# Monte Carlo tests of critical exponents in 3D Ising model

J. Kaupužs

Institute of Mathematics
and Computer Science
University of Latvia

29 Rainja Boulevard, LV–1459 Riga, Latvia
e–mail: kaupuzs@latnet.lv

# Critical exponents from a reorganized perturbation theory

J. Kaupužs, Critical exponents predicted by grouping of Feynman diagrams in $\varphi^4$ model, Ann. Phys. (Leipzig) **10** (2001) 299

$$\gamma = \frac{d+2j+4m}{d(1+m+j)-2j}$$
$$\nu = \frac{2(1+m)+j}{d(1+m+j)-2j},$$

where $m \geq 1$ and $j \geq -m$ are integers.

The Ising case: $m = 3$, $j = 0 \Rightarrow \gamma = 7/4$, $\nu = 1$ at $d = 2$ and $\gamma = 5/4$, $\nu = 2/3$ ($\alpha = 0$, $\eta = 1/8$, $\beta = 3/8$) at $d = 3$.

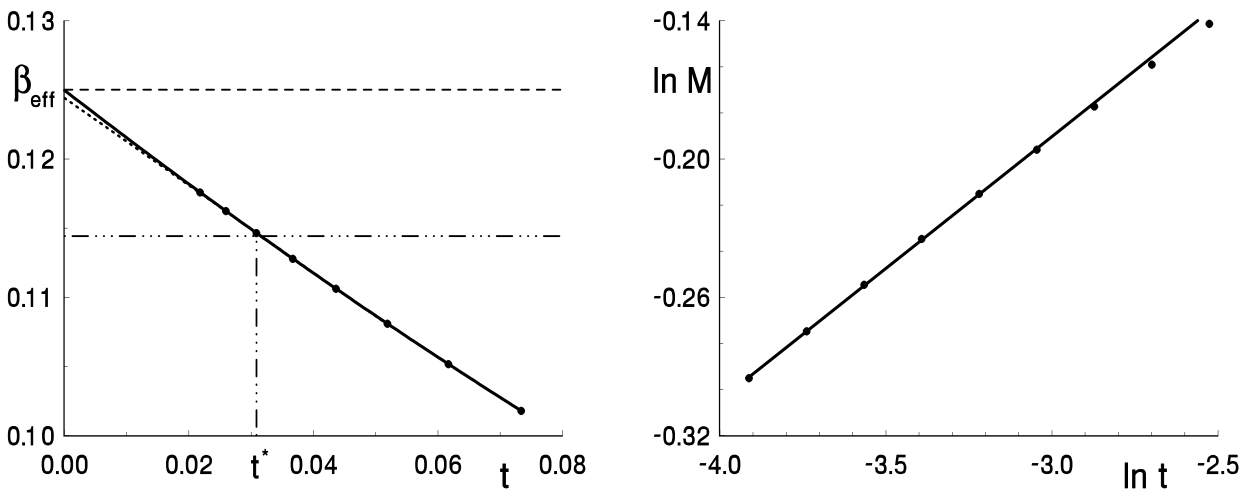
The conventional (RG) values: $\gamma \simeq 1.24$, $\nu \simeq 0.63$, $\alpha \simeq 0.11$ $\eta \simeq 0.0335$, $\beta \simeq 0.326$.

# Ising model

$$H/T = -K \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

where $\sigma_i = \pm 1$, $K$ – the coupling constant.

# How to estimate best the critical exponents?

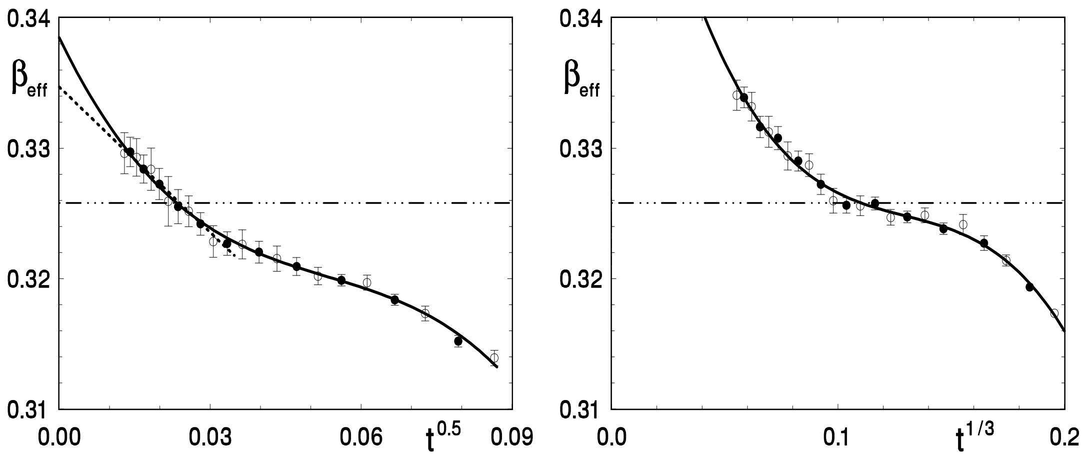



A test estimation of the critical exponent $\beta$ in 2D Ising model by the method of effective exponents (left) and by measuring the slope of the $\ln M$ vs $\ln t$ plot (right). The first method gives $\beta \simeq 0.12496$, the second one – $\beta \simeq 0.1144$.

# Estimation of $\beta$ in 3D Ising model

$$\beta_{eff}(t) = \frac{\ln\langle M^2(t_1, L(t_1))\rangle - \ln\langle M^2(t_2, L(t_2))\rangle}{2(\ln t_1 - \ln t_2)},$$

where $\ln t = (\ln t_1 + \ln t_2)/2$, $L(t)t^\nu = const.$
$t = (K - K_c)/K_c$ – the reduced temperature.




The effective critical exponent $\beta_{eff}$ vs $t^\theta$ with $\theta = 0.5$, $\nu = 0.63$, $K_c = 0.22165386(51)$ (left) and $\theta = 1/3$, $\nu = 2/3$, $K_c = 0.22165395(46)$ (right). The right–hand–side plot: $\beta = 0.366(16)$ in agreement with our analytical value 3/8. The dot–dot–dashed line: RG prediction $\beta \simeq 0.3258$. The simulation range: $t \geq 0.000086$, $L \leq 410$; sizes $48 \leq L \leq 384$ used for estimation of $K_c$ from Binder cumulant data $(\langle m^4\rangle/\langle m^2\rangle^2 = 1.6 \Rightarrow$ pseudocritical couplings). In literature: $K_c = 0.2216544(3)$, $K_c = 0.22165459(10)$ (MC); $K_c = 0.221654(1)$ (HT).

## Singularity of specific heat

**Conventional statements for 3D case:**

$$C_V \sim t^{-\alpha} f\left(L^{1/\nu} t\right)\left(1 + \mathcal{O}\left(t^{\nu\omega}\right)\right)$$

$$C_V^{\text{max}}(L) \propto L^{\alpha/\nu}\left(1 + \mathcal{O}\left(L^{-\omega}\right)\right)$$

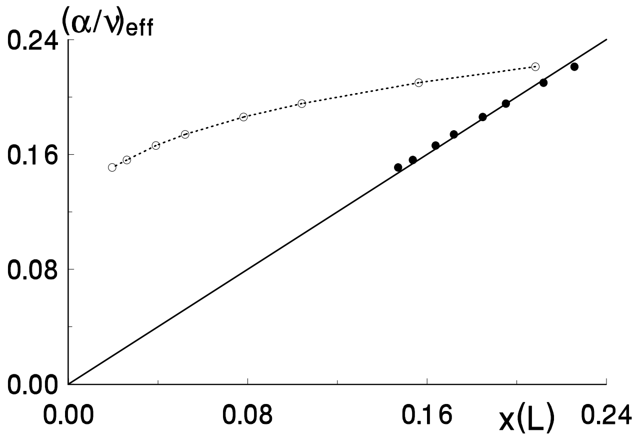
$$\Longrightarrow (\alpha/\nu)_{eff} \simeq (\alpha/\nu) + const \cdot L^{-\omega}$$

where

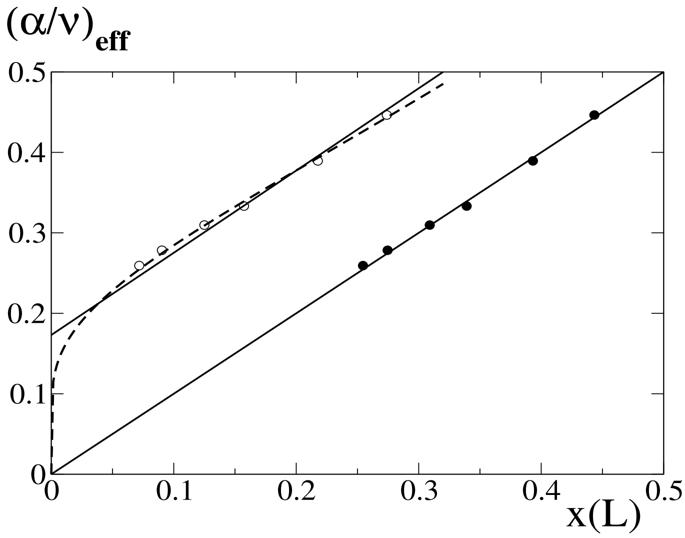
$$(\alpha/\nu)_{eff}(L) = \ln\left[C_V^{\text{max}}(2L)/C_V^{\text{max}}(L/2)\right] / \ln 4$$

**We allow a logarithmic singularity (like in 2D case):**

$$(\alpha/\nu)_{eff} \simeq \frac{1}{\ln(L/L_0)}$$




The effective exponent $(\alpha/\nu)_{eff}$ of 2D Ising model depending on $x(L) = 1/\ln(L/L_0)$ with $L_0 = 0.572$ (solid circles) and $x(L) = 10L^{-1}$ (empty circles).




We find from MC simulations $C_V^{\max}(L)$ together with the pseudo–critical couplings in 3D Ising model and evaluate $(\alpha/\nu)_{eff}$ depending on $x(L) = 1/\ln(L/1.258)$ (solid circles) and $x(L) = 2L^{-0.8}$ (empty circles).

# Other kind of estimations

$$\alpha = 2 - d\nu$$

$$\frac{dU}{dK} \propto L^{1/\nu} \quad \text{at} \quad U = \frac{\langle m^4 \rangle}{\langle m^2 \rangle^2} = const$$

The RG value $1/\nu \simeq 1.587$, the logarithmic singularity of specific heat: $\alpha = 0$, $1/\nu = 1.5$.

Estimates of $1/\nu$ (effective exponents) from the mean slopes of log–log plots at $U = 1.6$

| | | |
|---|---|---|
| 1.5889(18) | | $L \in [16; 64]$ |
| 1.5901(27) | | $L \in [32; 128]$ |
| 1.5812(41) | **1.5817(41)** | $L \in [48; 192]$ |
| 1.5851(48) | **1.5821(38)** | $L \in [64; 256]$ |
| 1.5805(50) | **1.5802(47)** | $L \in [96; 384]$ |

# CONCLUSIONS

It has been claimed that the conventional RG exponents are false and the true values are simple fractions. The actual numerical (MC) tests in 3D Ising model show the following.

- The estimation of the effective exponent $\beta$ from the magnetization data much closer to criticality than in the published literature shows that the asymptotic value could be consistent with our prediction $\beta = 3/8$.

- The finite–size scaling of the maximum values of specific heat is in favour of the logarithmic singularity ($\alpha = 0$).

- The estimation of $1/\nu$ from the derivatives of the Binder cumulant is well consistent with the RG value 1.587 if not too large system sizes ($L \leq 128$) are considered. Larger sizes give smaller $1/\nu$.