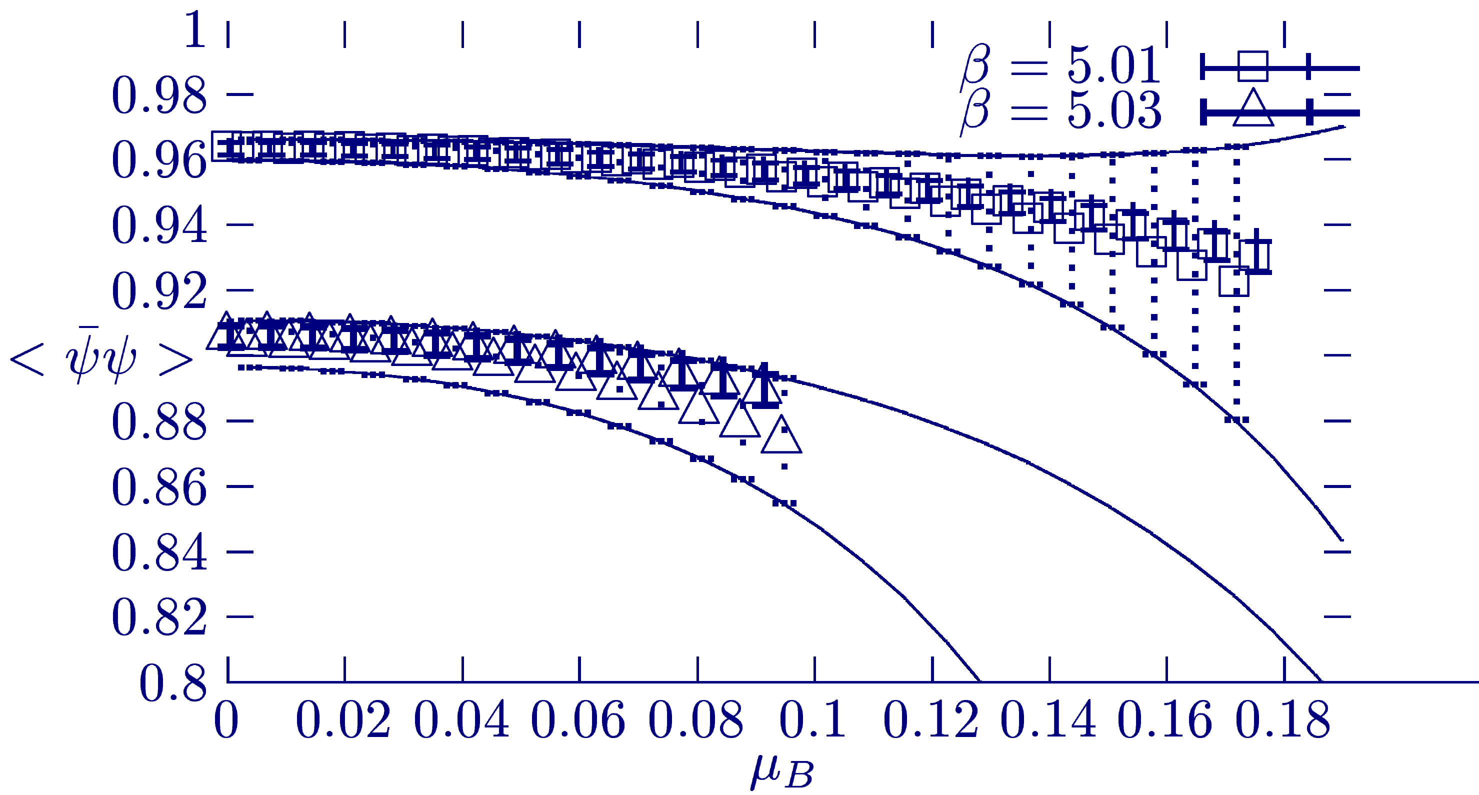
$$\frac{n(\mu,m_q+\Delta m_q)-n(\mu,m_q)}{n(\mu,m_q)} = 3 * N_T a_C / a_n \Delta m$$



$$\frac{\Delta n(\mu,m_q)}{n(\mu,m_q)} \simeq 2.53 \Delta m_q / T$$



$$\frac{\Delta n(\mu,m_q)}{n(\mu,m_q)} \simeq 4.03 \Delta m_q / T$$

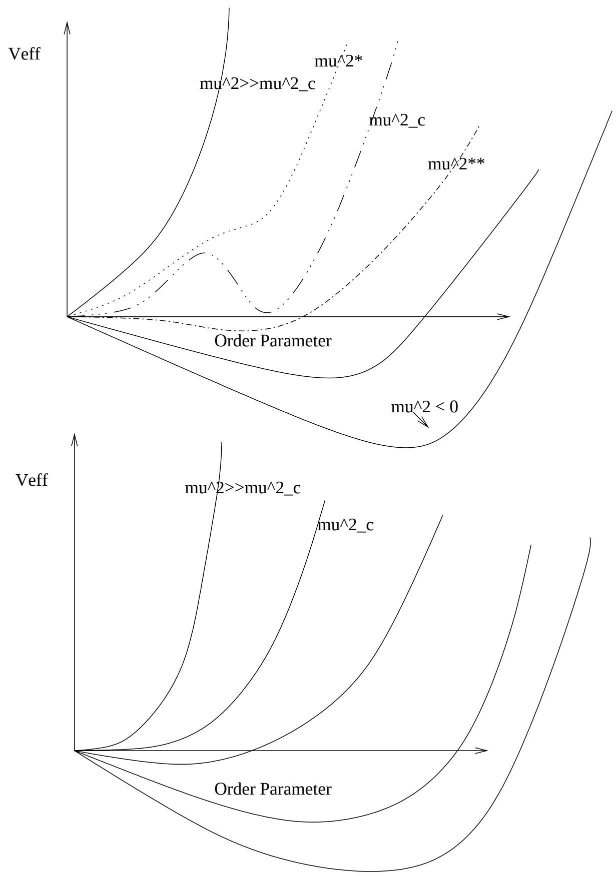


The chiral condensate $< \bar{\psi}\psi > (\mu_c) \neq 0$: first order transition for four flavor QCD, possibly weakaning a bit with temperature.

The metastable branch

The analytic continuation is insensitive to a discontinuous phase transition since it lives on the metastable branch : it follows the secondary minimum and determines the spinodal point.

$$< \bar{\psi}\psi >= A(\mu - \mu^*)^{\beta}$$



1st Order

2nd Order

The discontinuity can be related to $\mu - \mu^*$.

Both shrinks to zero at the endpoint of a first order transition

Hadron Resonance Gas Model

In general, when one Fourier component suffices

$$\frac{\partial <\bar\psi\psi>(\mu,m,T)}{\partial\mu} = \frac{\partial(n(\mu,m,T)}{\partial m}) = -kn(\mu,m,T)$$

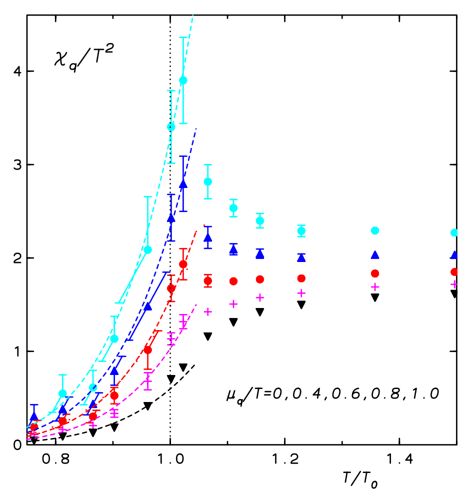
$$n(\mu,m,T) \propto Ae^{-m/T} sinh(3\mu/T)$$

The results can be contrasted with an Hadron Resonance Gas model

$$ln\mathcal{Z}(T,\mu) = \sum_{mesons} ln\mathcal{Z}^M(T,\mu) + \sum_{baryons} ln\mathcal{Z}^B(T,\mu)$$

$$(m_N - \mu_B) > T \rightarrow ln\mathcal{Z}(T,\mu) \simeq \frac{p_B}{T^4} \simeq F(T,m)cosh(3\mu/T)$$

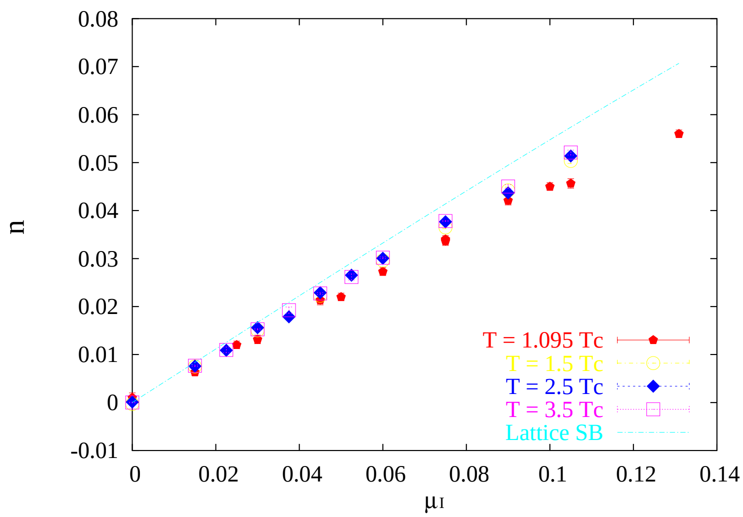
F. Karsch, K. Redlich and A. Tawfik (2003): Hadron Resonance gas model from expanded reweighting



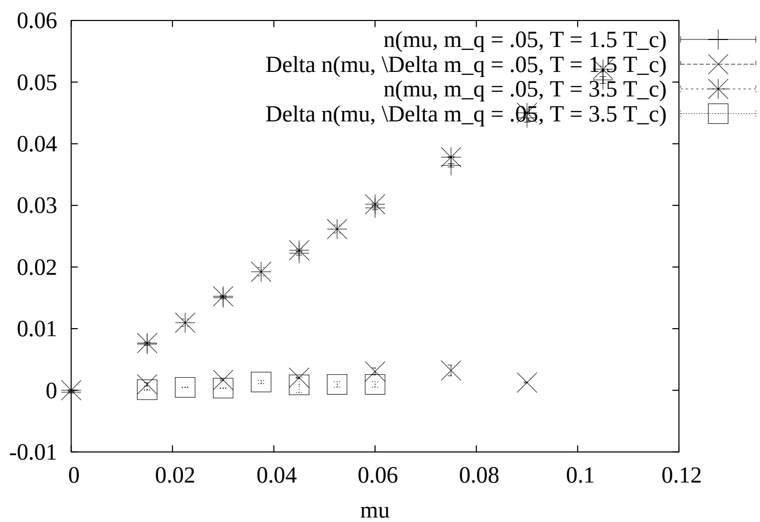
The Hot Phase $T > T_c$

Monitoring the approach to the SB (Lattice) behaviour:

analytic continuation from **real to imaginary** μ_B *of the SB lattice result*



Mass dependence from the derivative of the chiral condensate



Corrections to Free Field

A. Vuorinen 2004:

$\Delta P(\mu) = AT^2\mu^2 + B\mu^4 + \ldots.$

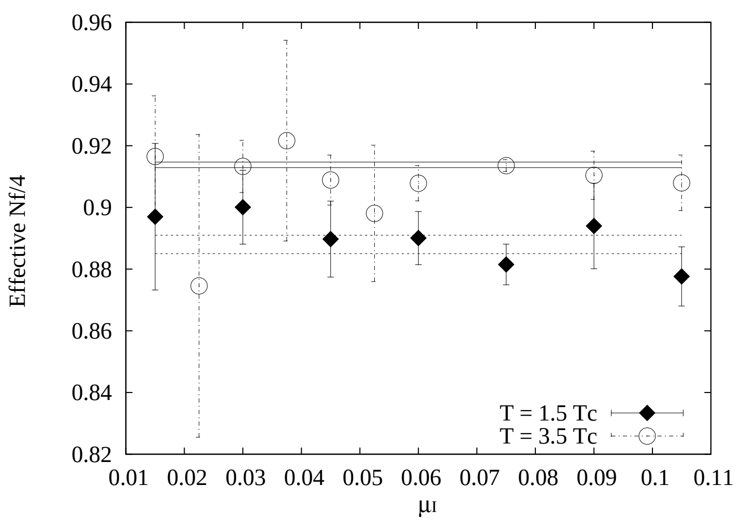
Alternatively (Rafelski, Letessier 2003, Quasiparticle models)

$\Delta P = f(\mu)(AT^2\mu + B\mu^3)$

Trivial posibility: $f(\mu)$: constant effective number of flavors

$$\Delta P = N_f^{eff}(AT^2\mu + B\mu^3)$$

Effective number of active flavors as estimated from the ratio of the lattice results to the lattice free field : appear to be constant for $T \geq 1.5T_c$

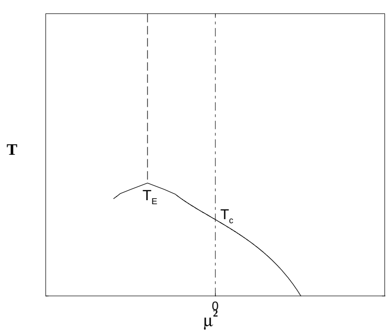


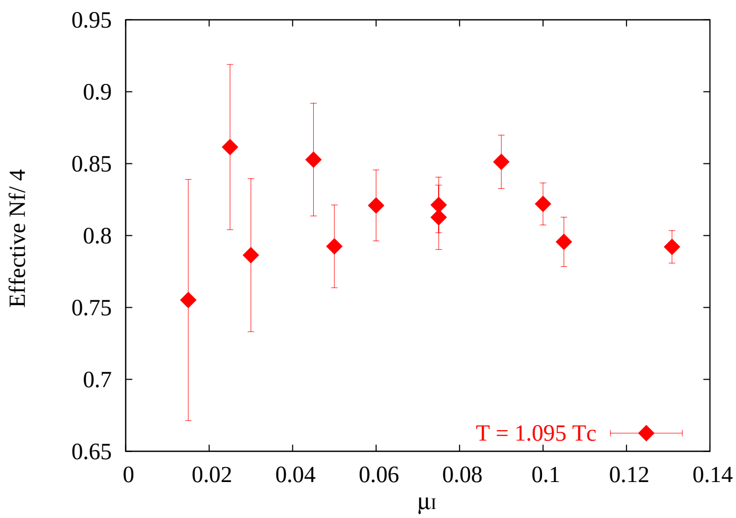
$T_c < T < T_E$

Interplay of thermodynamics and critical behaviour for $T_C < T < T_E \simeq 1.1 T_c$

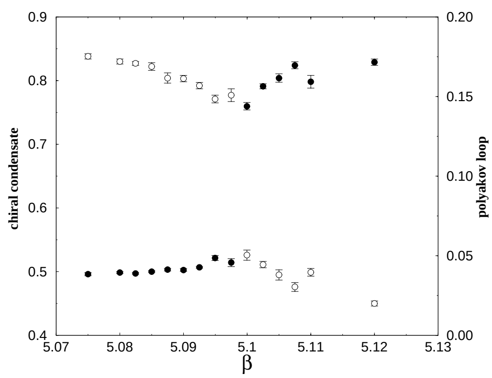
$$logP(\mu, T) \propto (\mu - \mu_c)^\eta$$

Incompatible with a free field for continuous transitions, and for first order transitions of finite strength





Chiral Symmetry and Confinement



Correlation between $<\bar{\psi}\psi>$ and Polyakov loop at $\mu_I = 0.15$, demonstrating the chiral and deconfining nature of the transition at nonzero *real* baryon density.

$$\beta_c(i\mu_I) - \beta_d(i\mu_I) = 0 \rightarrow \beta_c(\mu) - \beta_d(\mu) = 0$$

Summary

Strenght of the method: not limited by volume; gives access to critical values of observables.

Results for Four flavor QCD thermodynamics for
$.985T_c < T < 3.5T_c$

1. The critical line is of first order or a very sharp crossover

$$T/T_c^2 = 1 - 0.0021(2)(\mu/T)^2$$

$$\mu_B < 500Mev$$

.

2. Chiral and "deconfining" transition remain correlated at nonzero baryon density : $T_c^{chiral}(\mu) = T_c^{screening}(\mu)$
3. In the Hadronic Phase $\Delta P \propto\simeq cosh(\mu_B/T)$
4. $n(\mu_c, T = .985T_c, m_q = .05)/T^3 \simeq 0.5$, and the mass corrections $\Delta n = -4.03\Delta m_q/T$.
5. For $T \geq 1.5$ the results are compatible with lattice Stefan Boltzmann with an active fixed number of flavor 0.92 for T=3.5 T_c and 0.89 for $T = 2.5T_c$.
6. For $T \simeq 1.1T_c$ there is room for non trivial deviations fro free field, possibly connected with the chiral transition at $\mu^2 < 0$

Future possibilities for Im μ calculations

Hybrid Methods: combining the Im μ approach with derivatives/reweighting

Assessing the critical behaviour (tricritical point, endpoint) by monitoring the **discontinuities at the critical point**