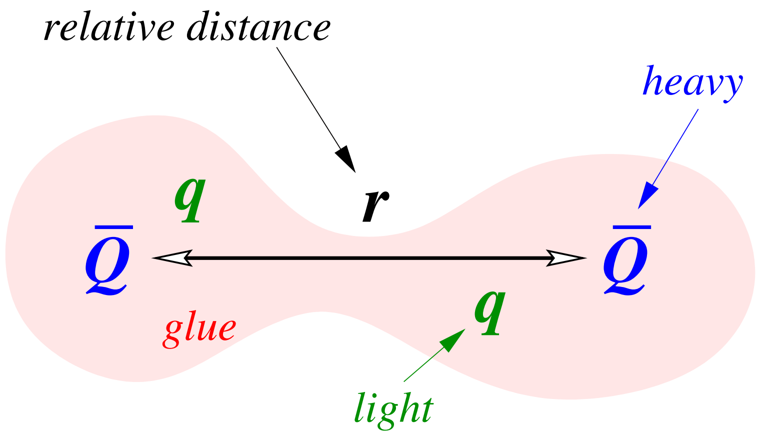
Interaction Mechanism in a Heavy-Light Meson-Meson System[a]

Merritt S Cook and HRF

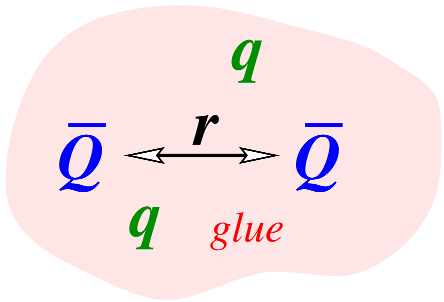
[a]This material is based upon work supported by the National Science Foundation under Grant No. 0073362 and upon resources provided by the Lattice Hadron Physics Collaboration LHPC through the SciDac program of the US Department of Energy.

Heavy-Light Meson-Meson System



? interaction…

…mechanism ?

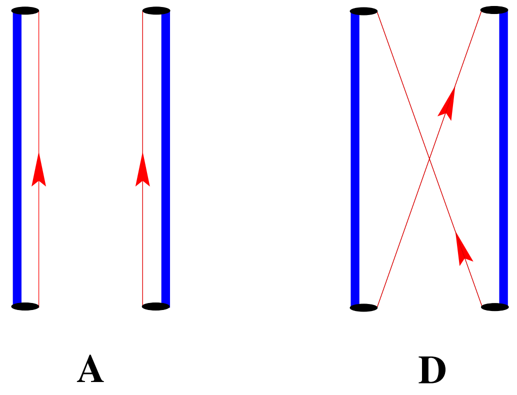


? importance of gluon vs quark d.o.f.

- heavy $Q \Rightarrow$ well-defined relative distance $\vec{r}$

$$
\begin{array}{llcl}
1-meson & \phi_{\vec{x}}(t) & = & \bar{Q}(\vec{x},t)\,\gamma_5\,q(\vec{x},t) \qquad Q\text{=heavy}, \quad q\text{=light} \\
2-meson & \Phi_{\vec{r}}(t) & = & \sum_{\vec{x},\vec{y}} \phi_{\vec{x}}(t)\phi_{\vec{y}}(t)\,\delta_{\vec{x}-\vec{y},\,\vec{r}} \qquad \vec{r} = \text{relative dist}
\end{array}
$$

$$
C^{(4)}_{\vec{r}\,\vec{r}}(t,t_0) = \langle \hat{\Phi}^{\dagger}_{\vec{r}}(t)\hat{\Phi}_{\vec{r}}(t_0)\rangle =
$$



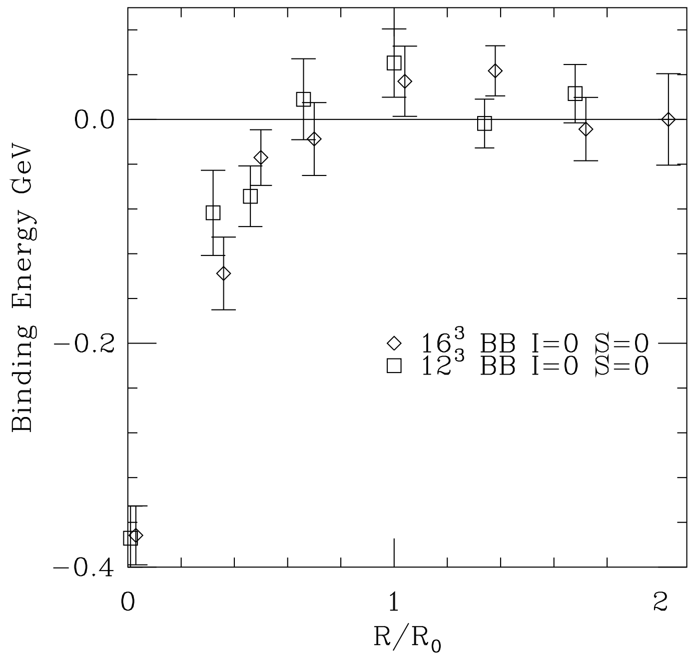
- *effective* correlator, diagonal

$$
\mathcal{C}_{\vec{r}\,\vec{r}}(t,t_0) = \frac{C^{(4)}_{\vec{r}\,\vec{r}}(t,t_0)}{C^{(2)}(t,t_0)C^{(2)}(t,t_0)} \sim e^{-[E(r)-2m](t-t_0)}
$$

$\Longrightarrow$ binding energy $[E(r)-2m] = V(r)$ potential (adiabatic)

- Early work on heavy-light:
  D.G. Richards, Nucl. Phys. (Proc. Suppl.) B 9 (1989) 181

- Example:
  UKQCD,
  C. Michael,
  P. Pennanen,
  hep-lat/9901007



$$\Phi_1(t) = \sum_{\vec{x},\vec{y}} \delta_{\vec{r},\vec{x}-\vec{y}} \overline{Q}_A(\vec{x}t)\gamma_5 q_A(\vec{x}t)\, \overline{Q}_B(\vec{y}t)\gamma_5 q_B(\vec{y}t)$$

$$\Phi_2(t) = \sum_{\vec{x},\vec{y}} \delta_{\vec{r},\vec{x}-\vec{y}} U_{P;AA'}(\vec{x}t,\vec{y}t)\, U^{\dagger}_{P';B'B}(\vec{x}t,\vec{y}t) \overline{Q}_A(\vec{x}t)\gamma_5 q_B(\vec{x}t)\, \overline{Q}_{B'}(\vec{y}t)\gamma_5 q_{A'}(\vec{y}t)$$

- local vs non-local color singlets
- link products along spatial paths P, P'

- $2{\times}2$ matrix $(\alpha,\beta=1,2)$ $\hat\Phi(t)=\Phi(t)-\langle\Phi(t)\rangle$

$$
\begin{aligned}
C_{\alpha\beta}(t,t_0)=\langle\hat\Phi^\dagger_\alpha(t)\hat\Phi_\beta(t_0)\rangle\ =\ & 2\delta^{(+)}_{\vec r,\vec r\,'}\langle\textstyle\sum_{\vec x,\vec y}\delta^{(+)}_{\vec r\,,\vec x-\vec y} \\
& \mathcal{U}_\alpha(t,\vec x\vec y)\,\mathcal{U}_\beta(t_0,\vec x\vec y) \\
& H^*(\vec y t,\vec y t_0)H^*(\vec x t,\vec x t_0) \\
& [G(\vec y t,\vec y t_0)G(\vec x t,\vec x t_0)-G(\vec y t,\vec x t_0)G(\vec x t,\vec y t_0)]\rangle
\end{aligned}
$$

- light-quark propagator $\rightarrow$

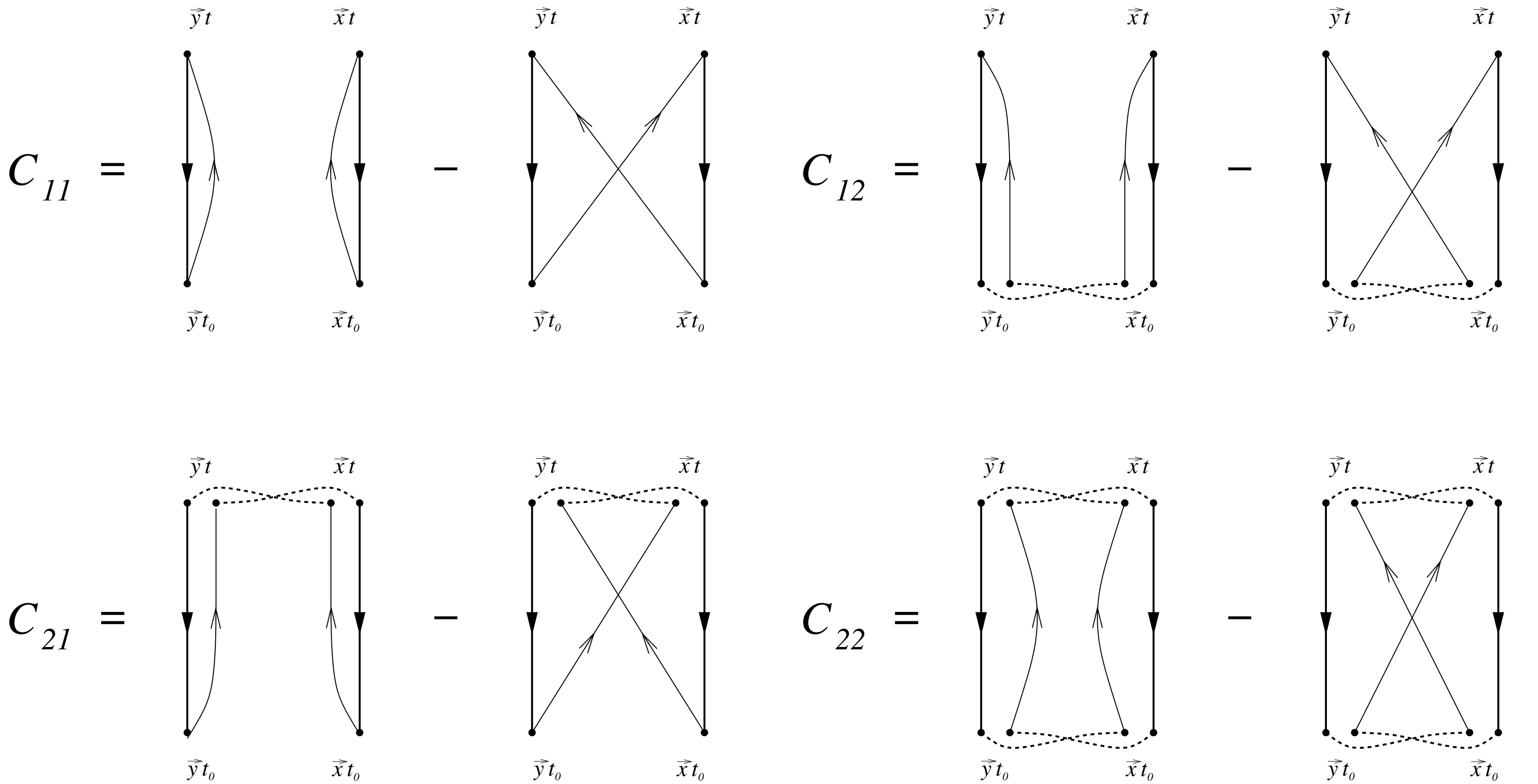
$$G(\vec y t,\vec x t_0)=\ldots\,\textit{stochastic estimator, BiCGStab}$$

- heavy-(anti)quark propagator, static$\rightarrow$

$$H^*(\vec x t,\vec y t_0)=\delta_{\vec x,\vec y}\,\frac{1}{2}(1+\gamma_4)U(\vec x t_0,\vec x t)$$

- nonlocality, spatial paths P,P' $\rightarrow$

$$\mathcal{U}_2(t,\vec x\vec y)=U_P(\vec x t,\vec y t)U^*_{P'}(\vec x t,\vec y t)+P\leftrightarrow P'$$

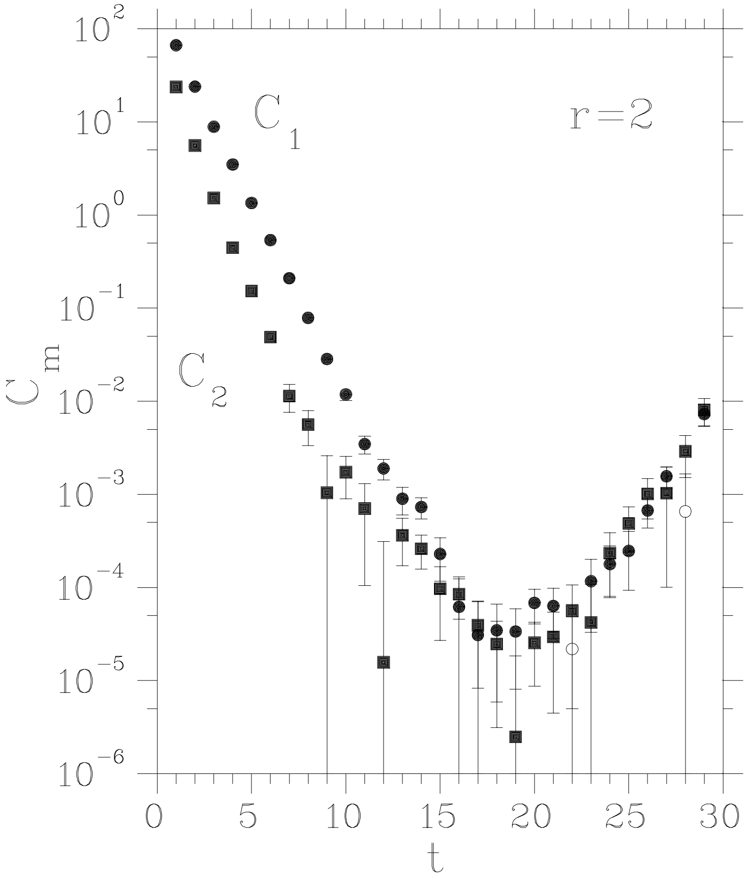


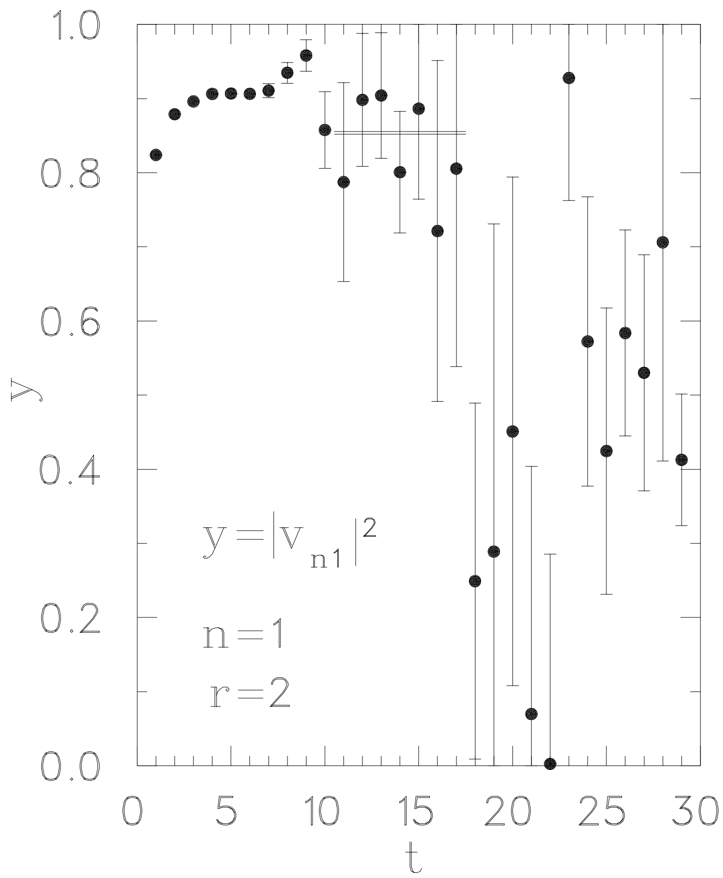
$C_{11} = $ $-$
$C_{12} = $ $-$
$C_{21} = $ $-$
$C_{22} = $ $-$
$\vec{y}\,t$ $\vec{x}\,t$
$\vec{y}\,t_0$ $\vec{x}\,t_0$

Lattice Tech Specs

- geometry $L^3 \times T = 10^3 \times 30$
- anisotropic $a_s \simeq 3a_t \simeq 0.25$fm
- tree-level tadpole improved Glue
- ditto, Wilson+Clover, $m_\pi / m_\rho \simeq 0.75$
- glue, APE fuzzing
- light quark operators, Gaussian smearing
- q-propagator, all–to–all $G(\vec{y}t, \vec{x}t_0)$
  random source estimator $(MG_R = R)$, 8 sources per color-spin
- Lüscher-Wolff type analysis: diag C on each t slice, use $t \to \infty$

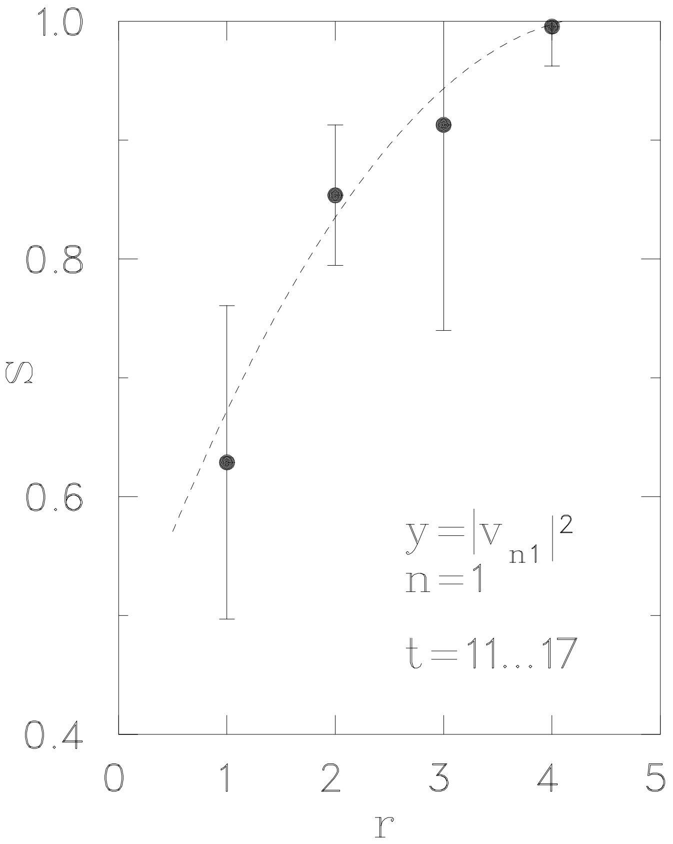
$$\begin{pmatrix} C_{11}(t,t_0) & C_{12}(t,t_0) \\ C_{21}(t,t_0) & C_{22}(t,t_0) \end{pmatrix} \begin{pmatrix} v_{m1}(t,t_0) \\ v_{m2}(t,t_0) \end{pmatrix} = C_m(t,t_0) \begin{pmatrix} v_{m1}(t,t_0) \\ v_{m2}(t,t_0) \end{pmatrix}$$

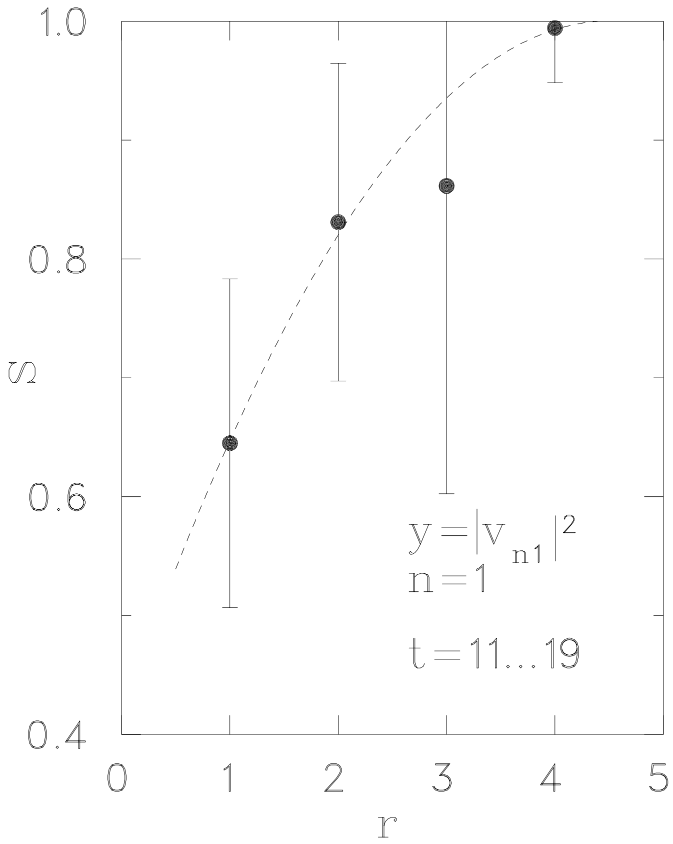




$$\bar{v}_1 = \begin{pmatrix} \sqrt{s} \\ \sqrt{1-s} \end{pmatrix} \quad \bar{v}_2 = \begin{pmatrix} \sqrt{1-s} \\ -\sqrt{s} \end{pmatrix}$$

fit $s = |v_{n1}|^2$ in asympt t range





- Use t-averaged $\overline{v}_m \Longrightarrow$ new operators, correlators

$$\Psi_m(t) \stackrel{\text{def}}{=} \sum_{i=1}^{M} \overline{v}_{mi} \Phi_i(t)$$

$$D_m(t,t_0) = \langle \hat{\Psi}_m^\dagger(t) \hat{\Psi}_m(t_0) \rangle = \sum_{n \neq 0} |\langle n | \hat{\Psi}_m(t_0) | 0 \rangle|^2 e^{-\omega_n (t-t_0)}$$

- Spectral model (analysis) $\Rightarrow$

$$F(\rho|t,t_0) = \int_{-\infty}^{+\infty} d\omega \, \rho(\omega) e^{-\omega(t-t_0)}$$

if exact $F(\rho|t,t_0) = D_m(t,t_0)$ then $\Rightarrow$

$$\rho(\omega) = \sum_{n \neq 0} \delta(\omega - \omega_n) \, |\langle n | \hat{\Psi}(t_0) | 0 \rangle|^2$$

- Discretization $\omega_k = \Delta\omega\, k, \quad k = K_1 \dots K_2$

$$F(\rho|t,t_0) \simeq \sum_{k=K_1}^{K_2} \rho_k\, e^{-\omega_k(t-t_0)} \qquad \text{parameters} \mapsto \rho_k$$

# of parameters (~ 200) $\gg$ # of lattice data (~ 30)

- Bayesian inference: given set of data D, fixed

→ find conditional `pdf` of parameters ρ

$$\mathcal{P}(\rho \leftarrow D) \propto \mathcal{P}(D \leftarrow \rho) \times \mathcal{P}(\rho)$$
$$\text{posterior} \;\propto\; \text{likelihood} \times \text{prior}$$

→ maximize posterior probability ⇒ best "fit"

→ more parameters than data? ... no problem!

Likelihood

lattice data ↓ ↓ spectral model

$$\chi^2(D,\rho) = \sum_{t_1,t_2} [D(t_1,t_0) - F(\rho|t_1,t_0)]\, \Gamma^{-1}(t_1,t_2)\, [\,..\,t_1 \to t_2\,..\,]$$

↑ covar matrix

given $[\rho]$ make (large number of) measurements of $[D]$,
then (central limit theorem) $\Rightarrow$ Gaussian `pdf`

$$\mathcal{P}(D \leftarrow \rho) = e^{-\chi^2(D,\rho)/2}$$

Prior

Shannon-Jaynes Entropy

$$\mathcal{S}(\rho) = \sum_k \left(\rho_k - m_k - \rho_k \ln(\frac{\rho_k}{m_k}) \right) \leq 0 = \mathcal{S}(m)$$

measures (lack of) information relative to default model = $[m]$

$$\mathcal{P}(\rho) = e^{+\alpha\mathcal{S}} \quad \text{entropy weight} = \alpha$$

Posterior

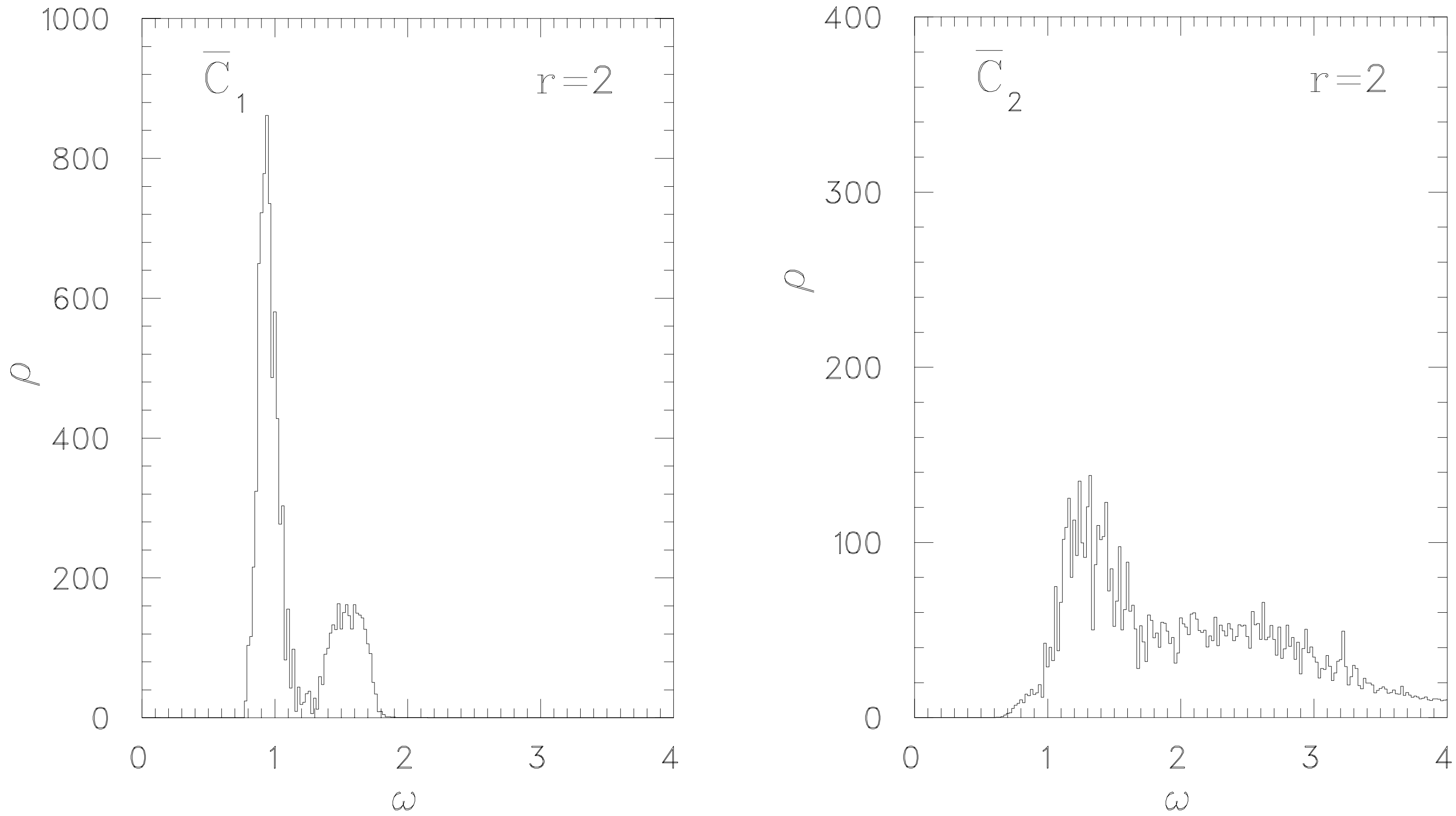
$$\mathcal{P}(\rho \leftarrow C) \propto \mathcal{P}(C \leftarrow \rho)\mathcal{P}[\rho] \propto e^{-W[\rho]} \quad \text{with} \quad W[\rho] = \chi^2/2 - \alpha\,\mathcal{S}$$

- Maximum Entropy Method (MEM)

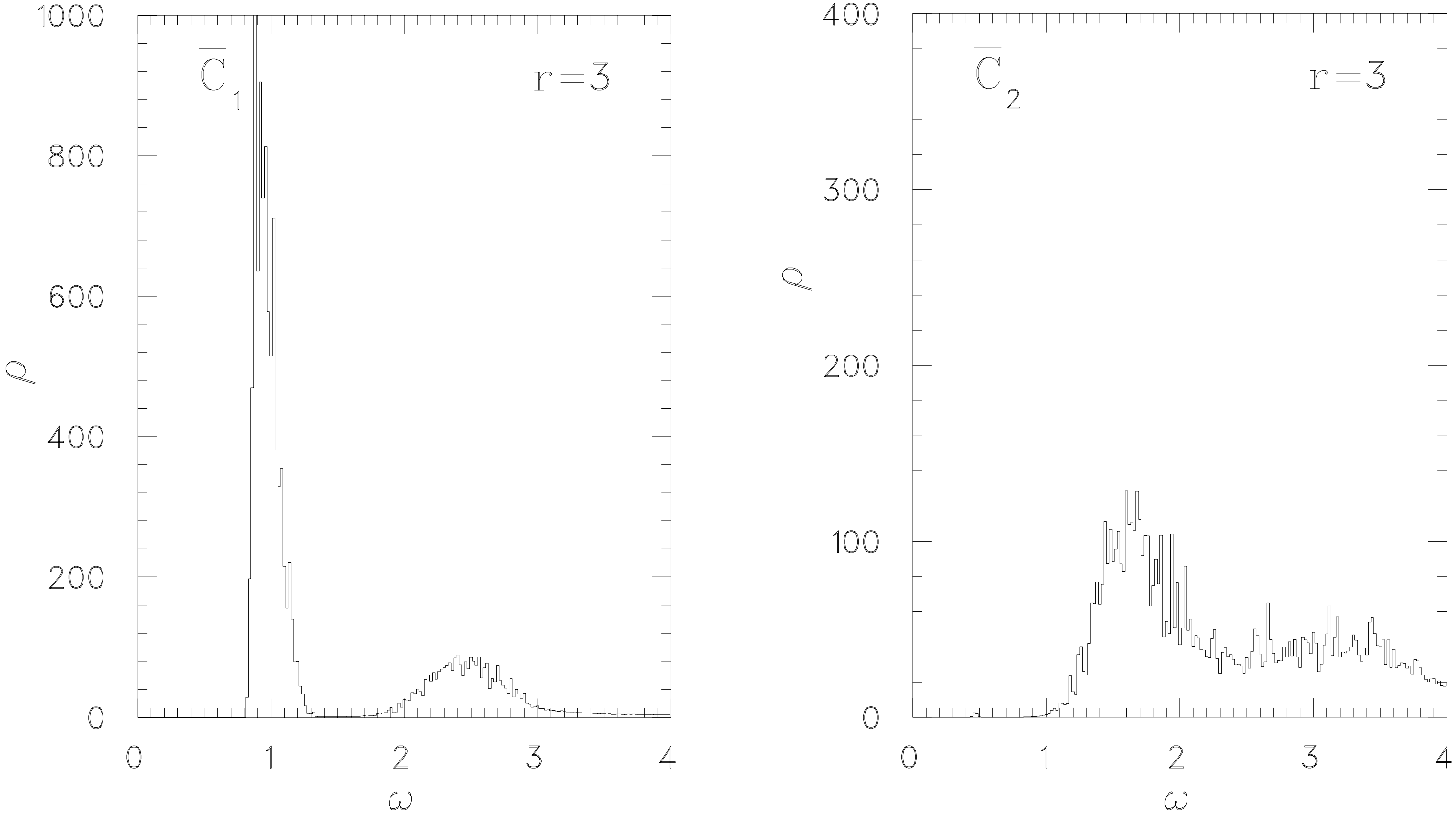
  given data $[D]$ $\Rightarrow$ solve $\mathcal{P}(\rho \leftarrow D) = \max \quad \equiv \quad W[\rho] = \min$

- Simulated Annealing (cooling)

$$Z = \int [d\rho]\, e^{-\beta W[\rho]}$$



1000
800
600
400
200
0
ρ
$\bar{C}_1$
r=2
0
1
2
3
4
ω
400
300
200
100
0
ρ
$\bar{C}_2$
r=2
0
1
2
3
4
ω



1000
800
600
400
200
0
ρ
$\overline{C}_1$
r=3
0
1
2
3
4
ω
400
300
200
100
0
ρ
$\overline{C}_2$
r=3
0
1
2
3
4
ω

Spectral observables

loosely define

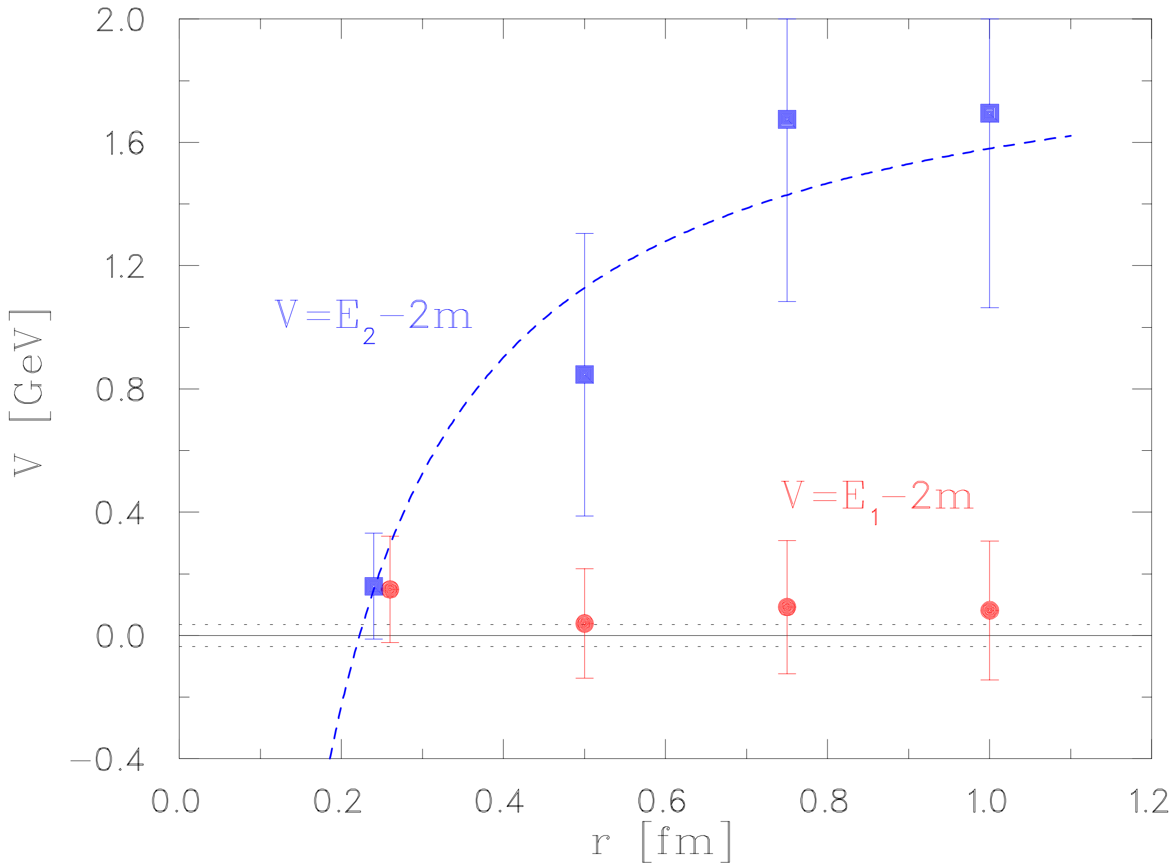
$$\delta_n = \{\omega : \omega \in \text{peak } \#n\} \quad n = 1, 2 \ldots .$$

then, for each peak n

$$\begin{array}{llcll} \text{volume} & Z_n & = & \int_{\delta_n} d\omega \rho(\omega) & \cong \quad |\langle n|\hat{\Psi}(t_0)|0\rangle|^2 \\ \text{mass} & E_n & = & Z_n^{-1} \int_{\delta_n} d\omega \rho(\omega) \omega & \\ \text{width} & \Delta_n^2 & = & Z_n^{-1} \int_{\delta_n} d\omega \rho(\omega) \left(\omega - E_n\right)^2 & \end{array}$$

→ moments $\langle \omega^p \rangle_\rho$ low $p = 0, 1, 2 \Longleftarrow$ extractable INFORMATION
→ smoothing $\int \ldots d\omega$ micro structure ...
→ annealing average over random ρ starts
→ tuning $[m]$ and α, insensitive over many orders of magnitude

r	$\bar{Z}_1$	$\bar{E}_1$	$\bar{\Delta}_1$	$\bar{Z}_2$	$\bar{E}_2$	$\bar{\Delta}_2$
1.0	107.(1)	0.997(3)	0.072(5)	67.2(9)	1.002(5)	0.072(9)
2.0	141.(2)	0.951(4)	0.074(7)	44.(1)	1.29(1)	0.191(8)
3.0	175.7(9)	0.974(3)	0.090(7)	60.(2)	1.63(1)	0.246(9)
4.0	162.6(6)	0.969(2)	0.094(3)	65.(1)	1.64(1)	0.263(9)



Assessment

- Problem areas

  operator noise: nonlocal ops, double looping, loop-loop corrs

  propagator noise: random source estim for q-props

  eigenvector noise: very strong

  level crossing: barely seen

- Remedies

  new lattice with smaller a_s

  avoid stochastic prop estim, one G for each $\vec{r}$

  different extraction strategy for eigenvectors, early time slices

- Glimps at physics

  signature of mixing quark and gluon d.o.f.

  exchange of $\bar{q}q$ for $r > 0.2\text{fm}$

  gluons take over for $r < 0.2\text{fm}$