SU(3) calorons and their constituents

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in collaboration with:

- B. V. Martemyanov, A. I. Veselov (ITEP Moscow)
- C. Gattringer *et al.* (U Regensburg)
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Overview

- Introduction
- Calorons with non-trivial holonomy
- Dyonic structure of configurations in $SU(2)$ LGT
- Dyon recombination at low temperature
- $SU(3)$ calorons on the lattice
- Have calorons been observed in Monte Carlo $SU(3)$ samples ?
- Summary and outlook

I. Introduction

1. Is the instanton gas/liquid exhausting the QCD vacuum ?

   Criticism: confinement ignored, apparently unrelated.

   Not only no asymptotic string tension, also no Casimir scaling at intermediate distances (Simonov).

   Critical study of the instanton gas by Fukushima, EMI, and Toki [2001] (agrees, contrary to previous claims by the RCNP group).

2. Has the instanton content been undoubtedly found on the lattice ?

   Sizes and densities strongly method dependent (cooling, smoothing, cycling).

   If density (as estimated from topological susceptibility) is put in to stop cooling, sizes are in rough agreement with phenomenology (UKQCD).

   Restricted (overimproved) cooling shows: multiplicity and properties of topological clusters strongly dependent on "classicality". No strong correlations found which should/could "improve" the random (zero temperature) instanton liquid.

3. What other degrees of freedom are important ?

   If instanton content is determined by RG cycling (DeGrand, Hasenfratz, and Kovacs) the configurations are still confining, which is not explained by the instanton content.

   Kovacs: Hadronic correlators can be explained only with additional record of toron degrees of freedom (holonomies) of the lattice configurations.

   Guess : Are topological lumps and holonomies interrelated ?

4. Are there fractional instantons ? Are they desirable ? Can they be related to confinement ?

   Fractional instantons have been long with us:

   Fateev, Frolov, and Schwarz [1979] for the 2D σ-model.

   Callan, Dashen, and Gross: halfinstantons (singular merons) as the link between instantons and confinement in gluodynamics [1978] (recently revived by Negele et al.)

   Diakonov and Maul: instanton melting in the CP^{N-1} model ?

   In $\mathcal{N} = 1$ supersymmetric Yang Mills theory : gluino condensation and confinement (Davies, Hollowood, Khoze, and Mattis [1999]; Diakonov and Petrov [2003]).

5. Are fractional instanton solutions known ?

   Yes, but only at finite temperature: calorons with nontrivial holonomy (Kraan and van Baal, Lee and Lu), which have (anti) selfdual monopole (dyon) constituents.

   They are not covered by the instanton model extended to finite T (originating from Gross, Pisarski, and Yaffe [1981]) !

6. Could dissociating calorons be responsible for the onset of confinement ?

   Decomposition of calorons and anticalorons into dyonic constituents depends on the holonomy in the environment.

   The (partial and total) pressure of the constituents would depend on temperature and holonomy.

   Diakonov suggestively demonstrated: in the effective potential of the Polyakov loop the pressure of constituents can overcompensate the perturbative suppresion of non-trivial holonomy (prevailing in deconfinement) such that

   $\rightarrow$ average Polyakov loop $= 0$ at some T.

7. Could this be describable semiclassically ?

   Partly yes.

As a step toward this goal, see the recent paper Diakonov, Gromov, Petrov, and Slizovskiy QUANTUM WEIGHTS OF DYONS AND OF INSTANTONS WITH NONTRIVIAL HOLONOMY e-Print Archive: hep-th/0404042.

They calculated (for SU(2)) the conditions of caloron breaking into dyons:

at $T \approx \Lambda$ trivial holonomy becomes unstable, and the Polyakov loop "rolls" towards $L = 0$.

The dissociated gas is not semiclassically described.

What to do on the lattice ?

Investigate the temperature range where the new degrees of freedom might become manifest.

It remains to be asked whether nontrivial instantons (probably not dissociated, but coupled to the holonomy) are more appropriate for $T = 0$, where the random instanton liquid model gives a good phenomenological description.

This is closely related to the question how the near-to-zero fermionic modes propagate through space-time.

II. Calorons with non-trivial holonomy

KvBLL solutions :

T. C. Kraan and P. van Baal,

Phys. Lett. **B 428** 268 (1998),
Phys. Lett. **B 435** 389 (1998),
Nucl. Phys. **B 533** 627 (1998)

K. Lee and C. Lu

Phys. Rev. **D 58** 025011 (1998)

Here, the order parameter of deconfinement (untraced Polyakov loop) enters the construction of the classical background configuration

Example of $SU(2)$

in general, holonomy $\notin \mathbf{Z}(N_c)$:

$$P(\vec{x}) = P \exp(i \int_0^{b=1/T} A_4(\vec{x},t)dt)$$

$$\stackrel{|\vec{x}|\to\infty}{\longrightarrow} \quad P_\infty = e^{2\pi i \omega \tau_3} \notin \mathbf{Z}(2)$$

vector potential :

$$A_\mu^{\mathrm{KvB}} = \frac{1}{2}\bar{\eta}^3_{\mu\nu}\tau_3\partial_\nu \log\phi + \delta_{\mu,4}\ 2\pi\omega\tau_3$$

$$+\frac{1}{2}\ \phi\ \mathrm{Re}\left((\bar{\eta}^1_{\mu\nu} - i\bar{\eta}^2_{\mu\nu})(\tau_1 + i\tau_2)(\partial_\nu + 4\pi i\omega\delta_{\nu,4})\tilde{\chi}\right)$$

where

$$\phi(x) = \frac{\psi(x)}{\hat{\psi}(x)}$$

with

$$\psi(x) = -\cos(2\pi t) + \cosh(4\pi r\bar{\omega})\cosh(4\pi s\omega)$$
$$+\frac{r^2+s^2+\pi^2\rho^4}{2rs}\sinh(4\pi r\bar{\omega})\sinh(4\pi s\omega)$$
$$+\frac{\pi\rho^2}{s}\sinh(4\pi s\omega)\cosh(4\pi r\bar{\omega})$$
$$+\frac{\pi\rho^2}{r}\sinh(4\pi r\bar{\omega})\cosh(4\pi s\omega)$$

and

$$\hat{\psi}(x) = -\cos(2\pi t) + \cosh(4\pi r\bar{\omega})\cosh(4\pi s\omega)$$
$$+\frac{r^2+s^2-\pi^2\rho^4}{2rs}\sinh(4\pi r\bar{\omega})\sinh(4\pi s\omega)$$

finally

$$\tilde{\chi}(x) = \frac{1}{\psi}\left\{e^{-2\pi i t}\frac{\pi\rho^2}{s}\sinh(4\pi s\omega) + \frac{\pi\rho^2}{r}\sinh(4\pi r\bar{\omega})\right\}$$

The holonomy parameter

$$\bar{\omega} = 1/2 - \omega, \quad 0 \leq \omega \leq 1/2$$

determines the Polyakov loop $L = cos(2\pi\omega)$

The solution has two centers $\vec{x}_1$ and $\vec{x}_2$, and the potential depends on the two distances

$$r = |\vec{x} - \vec{x}_1|, \quad s = |\vec{x} - \vec{x}_2|$$

Properties of the KvB solutions :

1. periodic in $b = 1/T$

2. scale-size *vs.* distance: $\pi\rho^2\ T = |\vec{x}_1 - \vec{x}_2| = d$

3. limiting cases :
   - $\omega \to 0$ or $\bar{\omega} \to 0$ $\to$ "old" caloron
   - $|\vec{x}_1 - \vec{x}_2|$ large $\to$ solution dissociates into two static BPS monopoles (DD) with action ratio $= \bar{\omega}/\omega$, *i.e.* unbalanced for $L \neq 0$
   - $|\vec{x}_1 - \vec{x}_2|$ small $\to$ collapse into a single "caloron", irrespective of holonomy

4. (anti)selfdual with topological charge $Q_t = \pm 1$,

5. degenerate eigenvalues of holonomy, *i.e.* $L(\vec{x}) = \frac{1}{2}\mathrm{tr}P(\vec{x}) \to \pm 1$ close to centers $\vec{x} \simeq \vec{x}_{1,2}$

6. localization of a fermionic zero-mode on one of the centers :
   - time-antiperiodic b.c.: around the center with $L(\vec{x}_1) = -1$, with shape (for large d)

$$|\psi^-(x)|^2 = -\frac{1}{4\pi}\partial_\mu^2 \left[\tanh(2\pi r \bar{\omega})/r\right]$$

   - time-periodic b.c.: around the center with $L(\vec{x}_1) = +1$, with shape (for large d)

$$|\psi^+(x)|^2 = -\frac{1}{4\pi}\partial_\mu^2 \left[\tanh(2\pi s \omega)/s\right]$$

Constituents (if well-separated) can be described as

two BPS dyons (caloron) or two BPS anti-dyons (anti-caloron)

with opposite magnetic charge and electric charge and same sign of fractional topological charge

For $SU(2)$ we can call them N (north) and S (south).

$$N \rightarrow (q_{el} = +1\,;\,q_{mag} = +1\,;\,q_{top} > 0)$$

$$S \rightarrow (q_{el} = -1\,;\,q_{mag} = -1\,;\,q_{top} > 0)$$

$$\bar{N} \rightarrow (q_{el} = +1\,;\,q_{mag} = -1\,;\,q_{top} < 0)$$

$$\bar{S} \rightarrow (q_{el} = -1\,;\,q_{mag} = +1\,;\,q_{top} < 0)$$

Can they be considered in part of the moduli space as independent instanton "quarks" ?

This is now the subject of our lattice experiments in progress in Leiden, concentrating on various-Q caloron solutions for $SU(2)$.

III. Dyonic structure of configurations in $SU(2)$ LGT

Our aim was to show that in lattice configurations, eventually in a certain temperature interval, the carriers of unit topological charge appear partly dissociated into dyonic constituents.

Their shape could then approximately fitted by the analytical KvBLL solution.

Method :

- cooling to semiclassical levels (defined by minimal violation of lattice equations of motion)
- recording the configuration, including the value of "non-staticity"
- give a complementary description of these configurations by gluonic and fermionic observables (zero modes exist due to global topological charge $Q \neq 0$)
- detailed fits relating both aspects

Result :

Non-trivial configurations obtainable in this way only from lattices in the confinement phase ($L = 0$) !

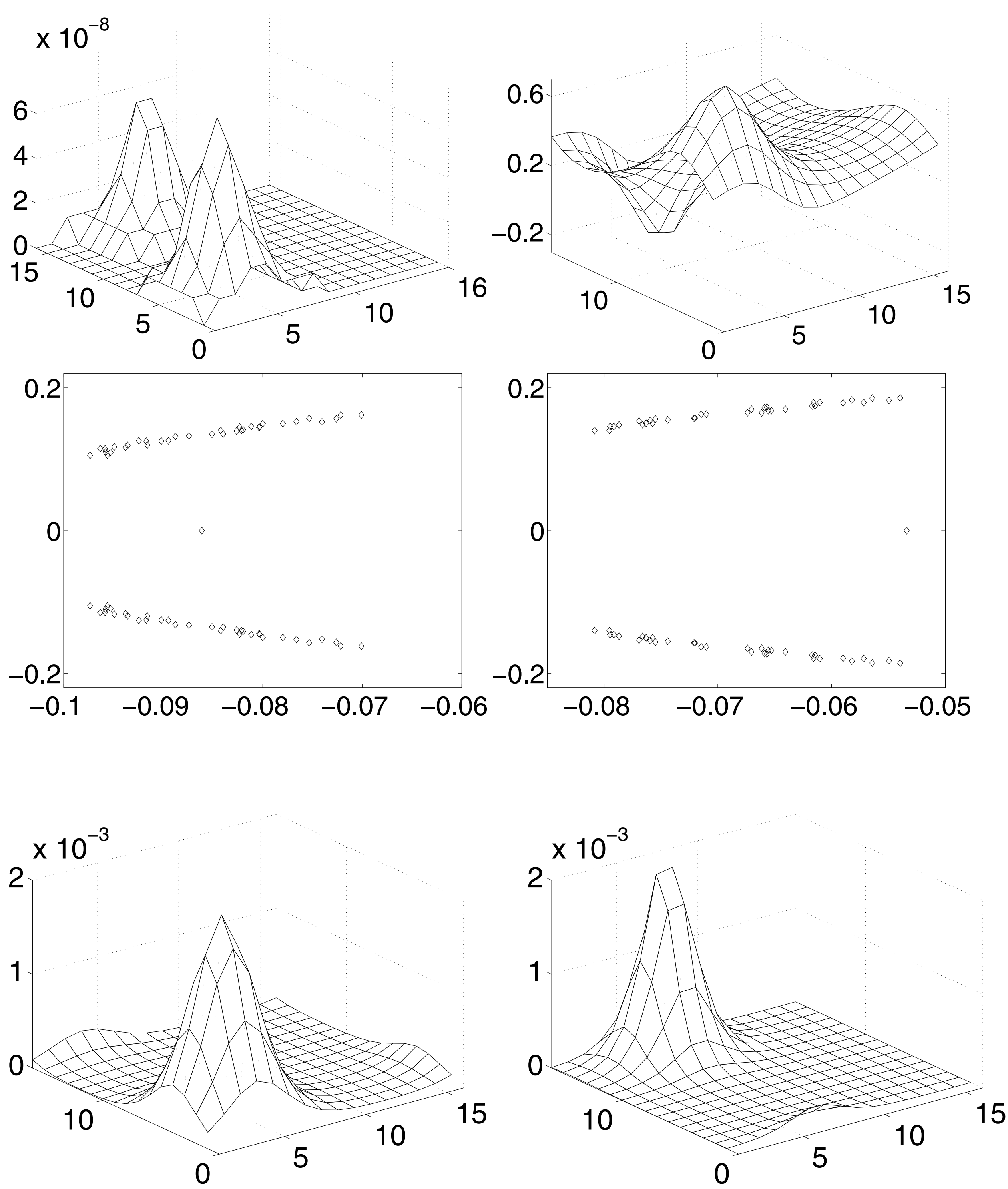

Figure 1: Portraits of a selfdual DD pair cooled from $\beta = 2.2$ on a $16^3 \times 4$ lattice. Upper row: cuts of the topological charge density (left) and of the Polyakov loop (right); lowest fermionic eigenvalues (middle row) and cuts of the real-mode scalar densities (bottom row), for time-periodic (left) and time-antiperiodic (right) fermionic boundary conditions.

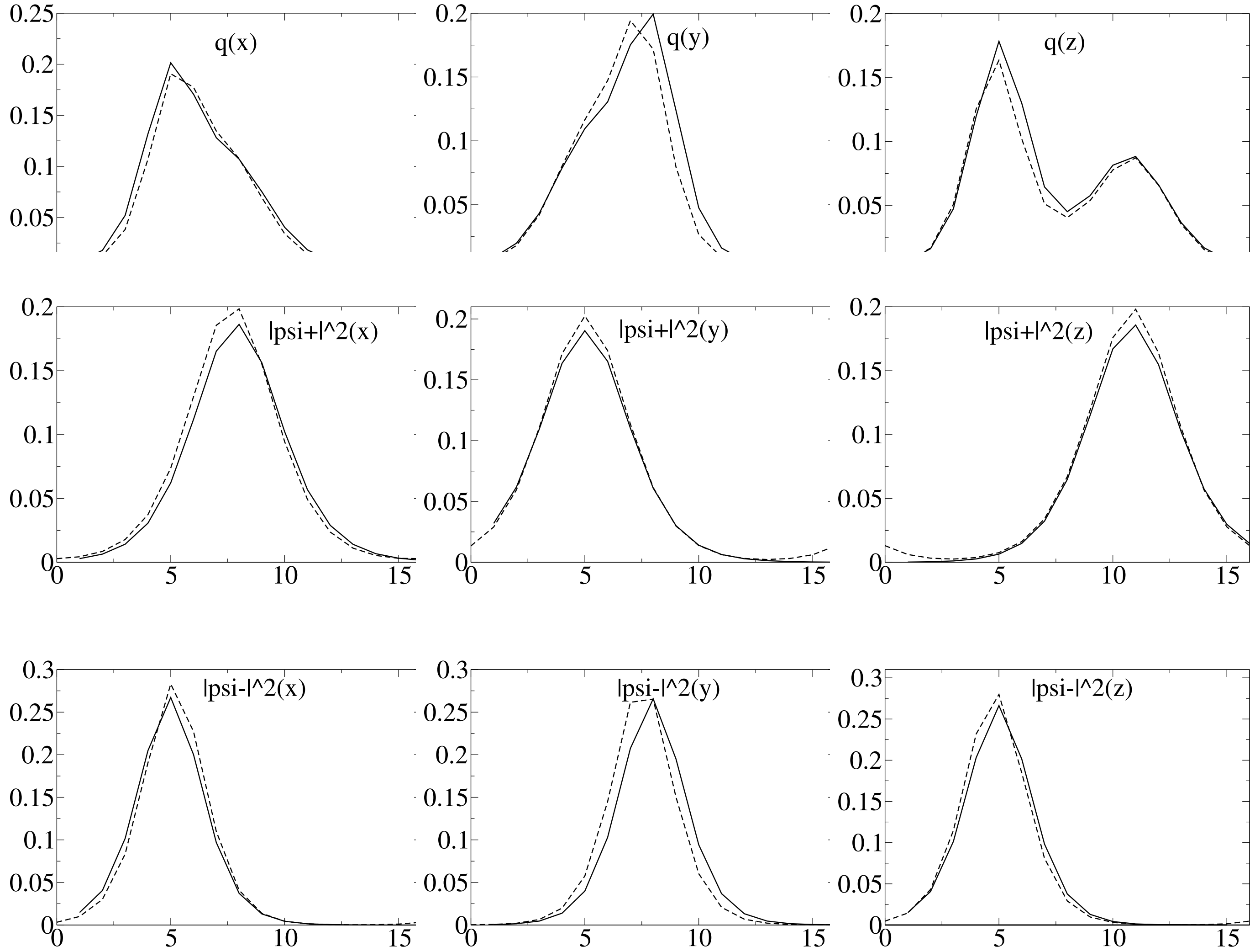

Figure 2: Fitting the action density of a selfdual DD pair cooled from $\beta = 2.2$ on a $16^3 \times 4$ lattice with the caloron solution. Upper row: predicted *vs.* measured topological charge density; middle row: predicted *vs.* measured scalar density of the fermion zero mode with periodic b.c.; bottom row: the same for antiperiodic b.c.

Conclusions for the new calorons from the $SU(2)$ example

- Constituents might be well separated or not, corresponding to $|\vec{x}_1 - \vec{x}_2| >> b$ or not
- Constituents might be well separated or not, depending on the asymptotic holonomy
- with a "non-staticity" $\delta_t < 0.27$ ($\delta_t > 0.27$) constituents can be (cannot be) distinguished by action profile (this limit estimated from latticized $SU(2)$ calorons)
- If not well-separated, a rapid change of Polyakov loop inside a lump of action is the signal
- Localized fermionic zero-mode, as required by the index theorem, is hopping from one dyon to the other with change of temporal boundary condition (say, from periodic to antiperiodic)

Cooling with periodic b.c. preserves holonomy only in average

Therefore, if no cut is applied, one may also find calorons resembling the trivial one !

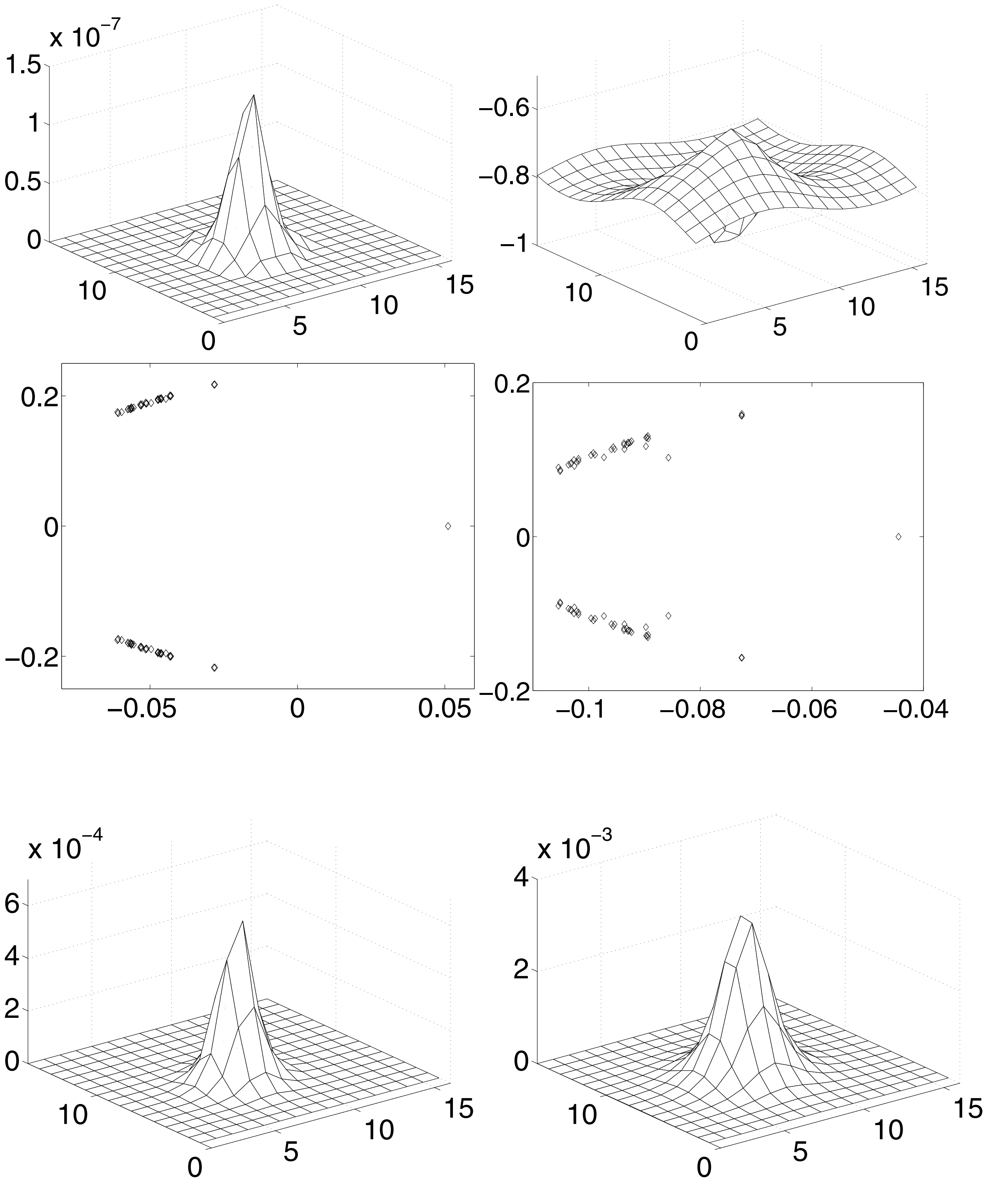

Figure 3: Portraits of a selfdual CAL configuration cooled from $\beta = 2.2$ on a $16^3 \times 4$ lattice. Upper row: cuts of the topological charge density (left) and of the Polyakov loop (right); lowest fermionic eigenvalues (middle row) and cuts of the real-mode scalar densities (bottom row), for time-periodic (left) and time-antiperiodic (right) fermionic boundary conditions. Only the width is changing.

Unexpected findings ...
.... beyond exact solutions

- Dyon-anti-dyon pairs ($D\bar{D}$)

  Such configurations have also emerged under cooling down from $\beta = 2.2$ (confinement) with periodic boundary conditions on a $16^3 \times 4$ lattice

- Higher action semiclassical configurations

  On a larger ($24^3 \times 4$) lattice, cooling down from $\beta = 2.2$ (confinement) with fixed holonomy boundary conditions we found cascades of multi-dyon-antidyon configurations

  At the $S = 3S_{inst}$ level, a $Q = 2$ configuration shows the necessary real (zero) modes, as well as near-zero modes

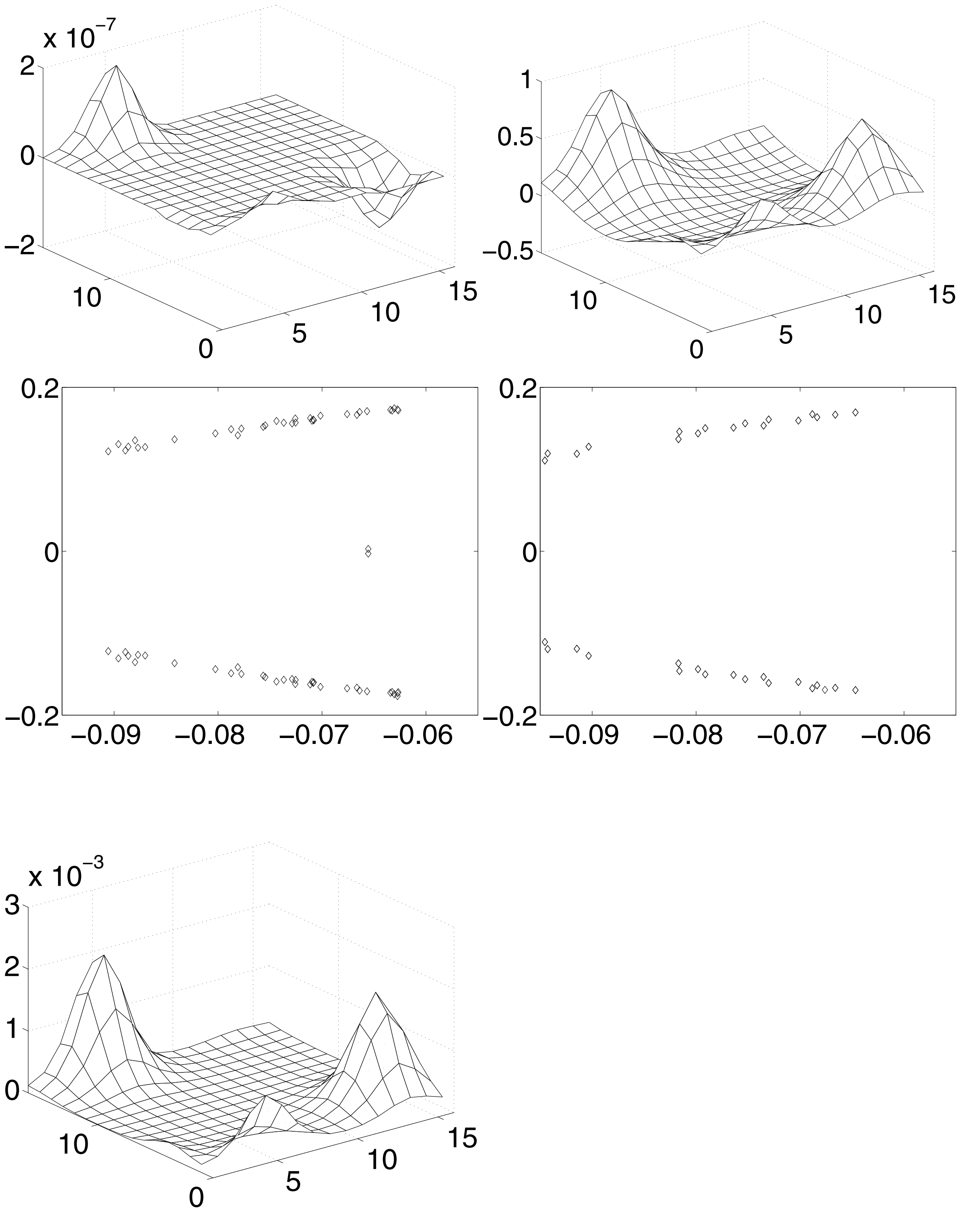

Figure 4: Portraits of a mixed-duality $D\bar{D}$ pair cooled from $\beta = 2.2$ on a $16^3 \times 4$ lattice. Upper row: cuts of the topological charge density (left) and of the Polyakov loop (right); middle row: lowest fermionic eigenvalues showing a pair of almost-real modes only in the case of periodic b.c. (left); bottom row: the scalar densities of the two almost-real modes.

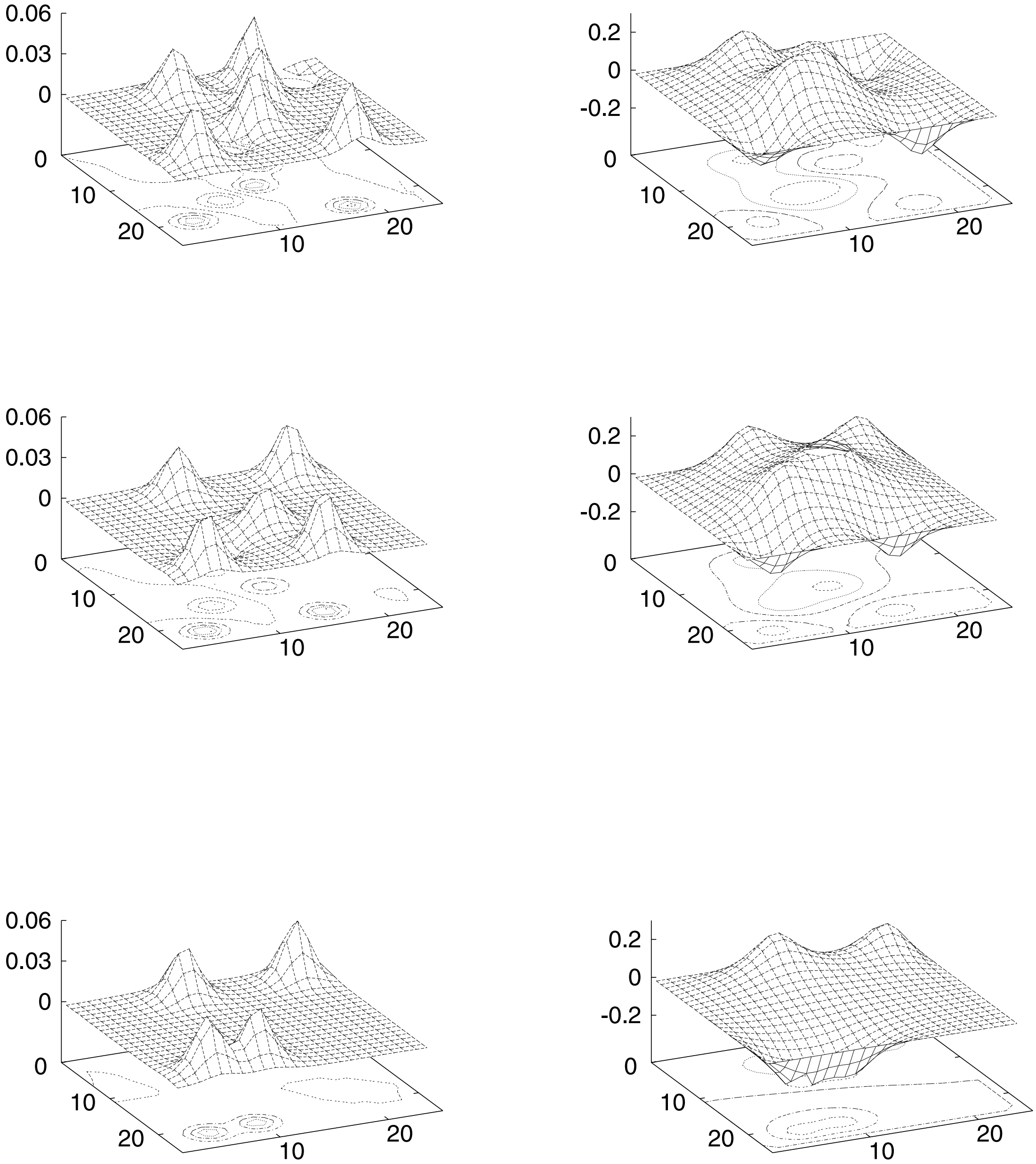

Figure 5: Topological charge density and Polyakov loop of a multi-dyon configuration cascading down in action. Upper row: at $S = 4S_{inst}$; middle row: at $S = 3S_{inst}$; bottom row: finally stabilized as a $Q = 2$ caloron

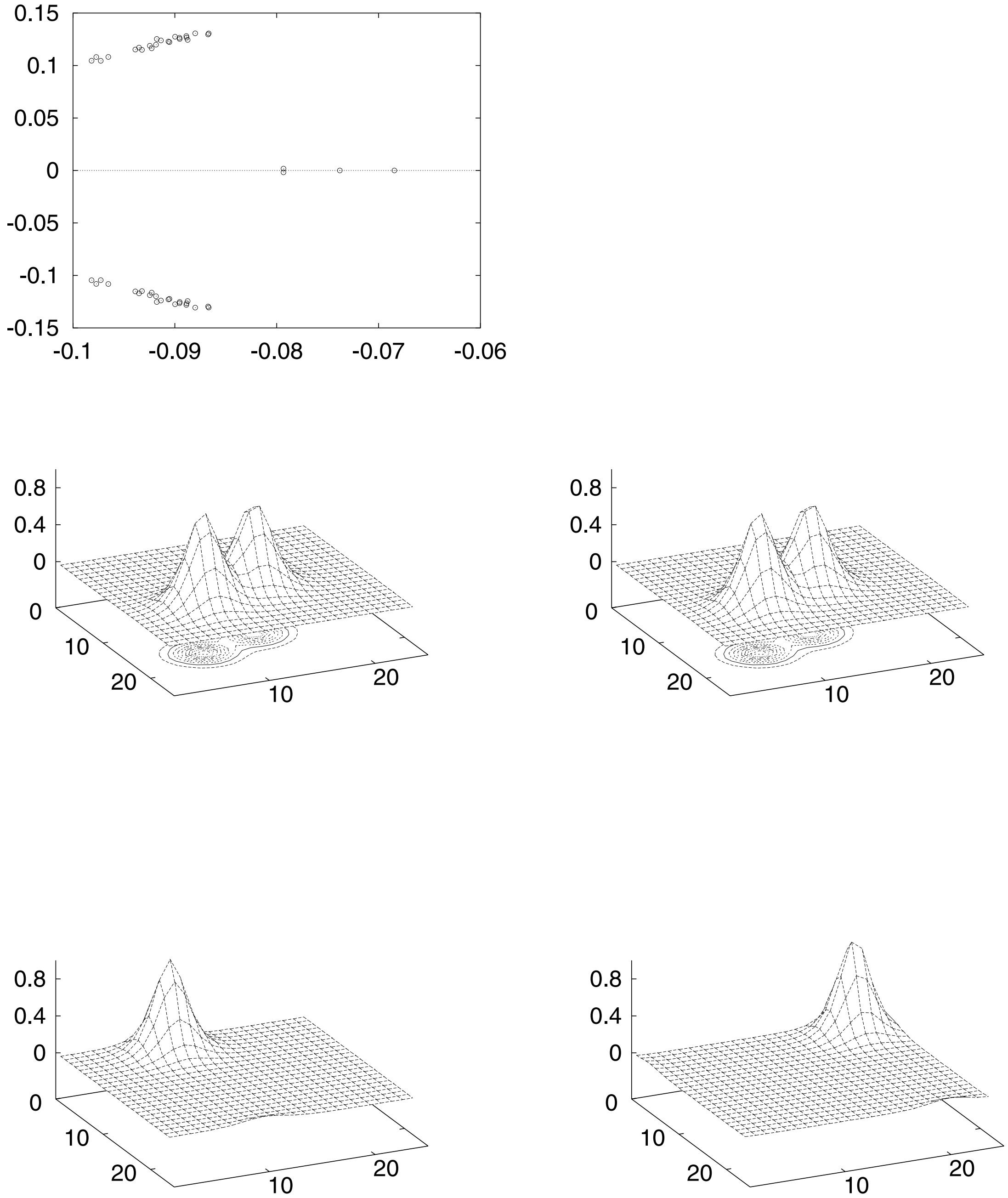

Figure 6: Low lying fermion modes for periodic fermionic b.c. Upper row: two real and one pair of almost-real eigenvalues; middle row: localization of the almost real modes; bottom row: localization of the two distinct real modes.

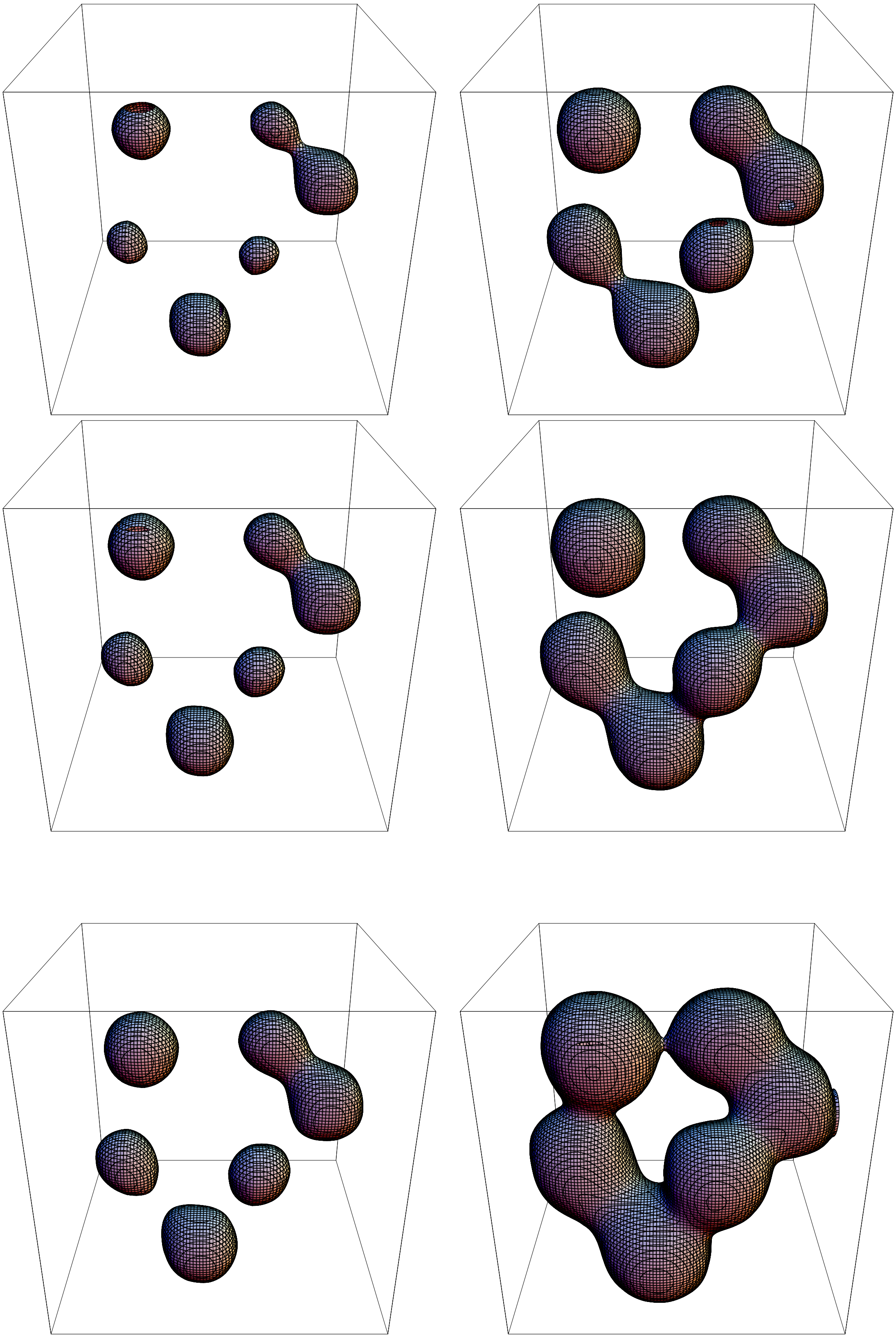

Figure 7: Iso-surfaces of topological density (with rising density) for a $Q = 3$ multi-caloron (in $SU(2)$) which is fully dissociated into a chain of dyons. (Visualization F. Bruckmann)

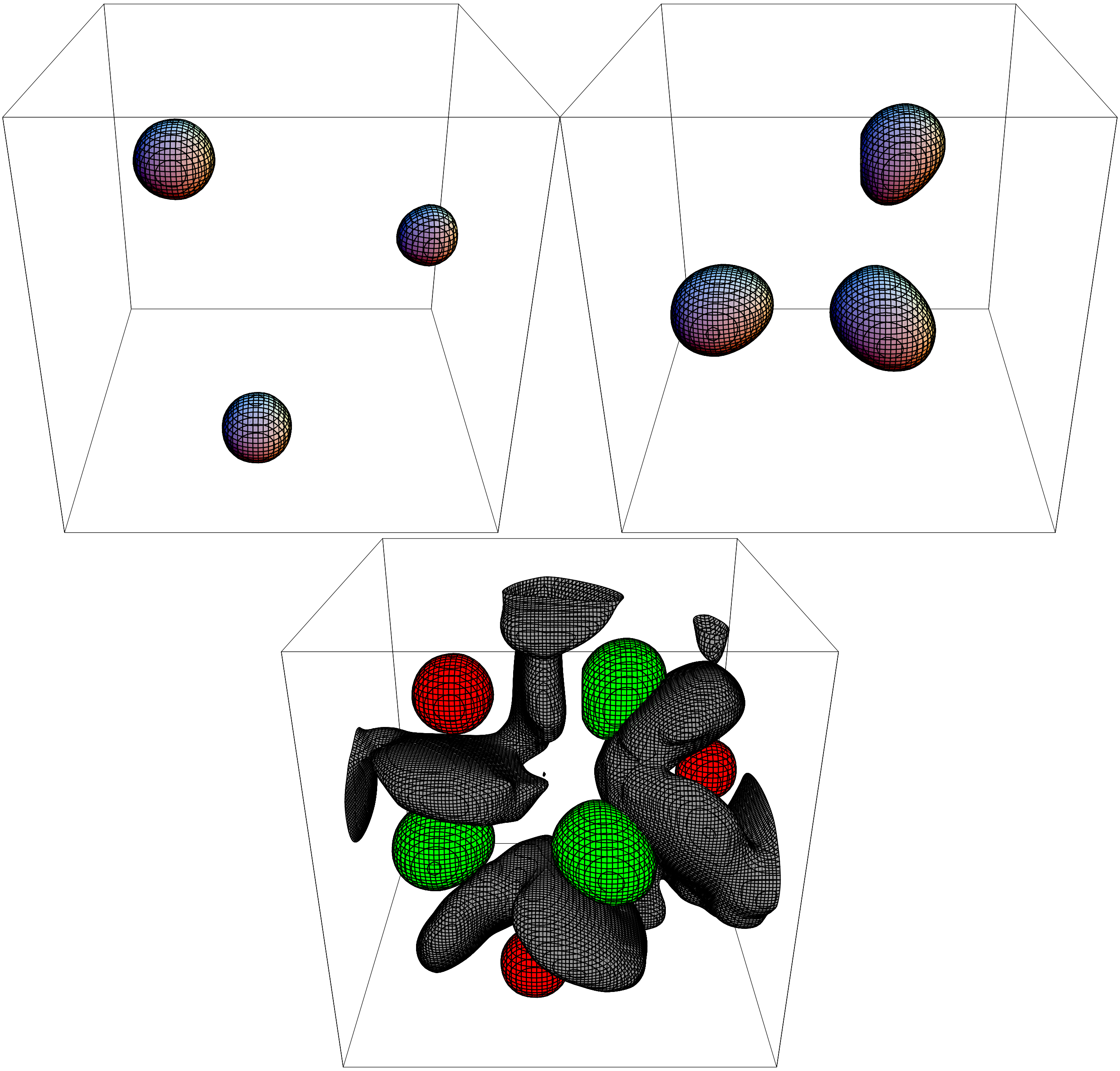

Figure 8: Above: Iso-surfaces of the Polyakov loop at positive (N dyons) and negative values (S dyons). Below: Plot together with the non-staticity, separating the alternating N and S dyons from each other. (Visualization F. Bruckmann)

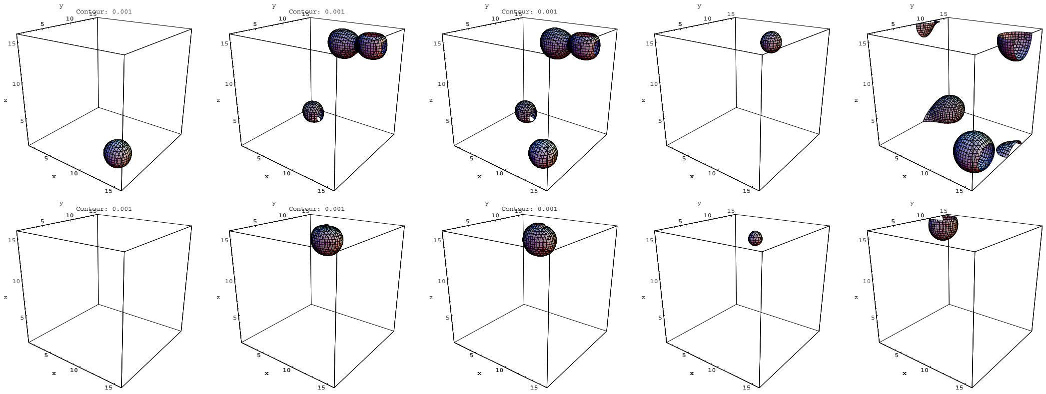

Figure 9: A showcase for constituent annihilation at finite temperature (configuration nr. 15 subject to Wilson cooling after 224 and 560 steps, respectively). Shown are contourplots of (from left to right) positive and negative topological charge density, action density and positive and negative Polyakov loop, respectively, for two consecutive plateaus which have $S = 1.92, Q = -0.89$ and $S = 0.99, Q = -0.86$. On the first plateau (top) one sees three anti-selfdual lumps (rear) and one selfdual lump (front), respectively. On the second plateau (bottom) the two upper antiselfdual lumps merged, while at the bottom of the figure an anti-selfdual lump annihilated with the selfdual lump.

A stable $|Q| = 2$ caloron (stable under overimproved cooling).

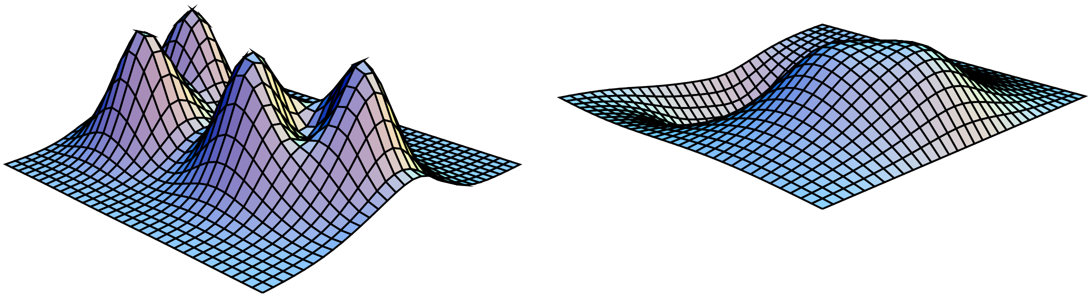

Figure 10: Action density (left) and Polyakov loop (right) for two "ring configurations" from cooling.

The nonlinear superposition of same-sign Polyakov loop constituents leads to ring-like structures of action (see the dips).

IV. Dyon recombination at lower temperature

The observation of dissociated calorons (and multi-calorons) on the lowest plateaux of action is only possible starting from the confinement phase, but not at too low temperature

Very recently, in

Recombination of dyons into calorons in $SU(2)$ lattice fields at low temperatures

E.-M. I., B. V. Martemyanov, M. Müller-Preussker, A. I. Veselov, hep-lat/0402010,

we have systematically studied how instanton-like (non-static) calorons predominantly emerge at lower temperature

At low temperature $\rightarrow$ a dense system with coalescent dyons. Calorons could be misinterpreted as "old" trivial calorons; only the behavior of the Polyakov loop shows that they are not instantons.

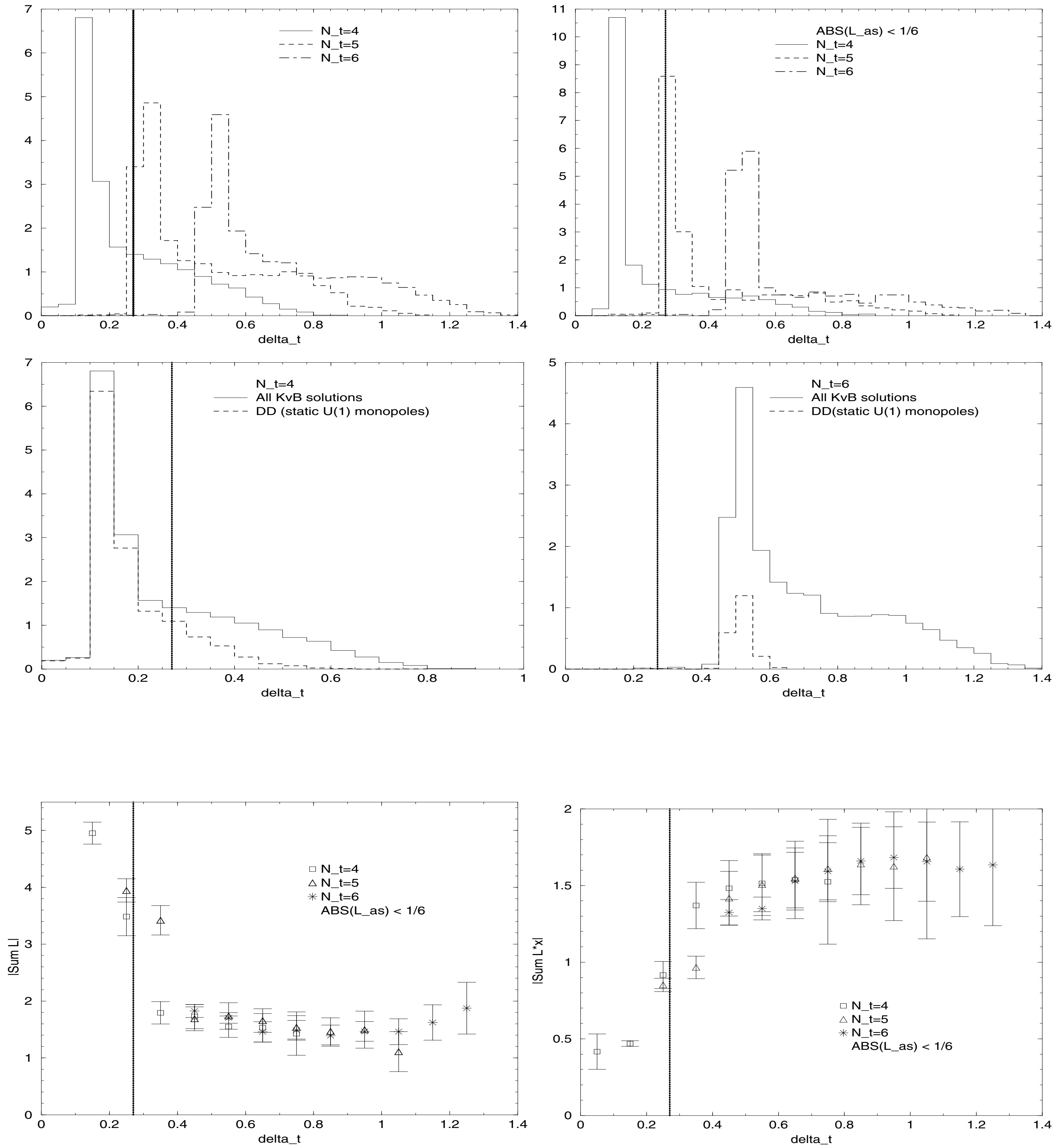

Figure 11: Getting more instanton-like at lower temperature. Upper row: histograms of non-staticity without cut (left) and with cut (right) restricting holonomy to $L \approx 0$; middle row: histograms of the full sample and the subsample with static monopoles at high T (left) and lower T (right); bottom row: the average holonomy (left) and the gradient of holonomy (right) inside a lump of action as function of staticity is independent of T. The vertical line marks the bifurcation into dyon constituents.

Could this be an artefact of cooling using Wilson action ?

We used the method of adiabatically lowering T

applying various actions (Wilson, improved, overim-proved)

- create a dissociated caloron at high temperature on an anisotropic lattice (with anisotropic coupling constants)

then iterate the following steps

- gradual expansion toward the symmetric torus, gradually lifting the anisotropy of the coupling con-stants
- lattice cooling to adjust the caloron to the changing lattice size
- record the classical solution for each box size

Conclusion :

Coalescence (with lowering T) into (unknown) non-dissociated instanton-like solutions, however with non-trivial holonomy, is a general feature.

V. $SU(3)$ calorons on the lattice

The parameters of a constructed caloron :

(i) eigenvalues $\exp(i2\pi\mu_i)$ of asymptotic holonomy : $\mu_1 < \mu_2 < \mu_3 < 1 + \mu_1$

(ii) positions of constituents $\vec{y}_1$, $\vec{y}_2$, $\vec{y}_3$

→ lumps appear with action in proportions

$$\mu_2 - \mu_1 : \mu_3 - \mu_2 : 1 + \mu_1 - \mu_3$$

Analytical expression for A_μ is used to construct the lattice links $U_{x,\mu}$ (= parallel transporter)

Lattice view of an analytic $SU(3)$ non-trivial caloron

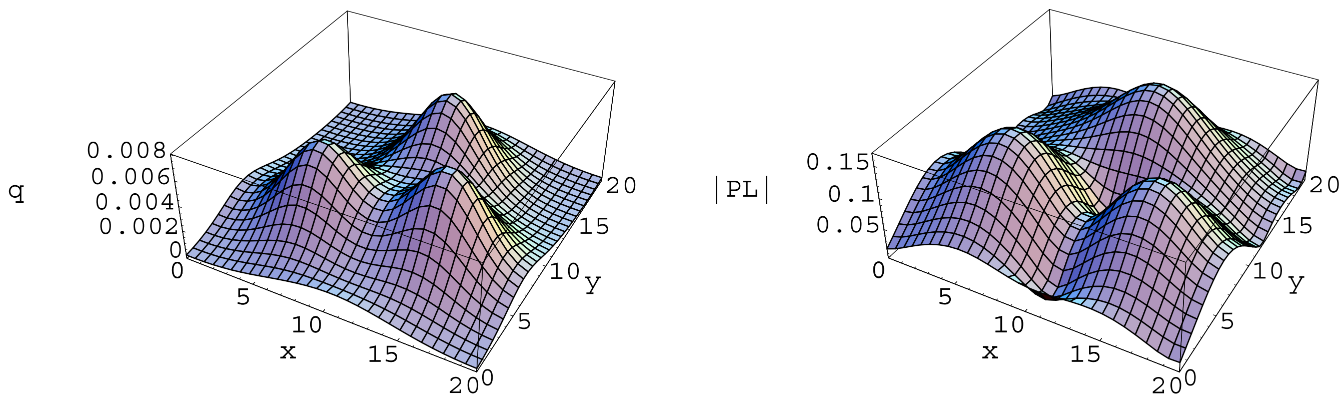

Figure 14: Topological charge density (left) and modulus of Polyakov loop (right) of a constructed symmetric non-trivial lattice caloron.

Signatures : trivial *vs* non-trivial

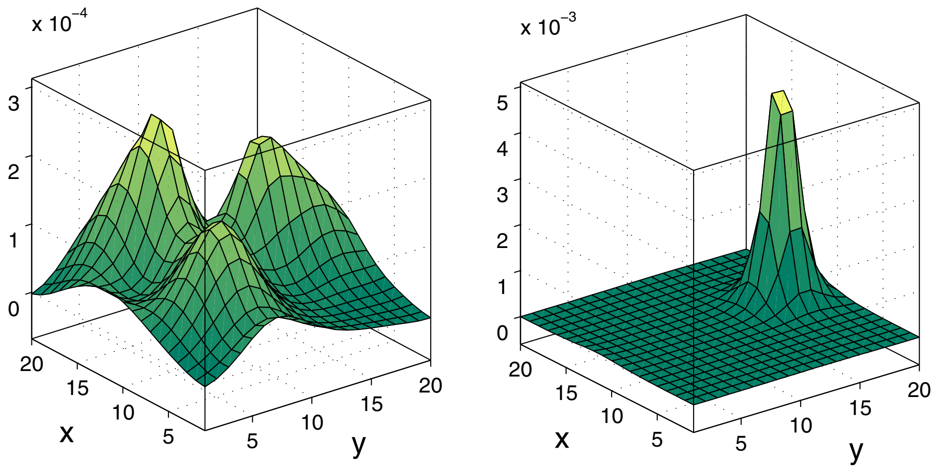

Figure 15: Topological charge density for a non-trivial (left) and a trivial (right) caloron. Notice the huge difference in scale due to the trivial holonomy !

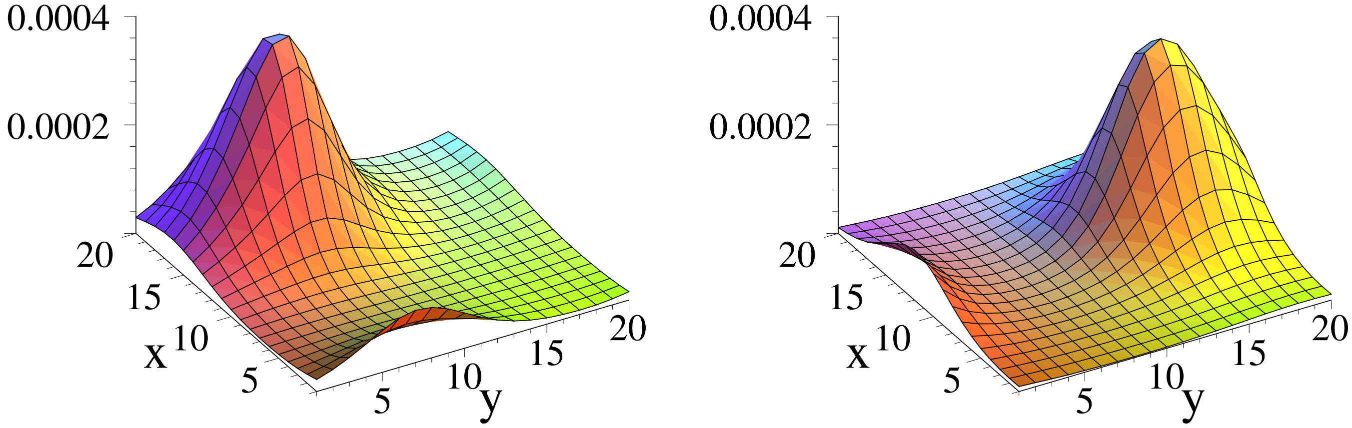

Figure 16: Scalar density of the fermionic zero mode on the non-trivial KvB lattice caloron sitting on different constituents for $\zeta = 0.5$ (left) and $\zeta = 0.8$ (right).

Generic classical $SU(3)$ configurations obtained by cooling

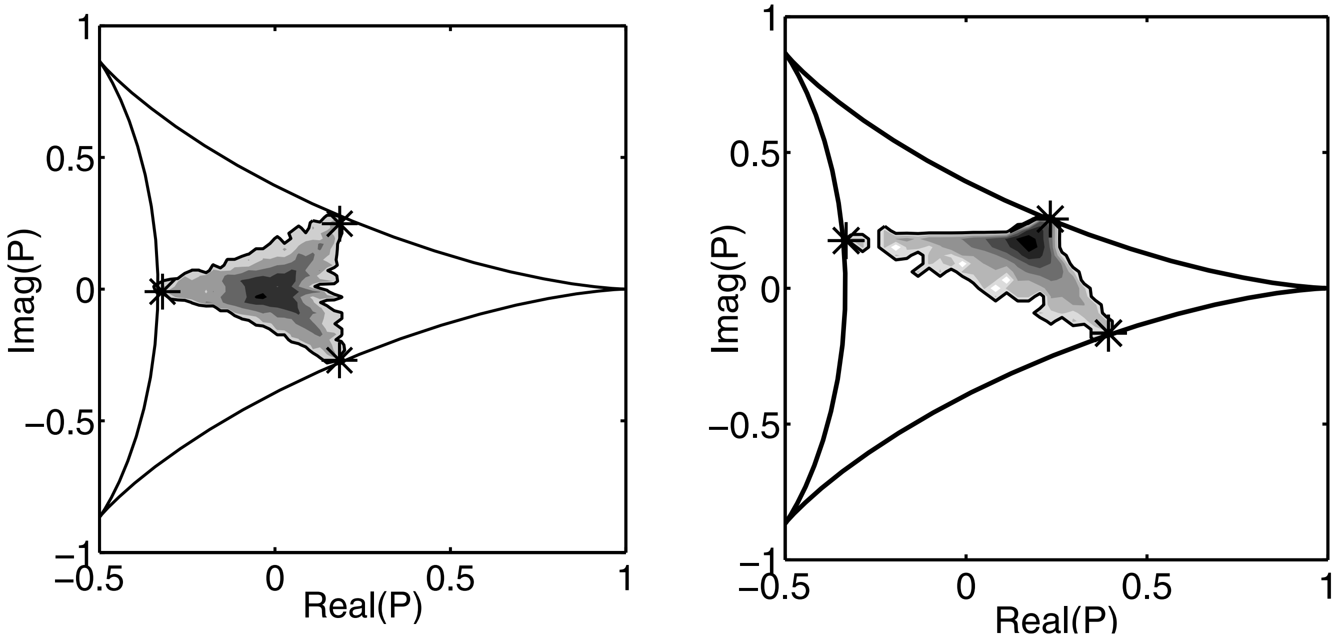

Figure 17: Scatterplot of the Polyakov loop in the Weyl chamber. Left: for the constructed maximally non-trival caloron; right: for a generic caloron obtained by cooling. Constituents are marked by stars, the asymptotic holonomy (dark spot) is the most frequent Polyakov loop in the plot.

For $|Q| = 1$, most of the cooled configurations are separated into only two lumps

The result of cooling depends on the final asymptotic holonomy (which was not under control) ! One could consider using fixed-holonomy boundary conditions (somewhat artificial).

A systematic investigation of cooling for $SU(3)$,

- classifying the caloron solutions
- studying their parameter space
- including cuts with respect to the asymptotic holonomy
- using the localization properties and spectral flow of zero and near-zero modes
- using the local holonomy to identify monopole constituents
- statistical features (numbers of separated constituent staticity etc.)

Some examples visualized with additional tools:

Portrait of the Polyakov loop (asymptotic and local) for a maximally nontrivial caloron

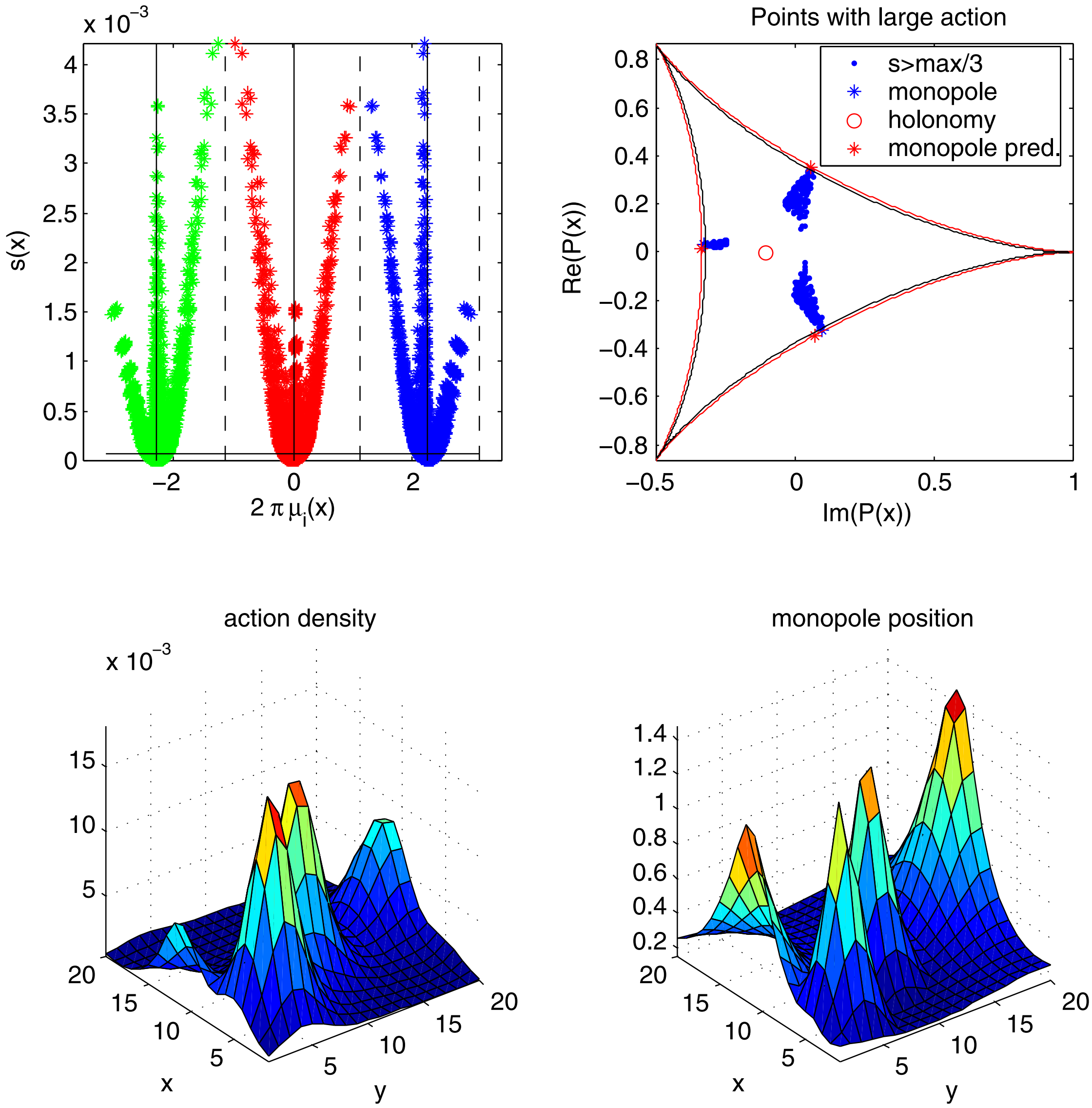

Figure 22: Asymptotic holonomy, clustering in the Weyl chamber, action profile and monopole positions for a fully dissociated caloron.

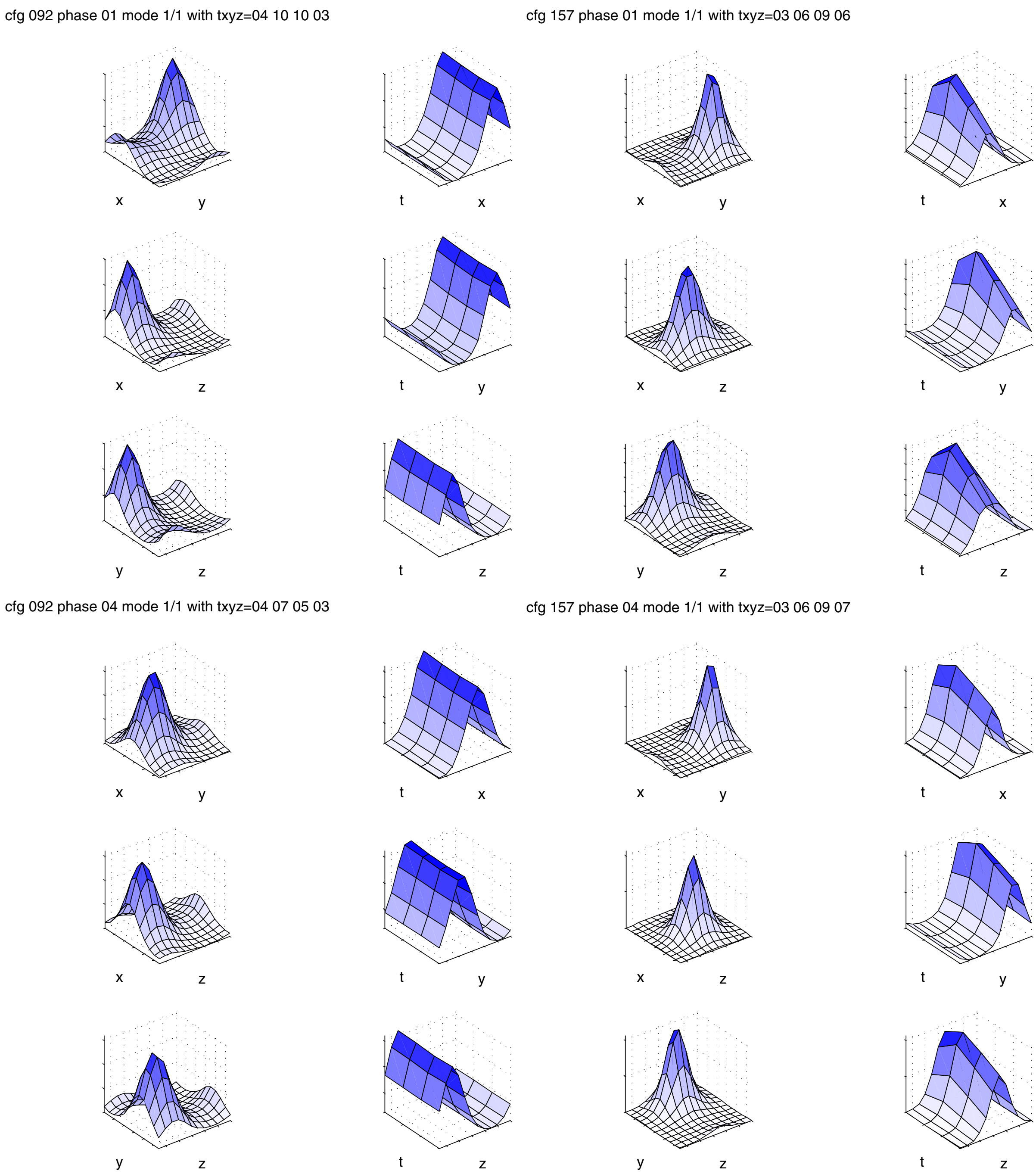

Figure 23: Jumping zero mode (left) of a static caloron with two well-separable constituents and non-jumping zero mode (right) of a less static (non-separated) caloron.

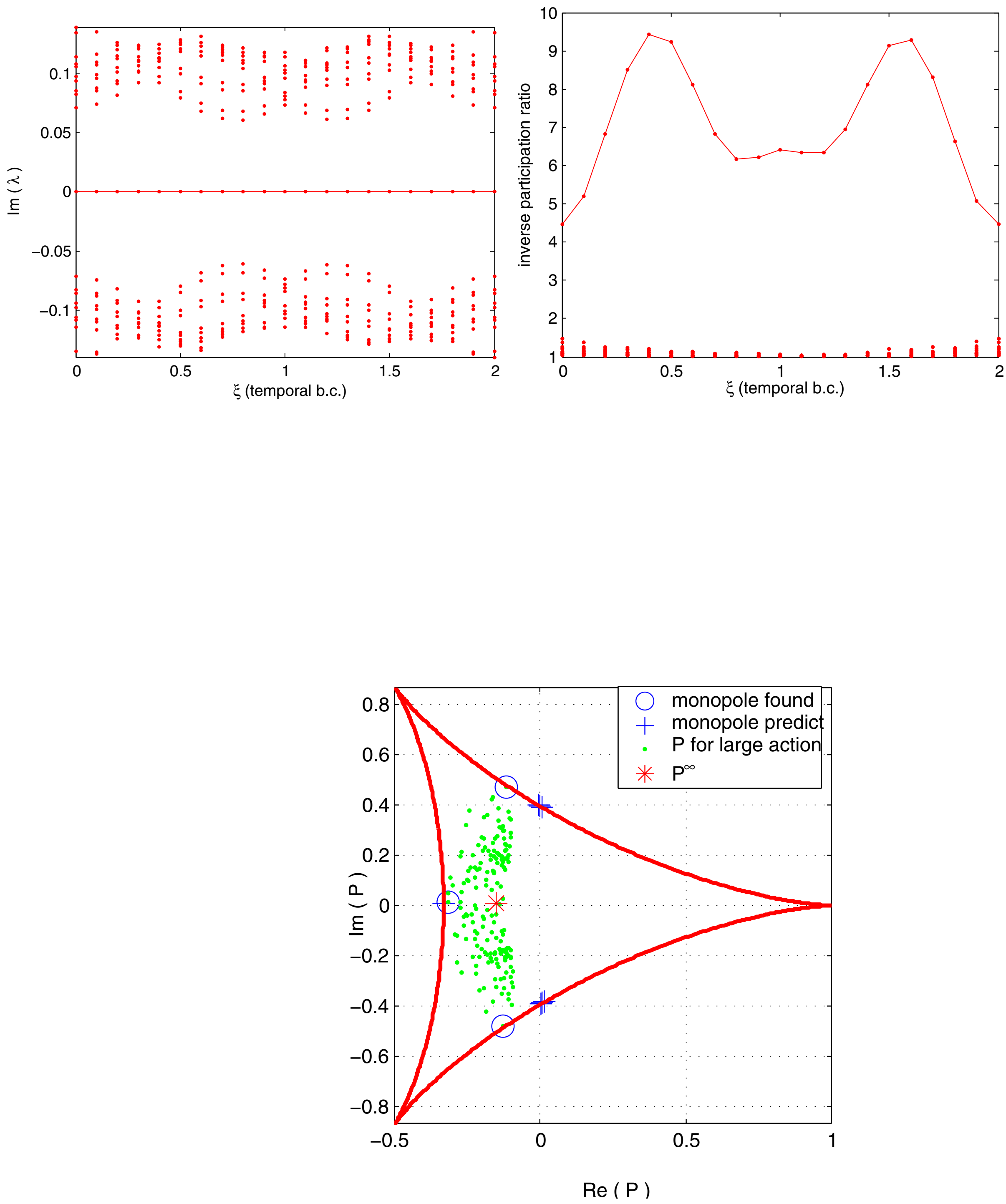

Figure 24: Spectral flow and variation of the inverse participation ratio (top) of the zero mode and the lowest non-zero modes with changing boundary conditions (for the non-jumping case in Fig. 23). Below the Weyl chamber plot of the same configuration.

One of the $Q = 0$ configurations

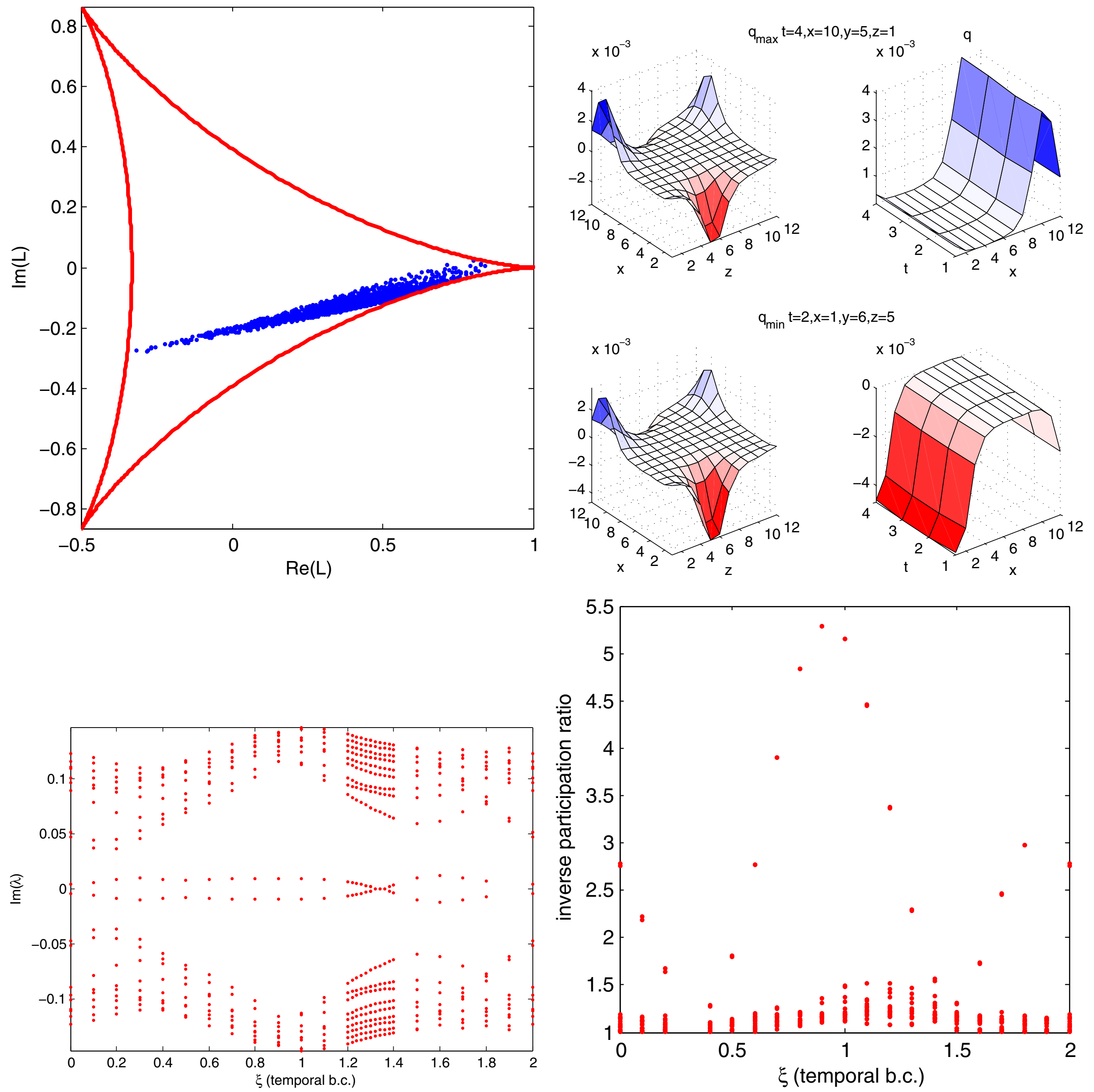

Figure 25: A Q=0 configuration in the Weyl chamber and its topological density, the lowest-lying fermion modes and their inverse participation ratios. The configuration is almost a pair of embedded SU(2) instanton and antiinstanton.

Ensemble properties:

Minimal violation of the lattice equations of motion

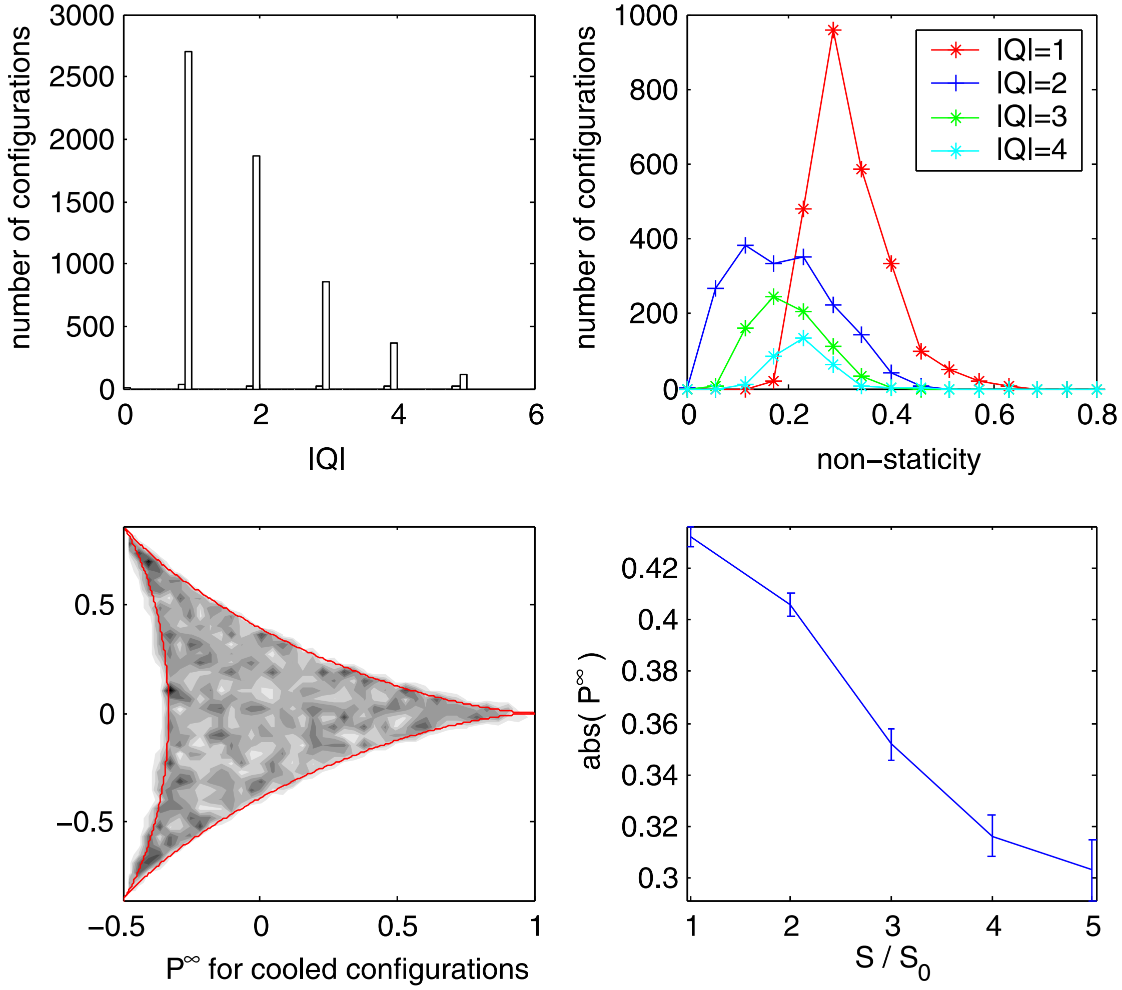

Figure 26: Q distribution, distribution of non-staticity for various $S/S_0 = |Q|$, the asymptotic holonomy distribution and the average various S/S_0. The ensemble is defined for minimal violation of the lattice equations of motion.

Maximally static (minimal δ_t)

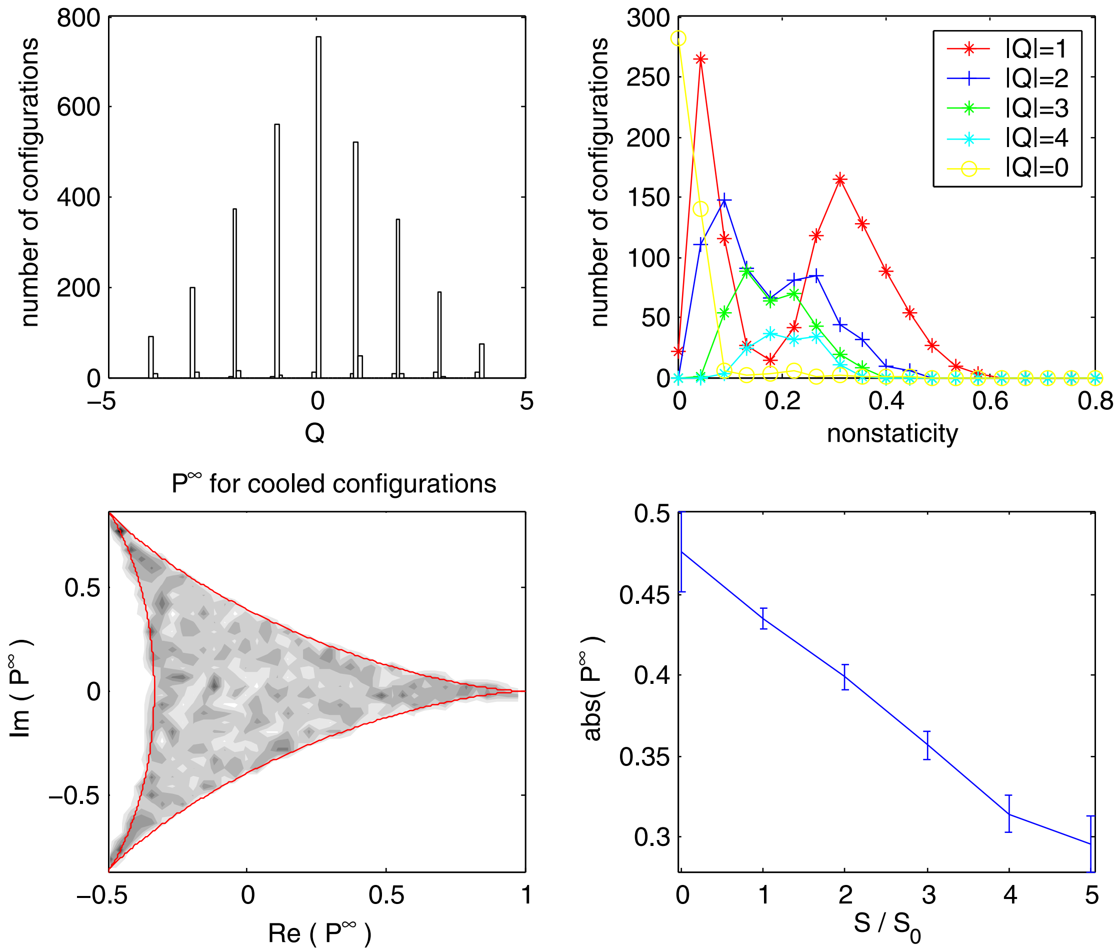

Figure 27: Q distribution, distribution of non-staticity for various $S/S_0 = |Q|$, the asymptotic holonomy distribution and the average various S/S_0. The ensemble is defined by another stopping criterion (see the occurrence of $Q = 0$).

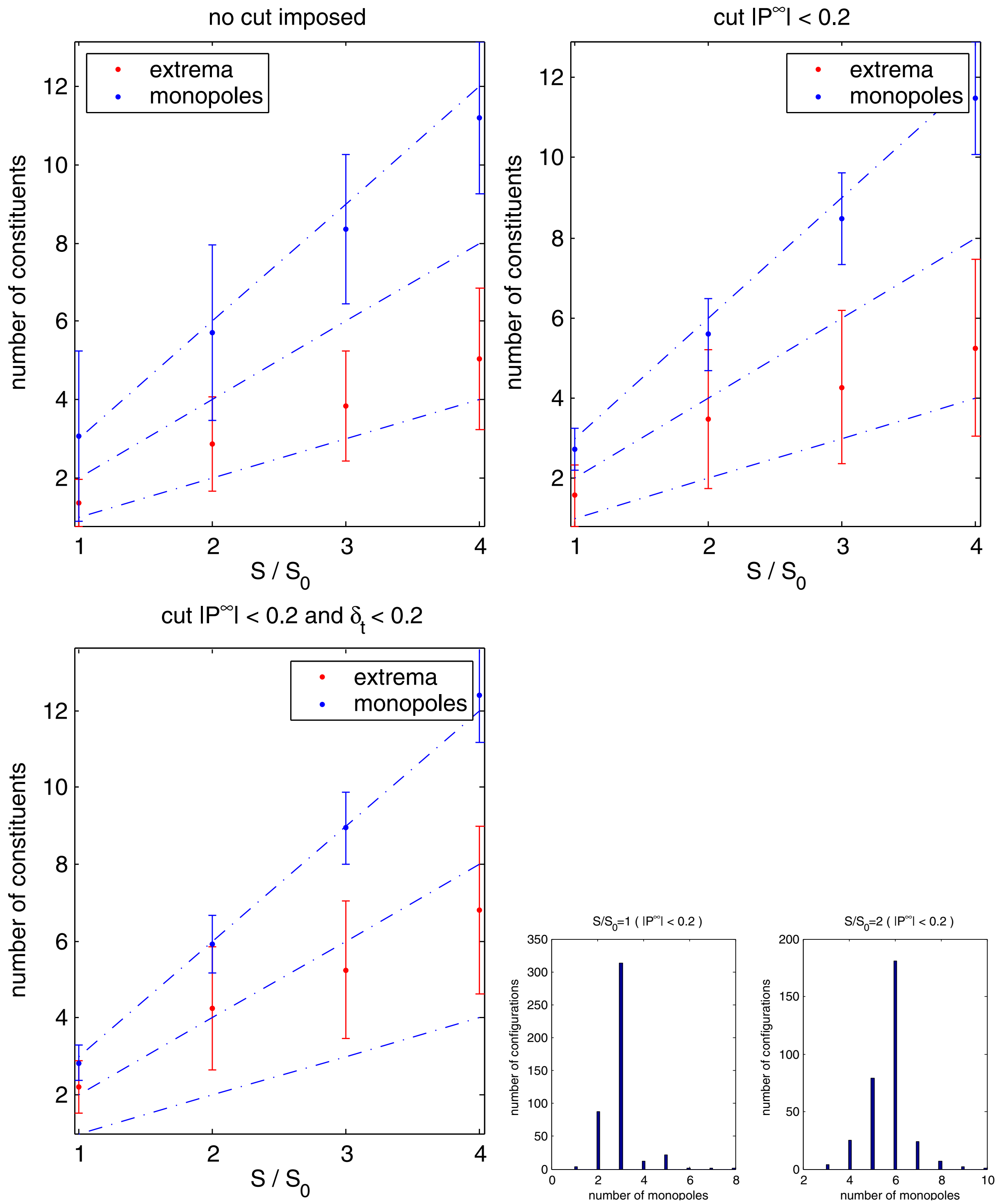

Figure 28: Number of peaks of action and number of monopoles (defined by coinciding eigenvalues of the local holonomy) vs. S/S_0 without cuts (top left) and with cut in the asymptotic holonomy (top right). Below with additional cut w.r.t. staticity (left) and the multiplicity distribution of monopoles for $|Q| = 1$ and 2 (right).

VI. Have calorons been observed in Monte Carlo $SU(3)$ samples ?

C. Gattringer and S. Schaefer

New findings for topological excitations in SU(3) lattice gauge theory, Nucl. Phys. **B 654**, 30 (2003)

They

- analyzed equilibrium lattice configurations
- tested the hypothesis that the behavior of fermionic zero modes follows the pattern known from KvB calorons

Two finite-T samples generated
using Lüscher-Weisz plaquette and rectangle action
on a $20^3 \times 6$ lattice
representing confinement and deconfinement :

$\beta = 8.20$ (confinement)
$a = 0.115$ fm $\quad T = 287$ MeV

$\beta = 8.45$ (deconfinement)
$a = 0.094$ fm $\quad T = 350$ MeV

Two technical advances made the goal feasible

- chirally improved lattice Dirac operator with good locality properties (still computationally cheap compared to the Neuberger overlap operator), constructed to fulfill the Ginsparg-Wilson equation (due to Gattringer, Hip, Lang)
- zero-mode counting allows to determine the topological charge Q without cooling

For a subsample of $Q = \pm 1$ lattice configurations, the localization of the single zero mode was studied depending on the periodicity angle $2\pi\zeta$:

$$\psi(\vec{x}, t + 1/T) = e^{2\pi i\zeta}\psi(\vec{x}, t)$$

$\zeta = 0.0$ or 1.0 for periodic, $\zeta = 0.5$ for antiperiodic b.c. in Euclidean time direction

After the observations for $SU(2)$,

zero mode jumping at angles ζ determined by the global holonomy

was hypothetically expected to occur.

Inspired by the caloron solutions, depending on the phase, the typical distribution of the "valence" topological charge ± 1 would be as follows :

- confinement : the holonomy

$$P \propto \text{diag}\left(\text{e}^{2\pi i \mu_1}, \text{e}^{2\pi i \mu_2}, \text{e}^{2\pi i \mu_3}\right)$$

  has maximal spacings between μ_i's and balanced fractional charges and actions of 3 constituents sharing the "valence" topological charge ± 1.
  If this is the case, jumping of zero modes at $\zeta = \mu_i$ between three diffent $3D$ positions of the constituents should happen.
- deconfinement : not one, rather $N = 3$ phases ($\mathbf{Z}(3)$ "phases") where the holonomy

$$P \propto \text{diag}\left(z_k, z_k, z_k\right) \quad (\ z_k = \text{e}^{2\pi i k/N}, k = 0, 1, 2 \ ).$$

  This should allow only one "massive" $Q = \pm 1$ (*i.e.* trivial) constituent, where the zero mode is localized for almost all ζ values, except in the vicinity of $\zeta \approx k/N$.

Observations

- zero mode jumping was observed for sizeable fraction of configurations;
- it happened at ζ_i consistent with maximal stability intervals (in confinement),
- it happened as an instability at one ζ, consistent with the phase of the Polyakov loop (in deconfinement);
- according to the distance between "constituents", two classes of events were present: close pairs and those with randomly distributed constituents;
- width of scalar density of zero modes "breathing" within the stability interval in ζ, in close analogy to the caloron properties.
- The gap width (between zero and near-zero eigenvalues) is regularly "breathing" with changing ζ which later has been understood to be similar for KvB calorons

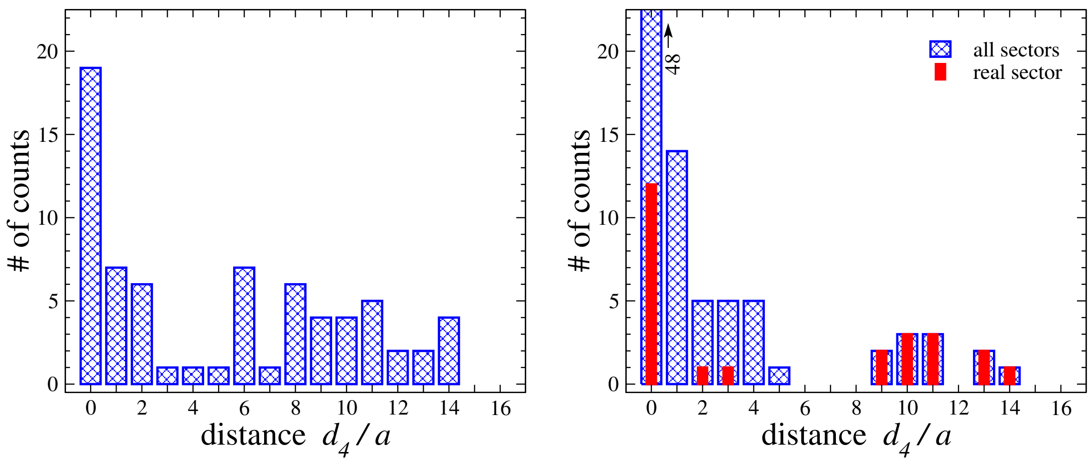

Figure 29: Distribution of distances between the peaks of the zeromode pinned down in a configuration due to zeromode jumping; in confinement (left) and deconfinement (right).

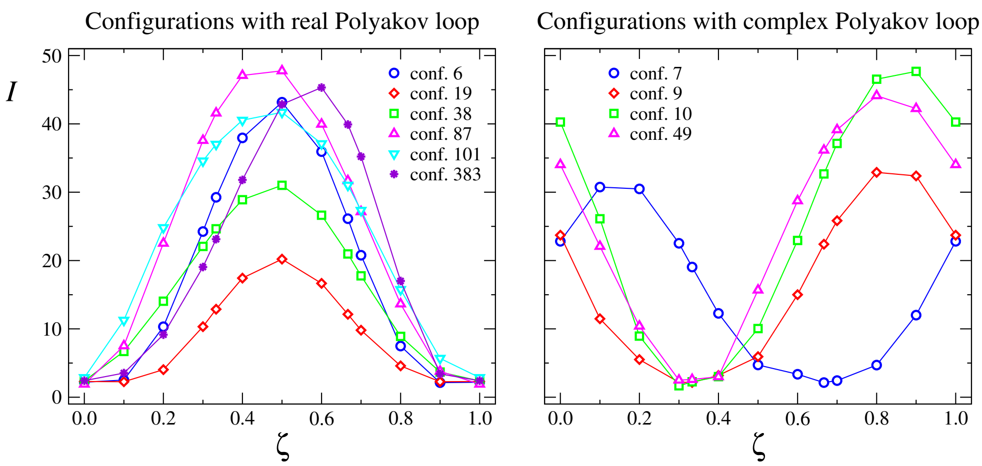

Figure 30: Inverse participation ratio I changing with ζ in deconfinement; with real Polyakov loop (left) and complex Polyakov loop (right).

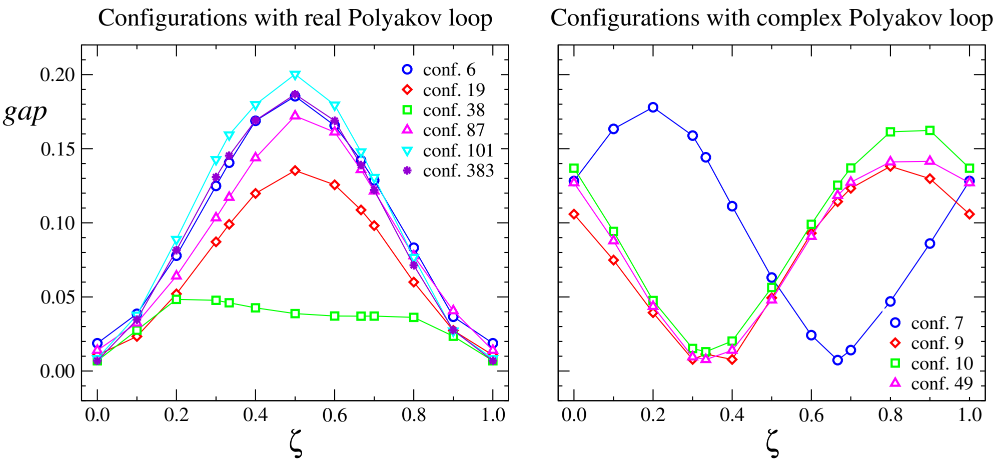

Figure 31: Gap changing with ζ in the deconfinement phase; with real Polyakov loop (left) and complex Polyakov loop (right).

This is challenging, ...

... because the distribution of topological charge in the Monte Carlo configurations is much more complex, even in the deconfinement phase, to be described by a quasiclassical background field

... possible that this a yet to be understood general feature going beyond the KvB caloron solutions ?

Comparison with the topological content revealed by APE smearing

goal of the Regensburg-Berlin collaboration

APE smearing has good properties

- the Dirac spectrum is not changed by moderate APE smearing
- the topological density shows lumps at the (one, two or three) positions of the zero mode and where the near-zero modes are localized

The smeared configurations are far from classical

- an improved operator of the topological density is needed to reproduce the topological charge Q known beforehand
- the topological density does not show the shape of a classical solution
- a fractional charge at the zero-mode position is difficult to establish

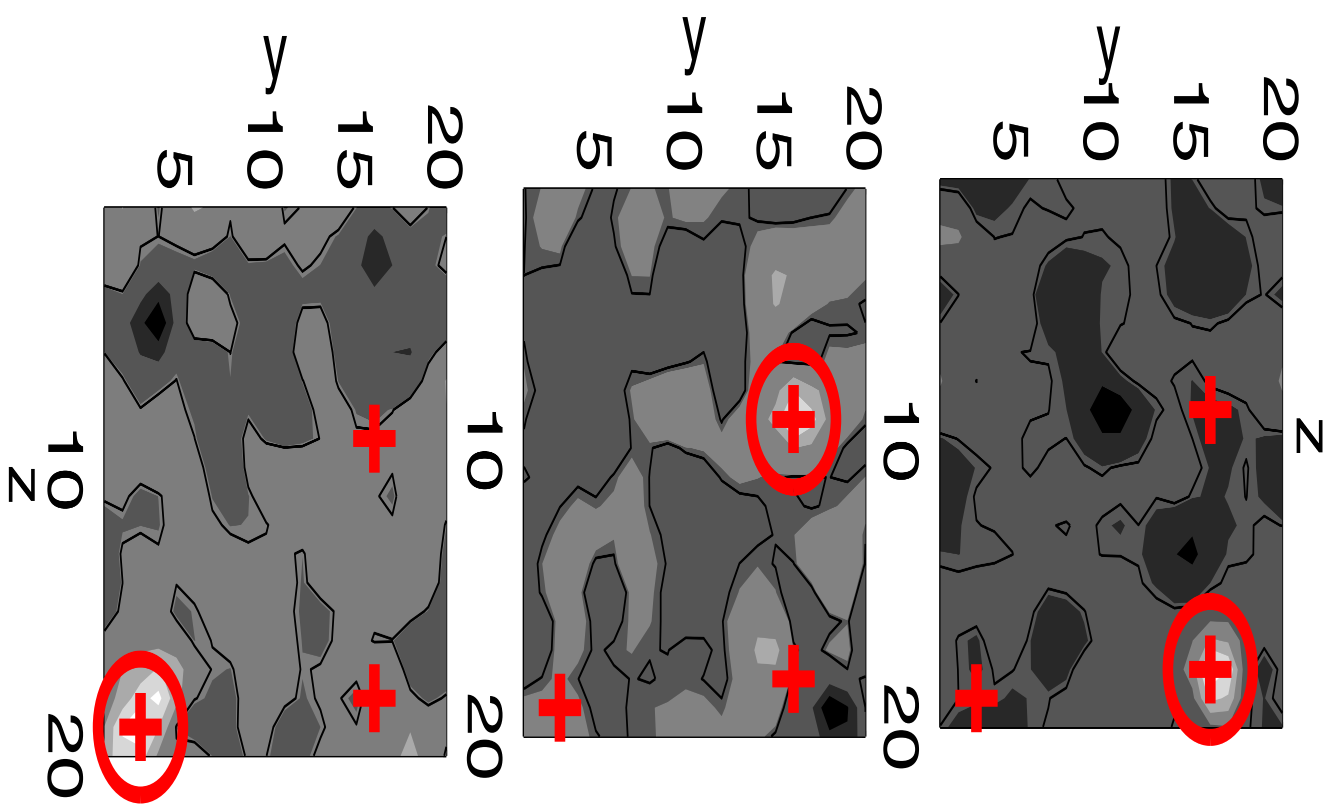

Figure 32: Topological density profile in cuts through the three peaks (marked by a cross) of the zeromode. Typical confinement configuration.

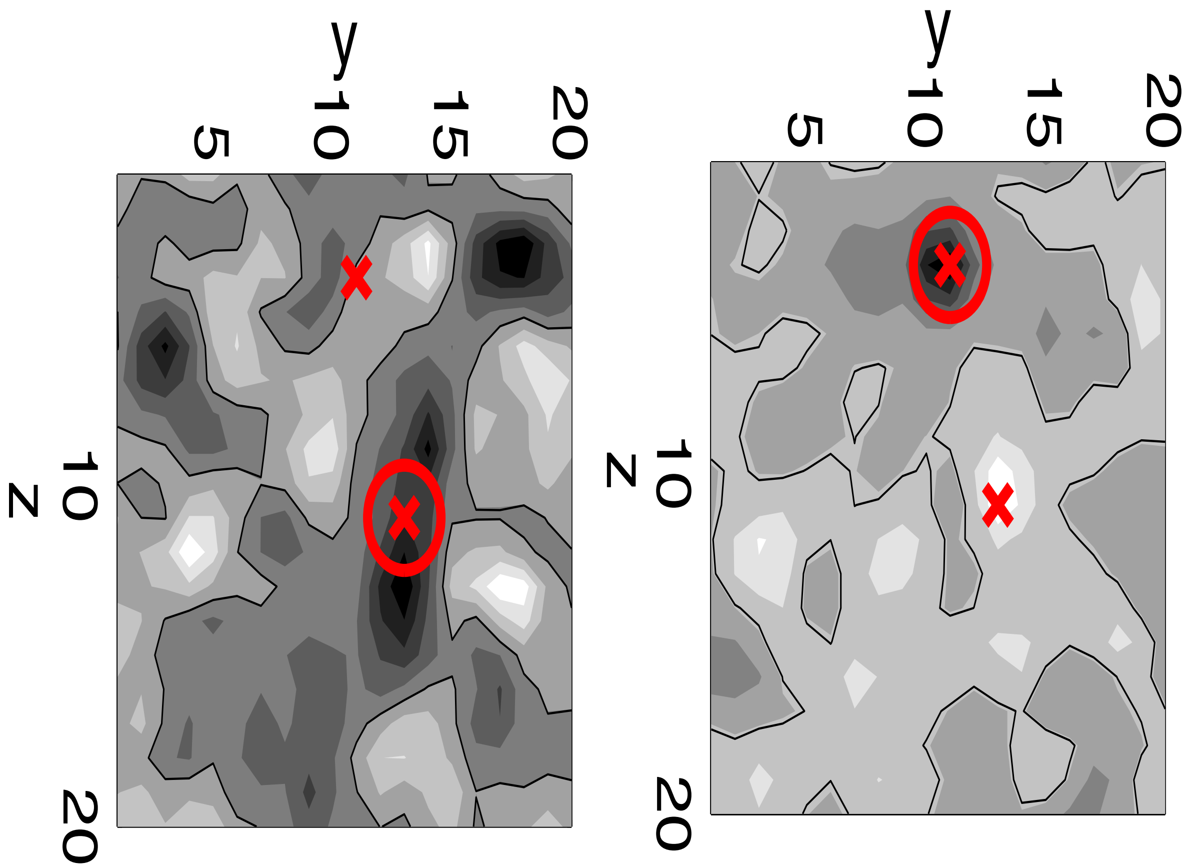

Figure 33: Topological density profile in cuts through the two peaks (marked by a cross) of the zeromode. Typical deconfinement configuration.

Summary and outlook

- If a semiclassical mechanism has a chance to be viable, it is near the onset of confinement.
- We gained insight in the appearance of $SU(2)$ calorons and multi-calorons at finite temperature, and some experience with $SU(3)$ calorons and multi-calorons for which no analytic results are available, on the lattice:
  1. spectral flow of zero and near-zero modes with ζ;
  2. the role of the Polyakov loop inside and outside.
- Continued toward $T = 0$ by *adiabatic cooling* we found confirmation for the (previous) observation that single calorons turn into instanton-like objects (apart from the Polyakov loop !) which are partly describable as KvB calorons. However, the scale is set more and more by the finite volume (not the temperature !).
- The same technique applied to $Q = \pm 2$ calorons at finite temperature, we have seen a multitude of solutions on the torus depending on the positions of constituents, the relative color orientation of the (sub)calorons etc., for which no explicit analytic expressions are known.

- Checking the results of zeromode jumping on $T > T_c$ and $T < T_c$ equilibrium configurations by direct observation of the topological charge distribution (APE smearing),
  1. we cannot exclude the interpretation as a coherent caloron background the scale of which is set by the temperature;
  2. the existence of fractionally charged clusters in this situation is not (not yet) unambiguously proven.
- Similar observations of zeromode jumping on $T = 0$ (torus) equilibrium configurations are more debatable: if they are to be interpreted as derived from a semiclassical background, the scales describing the corresponding "solutions" seem to grow with the size of the torus instead of being set by an intrinsic correlation length.