LEILAT04

Measuring interface tensions with a local observable

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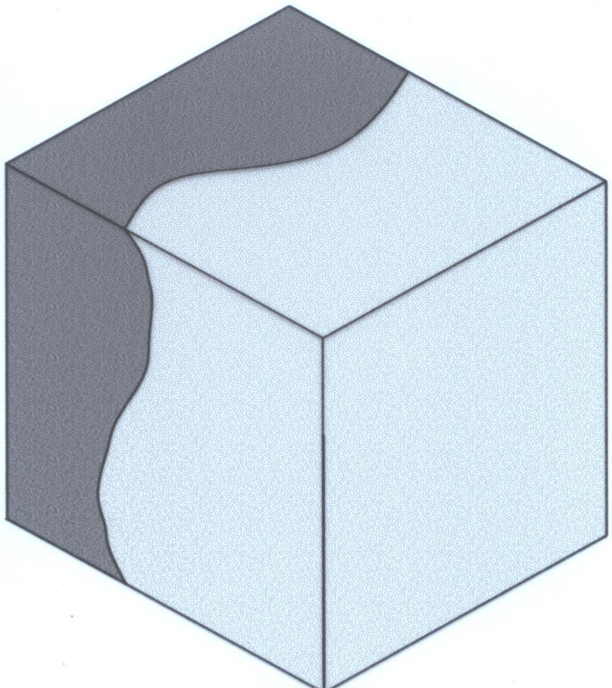
How to measure interface tension?



$$Z = e^{-[\beta]F_{\text{bulk}} - [\beta]F_{\text{interface}}}$$

$$\boxed{\frac{\sigma}{[T]} \equiv \lim_{A\to\infty} \frac{F_{\text{interface}}}{A}}$$

- must enforce interface
- must take limit $A \to \infty$
- must measure free energy

- Order-order interface: low T, two vacua
- Order-disorder interface: $T = T_c$ (1rst order PT), two phases

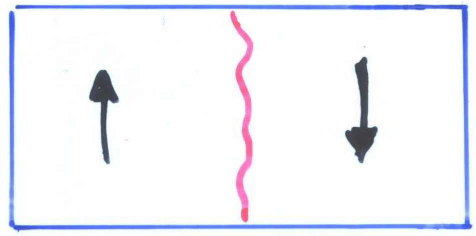
Both related via (perfect) wetting: $\sigma_{oo} \leq 2\sigma_{od}$

Here σ_{oo} only

① enforce interface:

- with external field h

  (p.b.c. → 2 interfaces)



  Must take limit $h \to 0$. Non-linearities. A mess!

- change b.c.: apbc (twisted) → 1, 3, .. interfaces

$$e^{-[\beta] F_{int.}} = \frac{Z_{apbc}}{Z_{pbc}}$$

② extrapolate $A \to \infty$:

- $F_{int} = \sigma L^{d-1} \left(1 + \frac{c_1}{L} + \frac{c_2}{L^2} + \cdots\right)$

- Translation invariance: $\mathcal{O}(L_\perp)$ interface positions

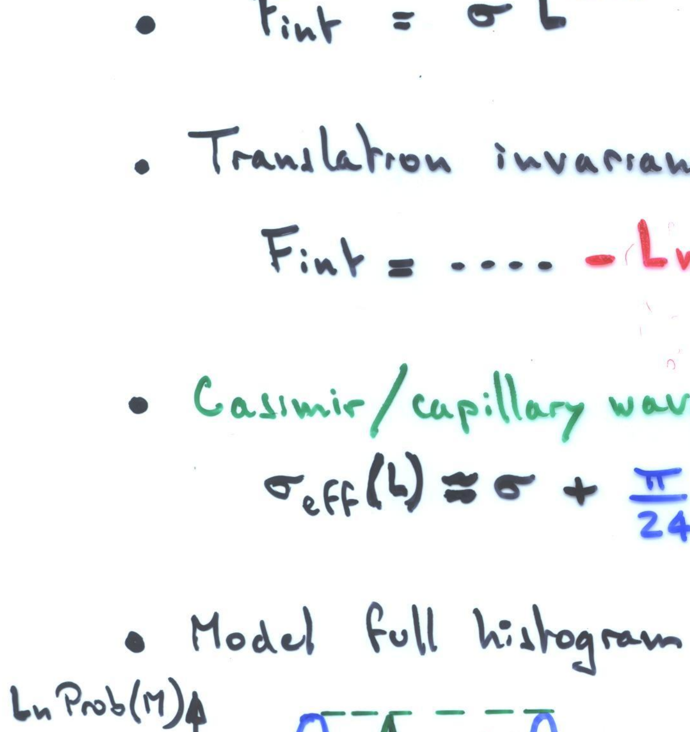
  $F_{int} = \cdots - \ln c L_\perp$ → difficult

  (c ↑ ?)

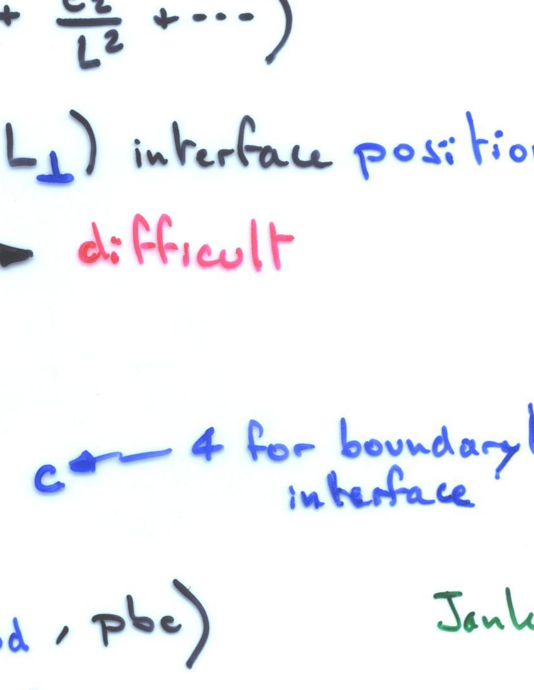
- Casimir / capillary wave / Lüscher

  $\sigma_{eff}(L) \approx \sigma + \frac{\pi}{24}(d-2)\frac{1}{L^2}\, c$ ← 4 for boundaryless interface

- Model full histogram (esp. σ_{od}, pbc) Janke



Ideal



Real world

③ measure free energy by Monte Carlo:

- density of states by multicanonical/multimagnetic

  Slowing down $\exp(L^{d-1}) \to L^{2d}$; tuning   Berg Neuhaus

- integrate derivative:

$$Z_{apbc} = \sum_{\{spins\}} e^{-\beta H_{apbc}} \longrightarrow -\frac{d}{d\beta} \ln Z_{apbc} = \langle H_{apbc} \rangle_{apbc}$$

$$\qquad \qquad \qquad \text{" " }\ {}_{pbc} = \langle H_{pbc} \rangle_{pbc}$$

$$-\ln \frac{Z_{apbc}}{Z_{pbc}} = \int_0^\beta d\beta' \left(\langle H_{apbc} \rangle_{apbc} - \langle H_{pbc} \rangle_{pbc} \right)(\beta')$$

$0 \leftarrow \beta_c - \varepsilon$

Run simulations by pair; integrate using Ferrenberg-Swendsen

Problem: cancellations $\langle H \rangle \sim L^d$, $F_{int} \sim L^{d-1}$

- direct estimate of $\frac{Z_{apbc}}{Z_{pbc}} \longrightarrow$ overlap problem

  * interpolate b.c. $J \to -J$ at boundary, in m steps

$$J_k = \left(1 - \frac{2k}{m}\right) J$$

  Measure histograms of E; combine together with F-S

$$\longrightarrow Z_m / Z_0$$

  error analysis?

  * interpolate number of flipped couplings: "snake"

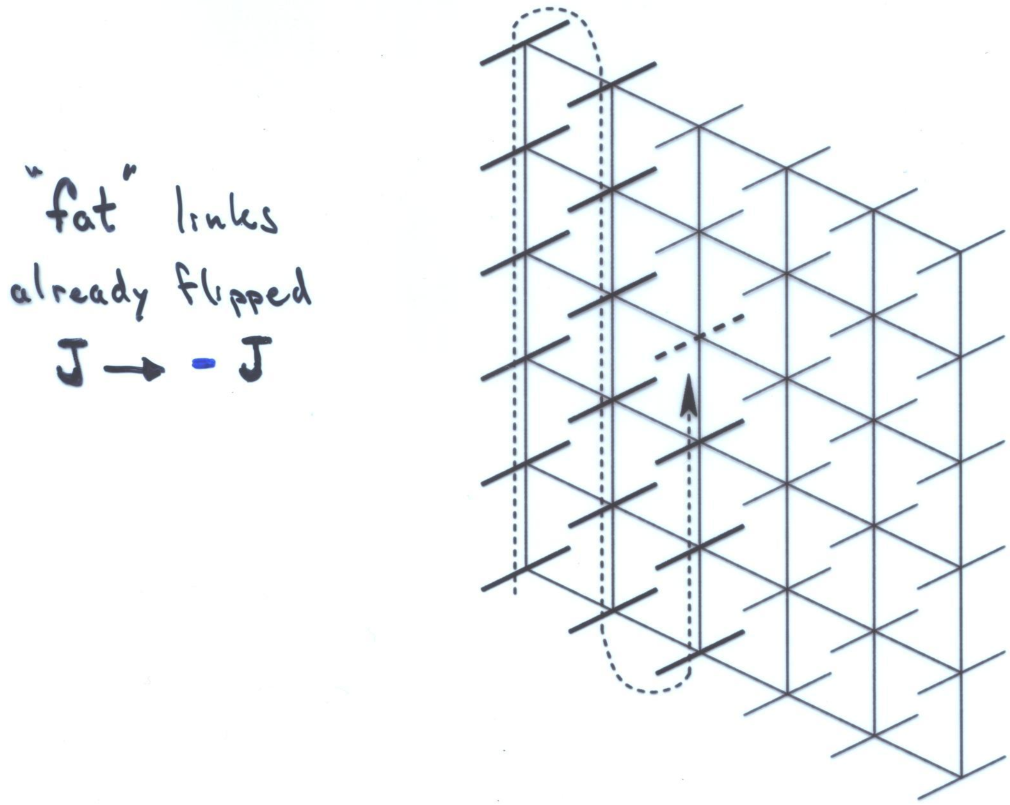
$$Z_0 = Z_{pbc} \ ; \ Z_{L^{d-1}} = Z_{apbc}$$

Factorize: $$\frac{Z_{apbc}}{Z_{pbc}} = \underbrace{\frac{Z_{L^{d-1}}}{Z_{L^{d-1}-1}}}\times \cdots \times \underbrace{\frac{Z_{k+1}}{Z_k}} \times \cdots \times \underbrace{\frac{Z_1}{Z_0}}$$

Each factor is $O(1)$, measured by independent simulation

↳ no overlap problem   ↳ error analysis trivial

Snake algorithm: 3d setup



- PRL 86 (2001) 1438 (D'Elia, Pepe & PdF) for 't Hooft loop
- 3d Ising interface tension in hep-lat/0110119 (Pepe & PdF)
- 3d Z_2 gauge theory in hep-lat/0211012 (Caselle et al)

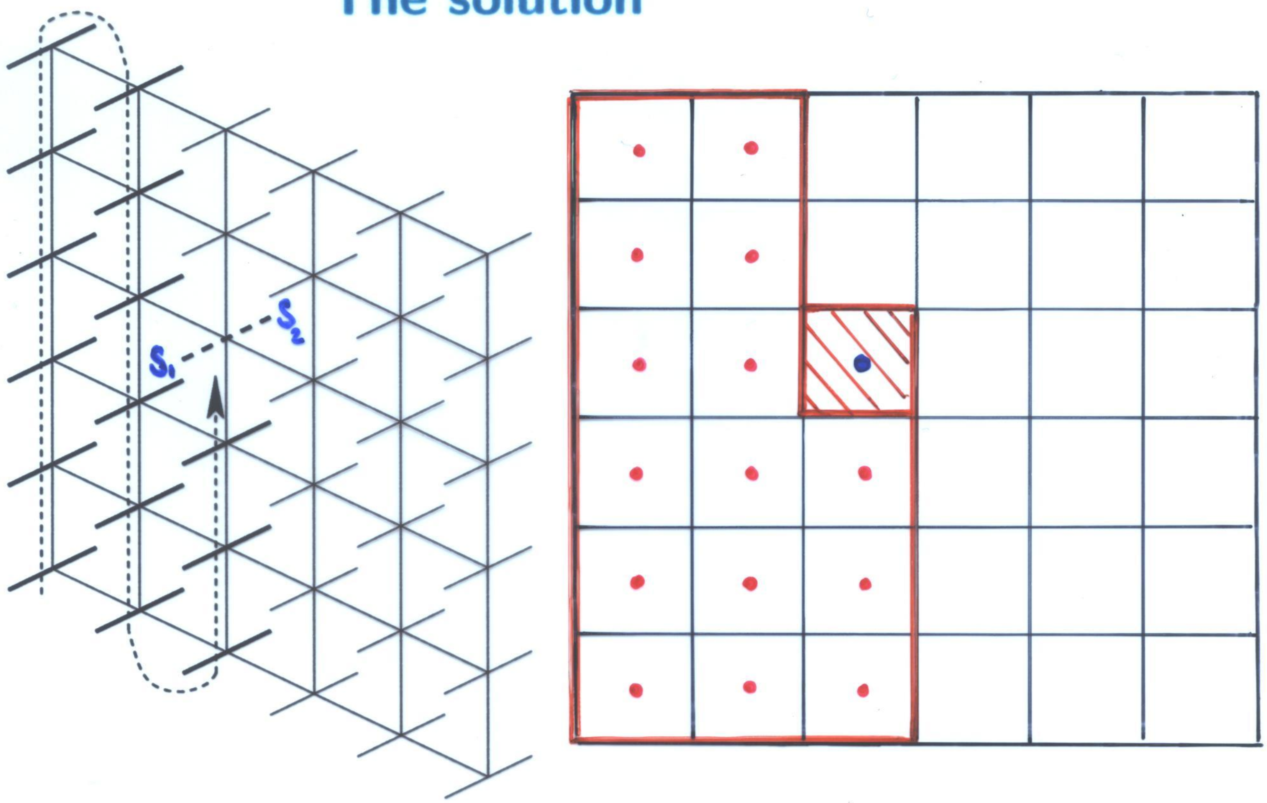
L^2 independent simulations, each with variance reduction
Intermediate results $Z_{R\times L}$ give $\exp(-LV(R))$

Computer effort for given accuracy on $V(R)$ indep. of R
$\leftrightarrow$ Wilson loop $\sim \exp(R)$ even with Lüscher-Weisz

Remaining difficulties:

- many simulations
- entropy from translation

The solution



Each $\frac{Z_{k+1}}{Z_k}$ increases area by a^2 $\longrightarrow$ **Measure ONE ratio**

$$\frac{Z_{k+1}}{Z_k} = \exp(-\sigma a^2) + \text{ finite-size corrections}$$

minimized when in the "middle" of the lattice

$$\frac{Z_{k+1}}{Z_k} = \frac{\langle \exp(+J s_1 s_2) \rangle}{\langle \exp(-J s_1 s_2) \rangle}$$

in ensemble where $J_{12} = 0$ and $J = -1$ for fat links

Work w.r.t. snake reduced by L^4 (# simul. + error on each)

Compatible with:

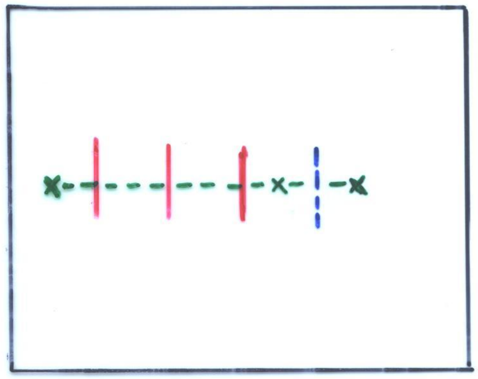
- cluster update
- multishell update
- reweighting

Partial interface is pinned $\Rightarrow$ no entropy

Leading finite-size correction known (Casimir/Lüscher)

2d Ising model

$$\frac{Z_4}{Z_3} = e^{-(V(4)-V(3))}$$
$$\approx e^{-a\text{Force}(r=3.5a)}$$



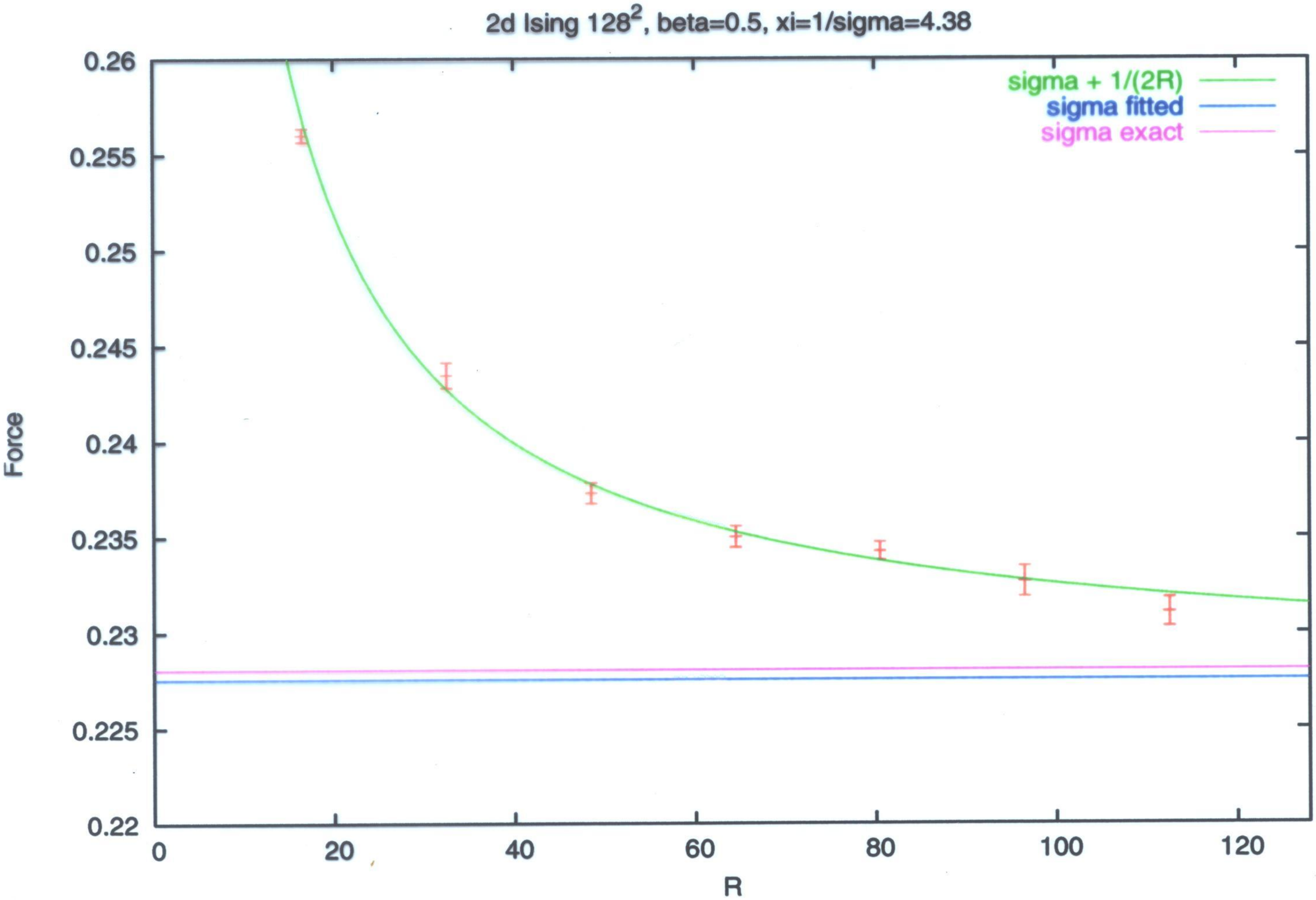
red links: J = -1

blue link: J = 0

Exact (Onsager): $\sigma(\beta) = 2\beta + \log(\tanh\beta)$

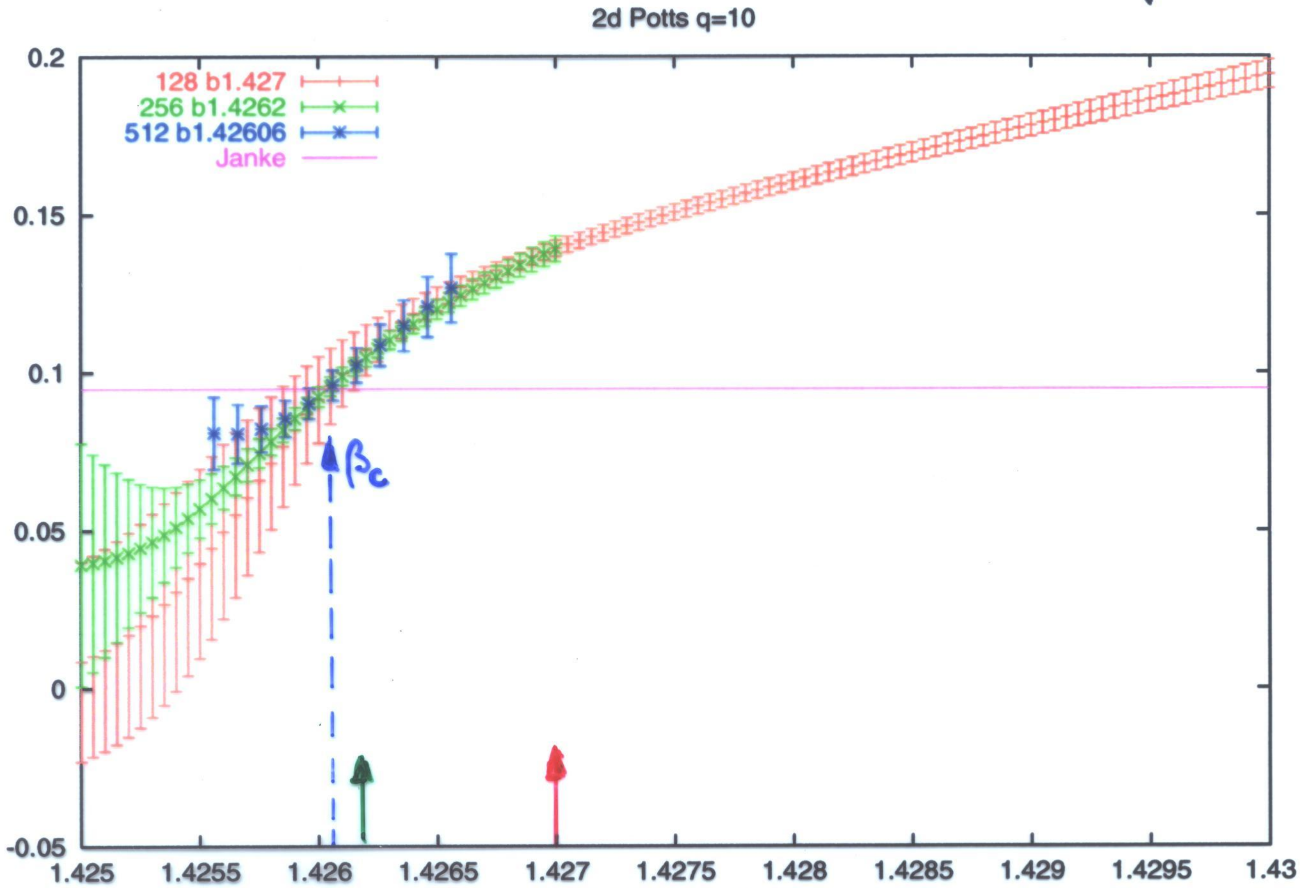
Two-point function of defects: $G(r) \sim \frac{e^{-\sigma r}}{r^{\frac{d-1}{2}}} = e^{-\sigma r - \frac{1}{2}\log r}$

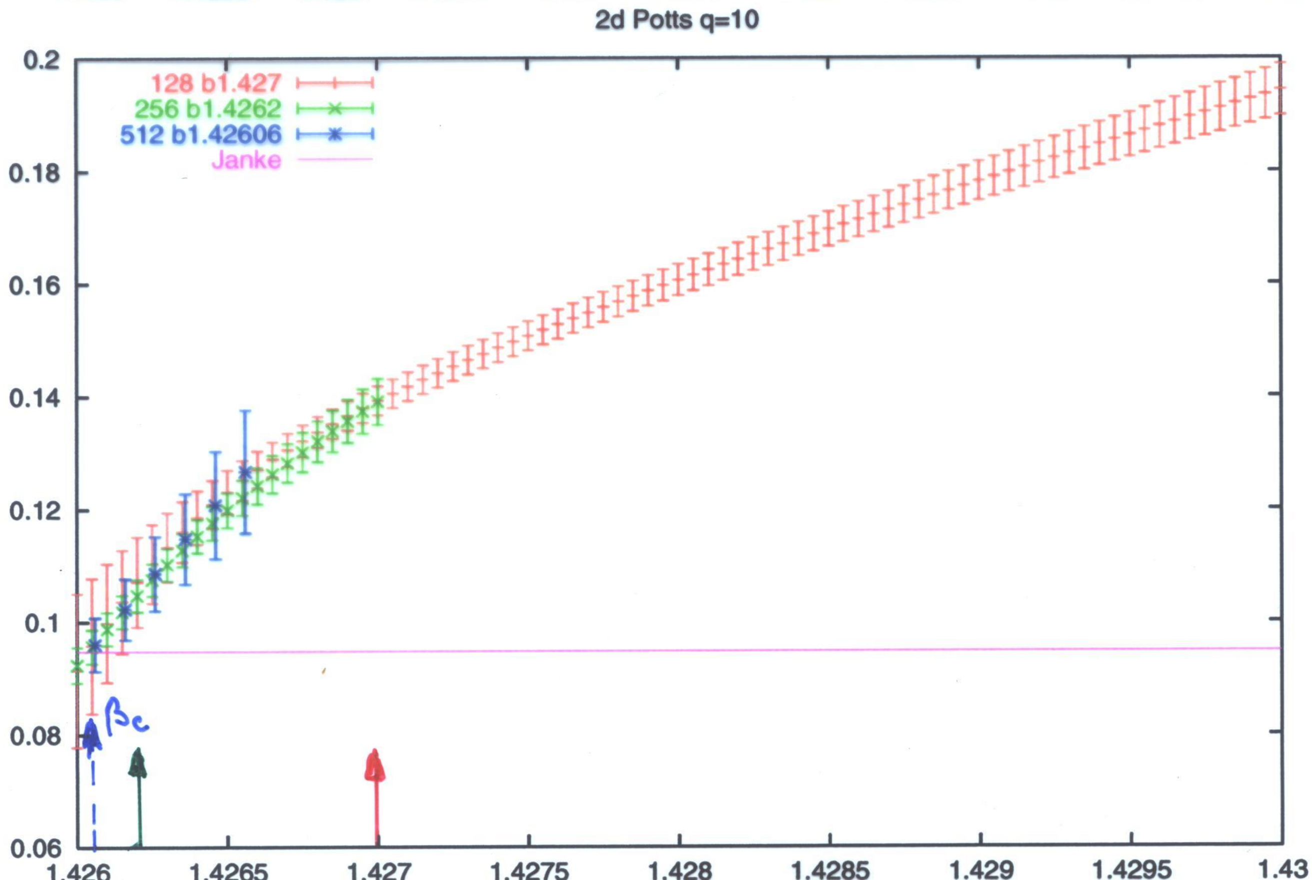
$\Rightarrow \text{Force}(r) = \sigma + \frac{1}{2r}$ $\leftarrow$ universal "Lüscher" term in 2d



2d Potts model $q = 10$

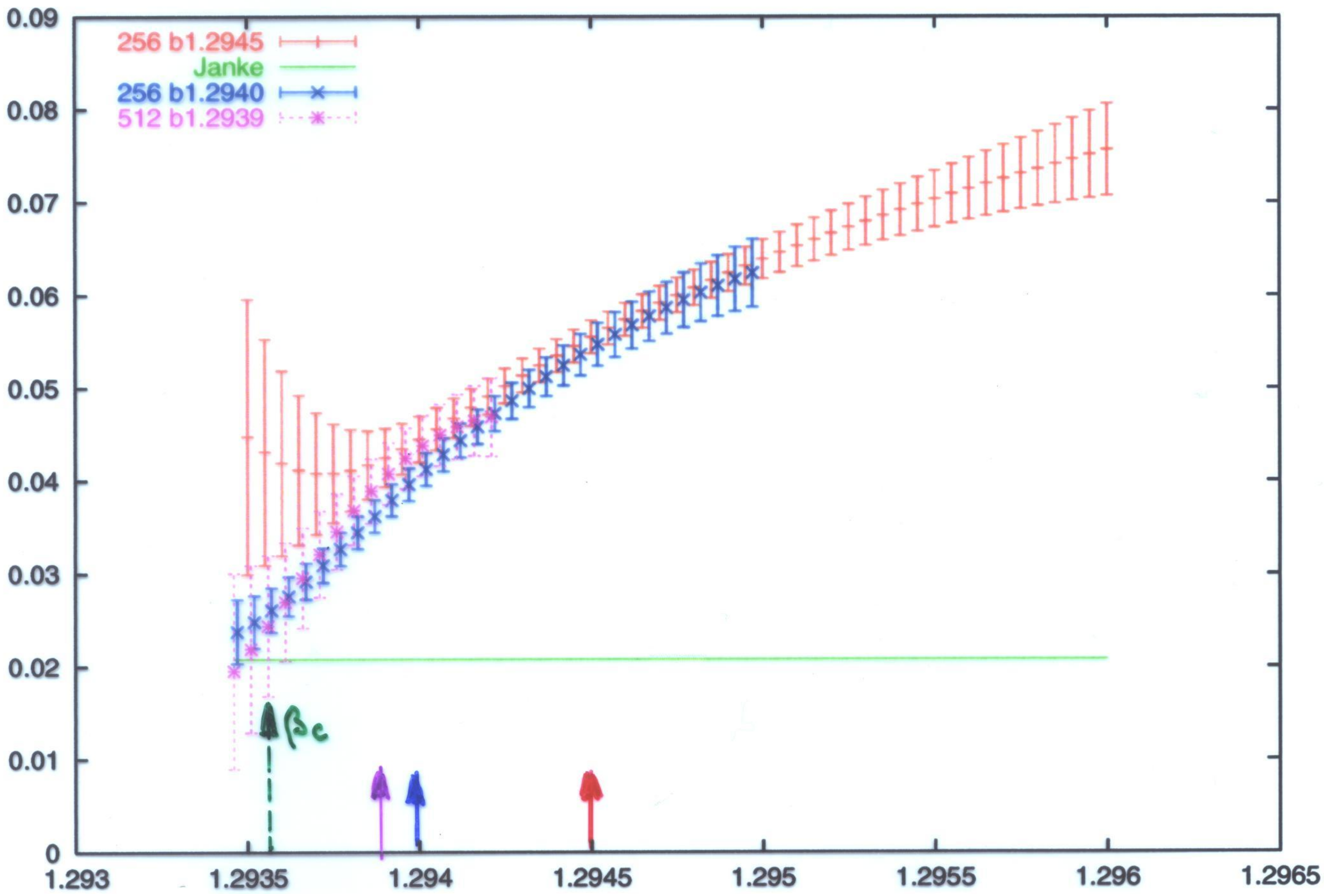
To get $\sigma(\beta_c)$, reweight from $\beta > \beta_c$ or take $L \gg \xi$ $\left(R = \frac{L}{2} \text{ fixed}\right)$





2d Potts model $q = 7$

$\beta_c = \log(1 + \sqrt{q}) = 1.29356..$
$L = 512$ simulation in progress



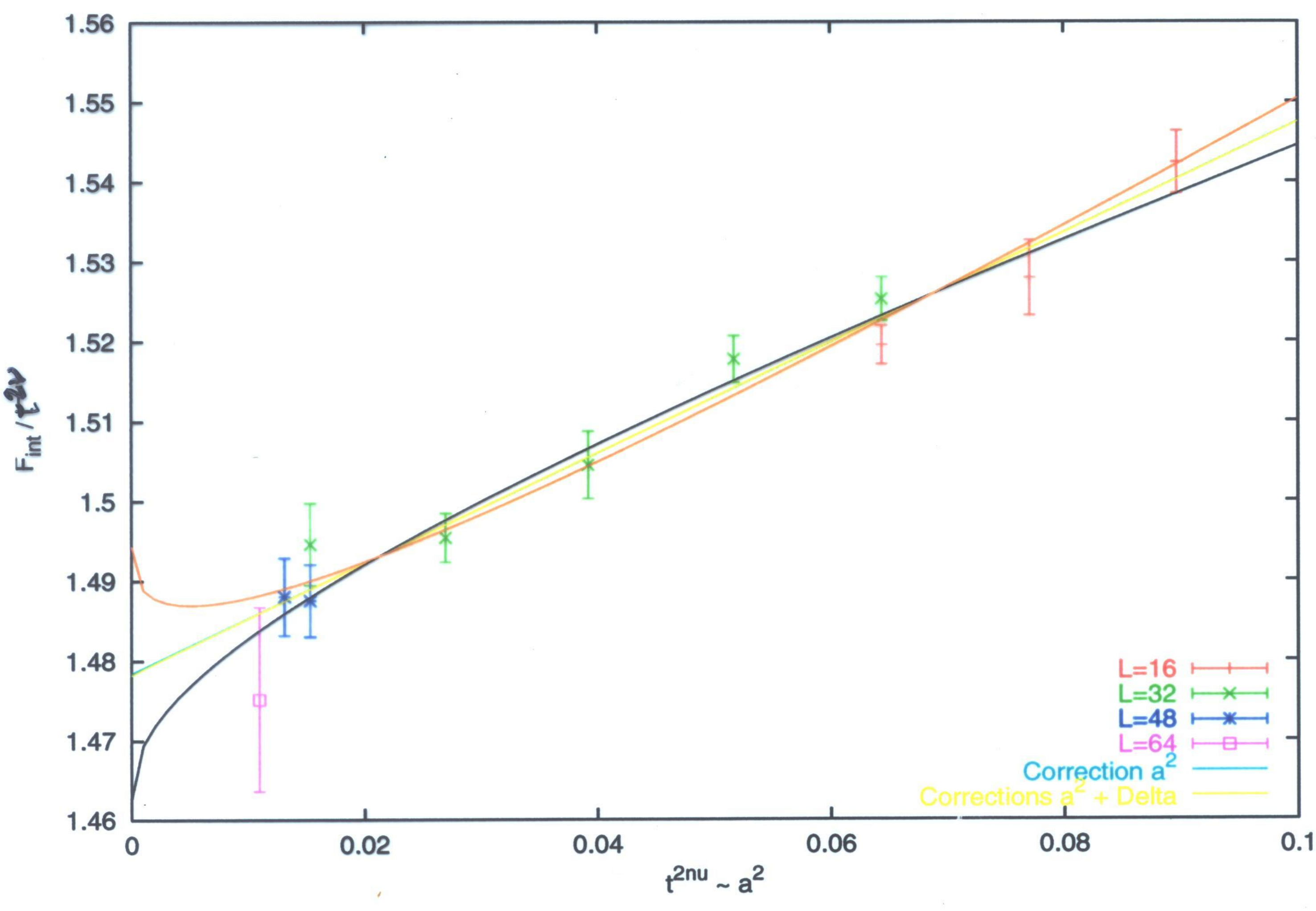
3d Ising model

$\xi \sim t^{-\nu}(1 + t^{\Delta})$ $F_{\text{interface}} \approx \sigma_0 \, t^{2\nu}$

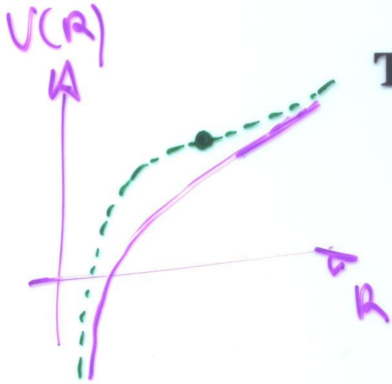
Literature for σ_0:

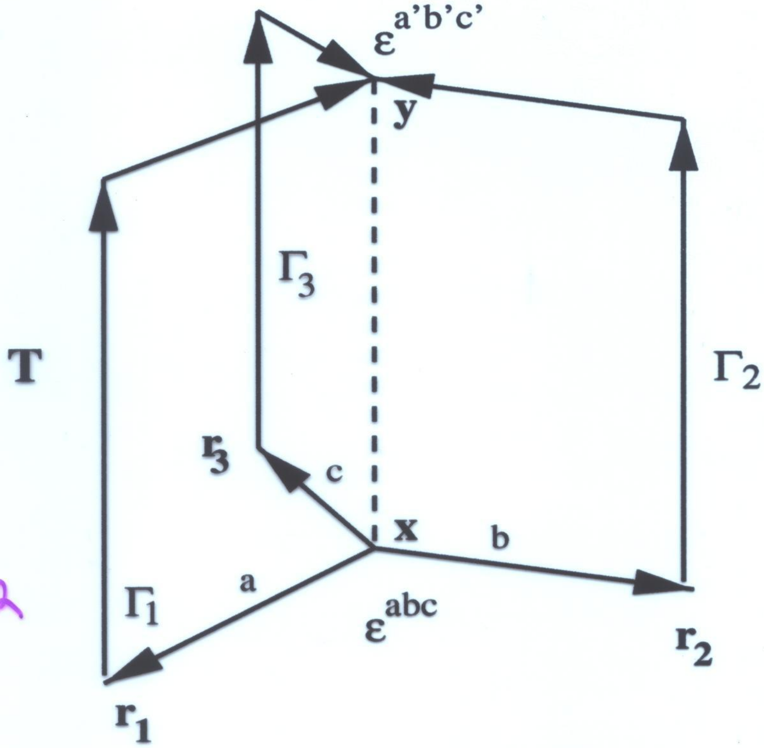
- 1.58(5) Mon PRL 1988 derivative
- 1.52(5) Berg et al.hep-lat/9206022 multimagnetic
- 1.55(5) Hasenbusch cond-mat/9704075 boundary flip
- 1.495(15) PdF and Pepe hep-lat/0110119 snake

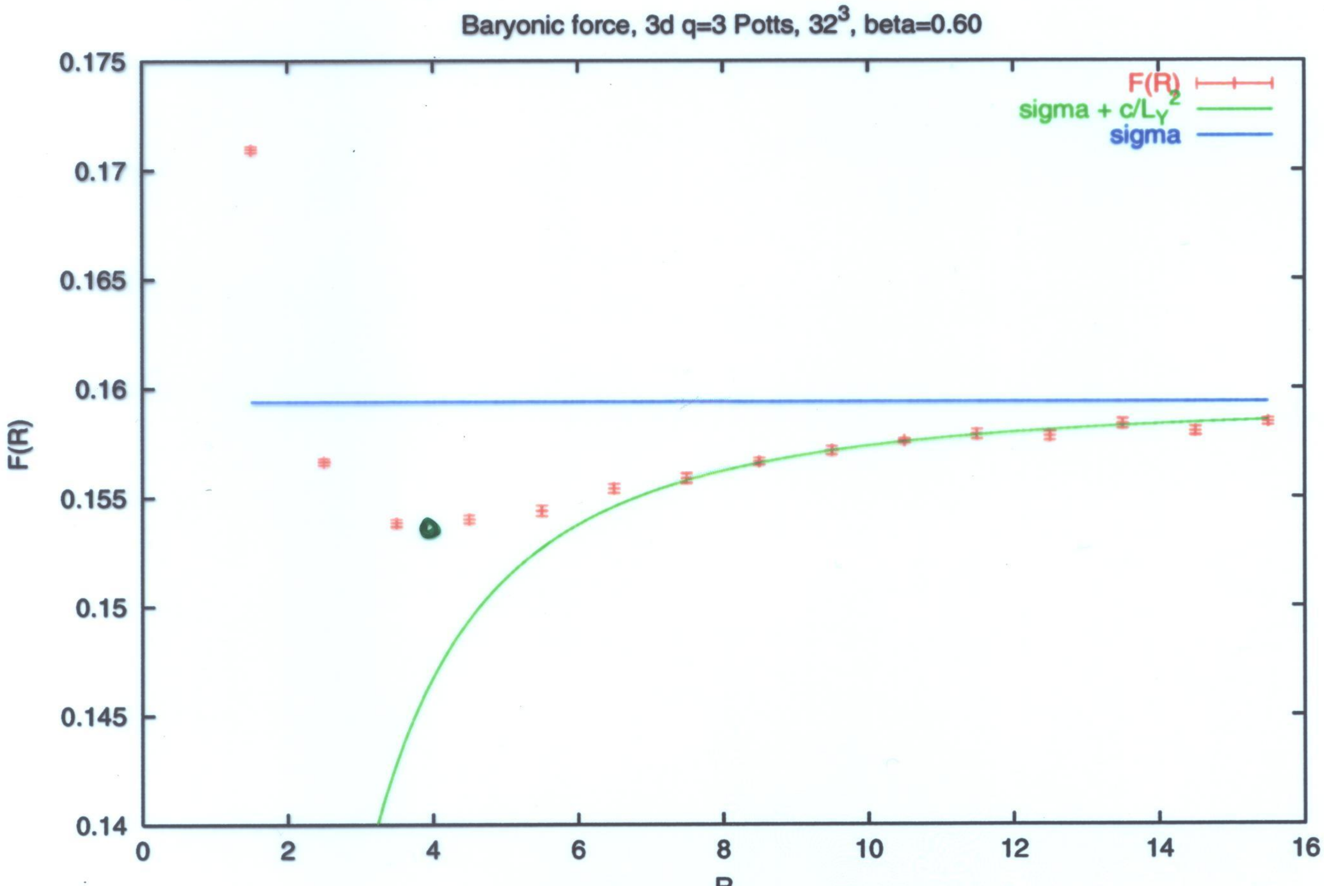
Here: 1.478(3)(18) (without/with correction to scaling)



Baryons in 3d q=3 Potts model







Free energy $\propto$ minimal interface area (Y-law) + corrections

Baryonic Lüscher term < 0 : hep-lat/0309115 O. Jahn & PdF