Phase structure and Photon propagator in the 3D-Abelian Higgsmodel

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References

main references:

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Outline

- Introduction
- The phase structure
- The Landau gauge fixing
- The photon propagator
- Summary & Outlook

Introduction

Lattice gauge theories in 3 dimensions coupled to matter fields:

- effective theories of strongly correlated fermions in 2D
- effective theories for superconductivity
- reduction of 4 dimensional models to 3 dimensions for computational reasons

Introduction

Lattice gauge theories in 3 dimensions coupled to matter fields:

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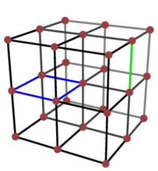
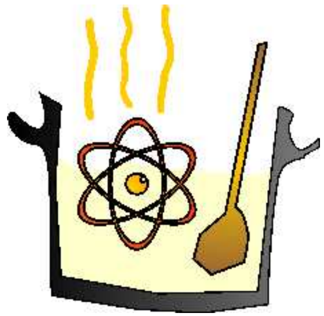
- reduction of 4 dimensional models to 3 dimensions for computational reasons

- U(1) Abelian Higgs: Interesting in its own right, since probably ”simplest” gauge theory with coupled matter fields



Introduction

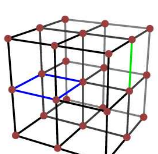


Main ingredients of the cAHM$_3$

- Higgs field with charge Q: $\phi_n = \rho_n e^{i\varphi_n}$
  angles $\varphi_n \in [-\pi, \pi)$, modulus $\rho_n \in [0, \infty)$
- Gauge field (angles): $\theta_{n,\mu}, \quad \sin(\theta_{n,\mu}) = agA'_\mu(n)$
- Gauge transformation:

$$\varphi_n \rightarrow [\varphi_n - Q\alpha_n]_{2\pi}$$

$$\theta_{n,\mu} \rightarrow [\theta_{n,\mu} - \alpha_n + \alpha_{n+e_\mu}]_{2\pi}$$



Introduction

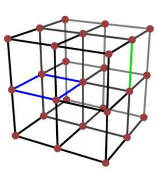
- The model: The action is defined as

$$S = -\beta \sum_P \cos\theta_P - \kappa \sum_{n,\mu} \rho_n \rho_{n+e_\mu} \cos(\underbrace{-\varphi_n + Q\theta_{n,\mu} + \varphi_{n+e_\mu}}_{\theta_L})$$

$$+ \sum_n (\rho_n^2 + \lambda(\rho_n^2 - 1)^2)$$

  - β gauge coupling, κ hopping parameter, λ self coupling
  - $\theta_P = d\theta + 2\pi k$ plaquette angle
- partition function:

$$Z = \int_{-\pi}^{\pi} \prod_n \frac{d\varphi_n}{2\pi} \int_0^\infty \prod_n \rho_n d\rho_n \int_{-\pi}^{\pi} \prod_{n,\mu} \frac{d\theta_{n,\mu}}{2\pi} \exp(-S)$$



Phase structure

compact gauge fields $\longrightarrow$ topological defects = (anti-)monopoles

2 phases:

- symmetric/confined phase: monopole plasma $\longrightarrow$ confinement of test charges; realized for small hopping parameter κ.
- Higgs phase: linear potential suppressed; remaining monopoles bound into dipole pairs; larger κ

Phase transition:

- for small $\lambda \rightarrow\ 1^{st}$ order
- large $\lambda \rightarrow$ crossover or 2^{nd} order or possibly Kosterlitz-Thouless type
- recent study at $\beta = 1.1$ (Wenzel): point of transition from 1^{st} order to continuous behaviour determined

Here $\beta = 2.0$ used (prel. end point $\lambda \approx 0.008$)

Phase structure

Algorithms:

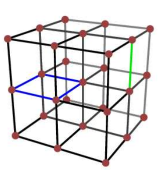
- Higgs field: Heat bath update (λ not to large)
- Gauge field: metropolis

Observables of interest:

- $\langle \phi^* \phi \rangle \rightarrow$ latent heat, interface tension
- monopole density $\rho_{\text{mon}} = \frac{1}{L^3} \sum_c |j_c|$
- Dirac string density $\rho_{\text{Dirac}} = \frac{1}{N_{\text{Plaq}}} \sum_P |k_P|$
- ANO string density $\rho_{\text{ano}} = \frac{1}{N_{\text{Plaq}}} \sum_P |\sigma_P|$

with $j = \frac{1}{2\pi} d\theta_P = dk, \quad \sigma = \frac{1}{2\pi}(d\theta_L - Q\theta_P) = dl - Qk$
Monopoles are the sources of Dirac strings and ANO strings

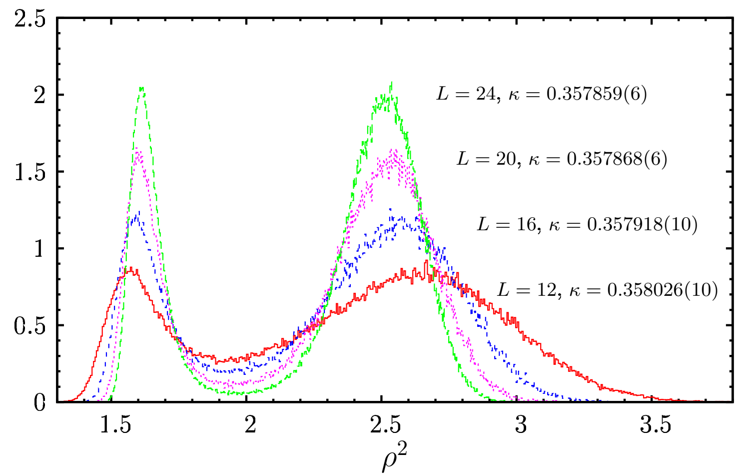
$$\delta * k = *j, \quad \delta * \sigma = Q * j$$

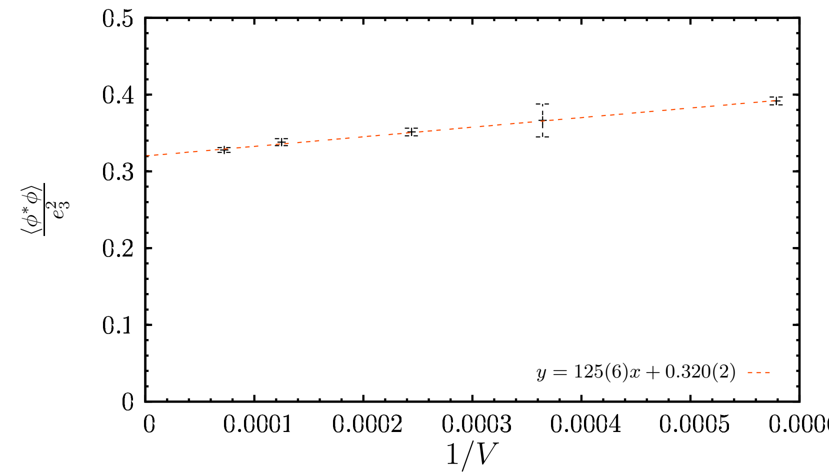


Phase structure

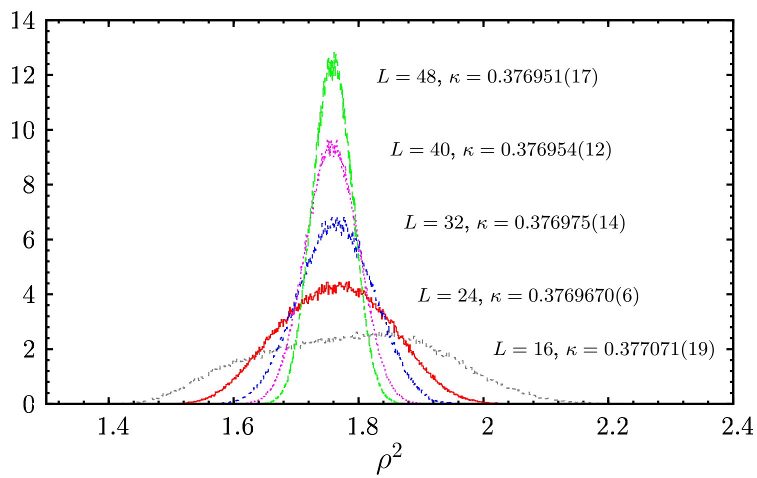
for propagator measurements → determine regions of 1^{st} order or cont. transition

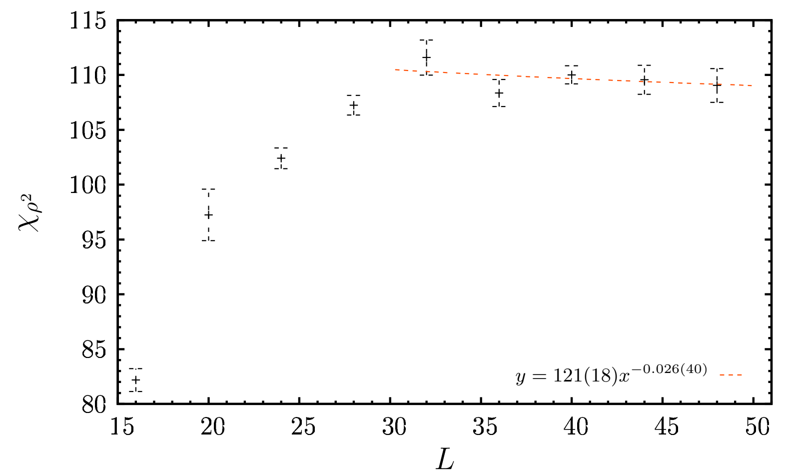
$\lambda = 0.005$ – 1^{st} order

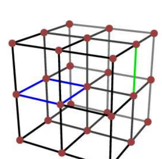




$\lambda = 0.02$ – continuous transition







Landau gauge fixing

- Photon propagator defined for a certain gauge $\rightarrow$ Landau gauge
- On the lattice $\rightarrow$ maximize functional $\mathcal{G}[\alpha] = \sum_{n,\mu} \cos(\theta_{n,\mu} - \alpha_n + \alpha_{n+e_\mu})$
- realized as sequence of localized gauge transformations, i.e. $\alpha_m = \omega \delta_{n,m}$
- each gauge transformation $\rightarrow$ sequence of pointwise gauge transformations
- optimal (greedy) ω (unique modulo 2π) can be determined on each site
- fundamental optimization problem: get stuck in local optimum

Landau gauge fixing

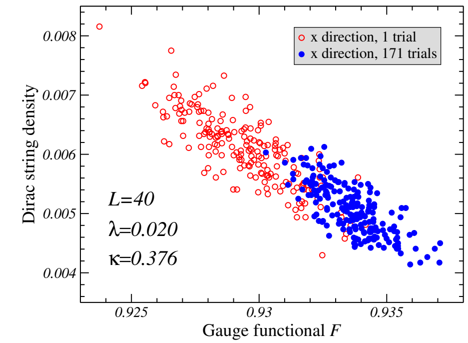
Approaches:

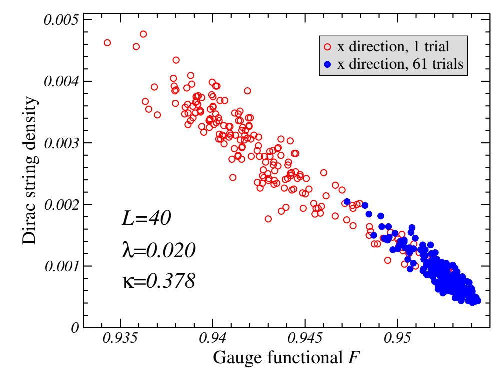
- several restarts from random gauge copies
- overrelaxation $\omega \rightarrow \eta\omega, \quad 1 < \eta < 2$, fastest convergence $\eta \approx 1.8$, $\eta = 2$ conserves gauge functional
- stop if change in gauge functional drops below limit

Landau gauge fixing

Found: Maximizing gauge functional $\leftrightarrow$ Minimizing Dirac string density

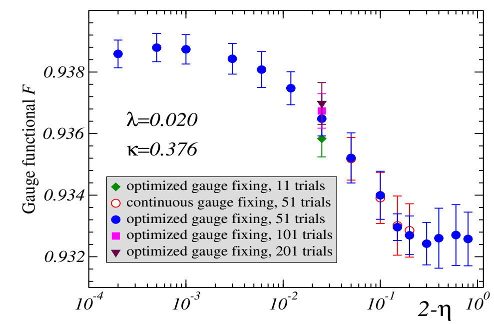
- $\rightarrow$ intuitive understanding of difficulties
- (strong) local maximum = no Dirac loops left, remaining Dirac lines between monopoles minimize their length
- global maximum = certain pairing with minimal Dirac string density
- Higgs phase: diluted dipole gas -> few pairings possible
- symmetric phase: dense monopole plasma -> gauge fixing more demanding



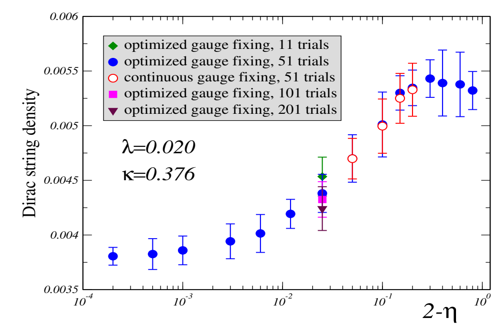


Landau gauge fixing

Study of gauge functional and Dirac string density dependence on η



- better results for η close to 2
- better results if $\eta = 2$ and $\eta < 2$ steps applied in alternating order
- both methods are more effective than increasing number of restarts (regarding number of loc. gauge transf.)



Landau gauge fixing

Gauge fixing very costly: refinements

- use discrete subgroup of U(1) (avoid calculation of trig. functions)
- use preselection: i.e. do several restarts, stop if a weak limit reached
- fine tuning with full U(1) and the best candidate from preselection

Photon propagator

The dimensionless, gauge dependent propagator $D_{\mu\nu}$ defined as expectation value of gauge field correlations

$$D_{\mu\nu}(\vec{p}) = \langle \tilde{A}_\mu(\vec{k}) \tilde{A}_\nu(-\vec{k}) \rangle$$

where

$$\tilde{A}_\mu(\vec{k}) = \frac{1}{L^3} \sum_n e^{2\pi i \frac{1}{L} \sum_{\nu=1}^{3} k_\nu (n_\nu + \frac{1}{2}\delta_{\mu\nu})} A_{n+\frac{1}{2}e_\mu,\mu}$$

is just a Fourier transform of the gauge field $A_{n+\frac{1}{2}e_\mu,\mu} = \sin(\theta_{n,\mu})$ and lattice momenta $\vec{p}$ are given by

$$p_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{L_\mu}\right)$$

Photon propagator

Assuming a rotational invariance and reality $\rightarrow$ Decomposition of the full propagator into transverse and longitudinal components

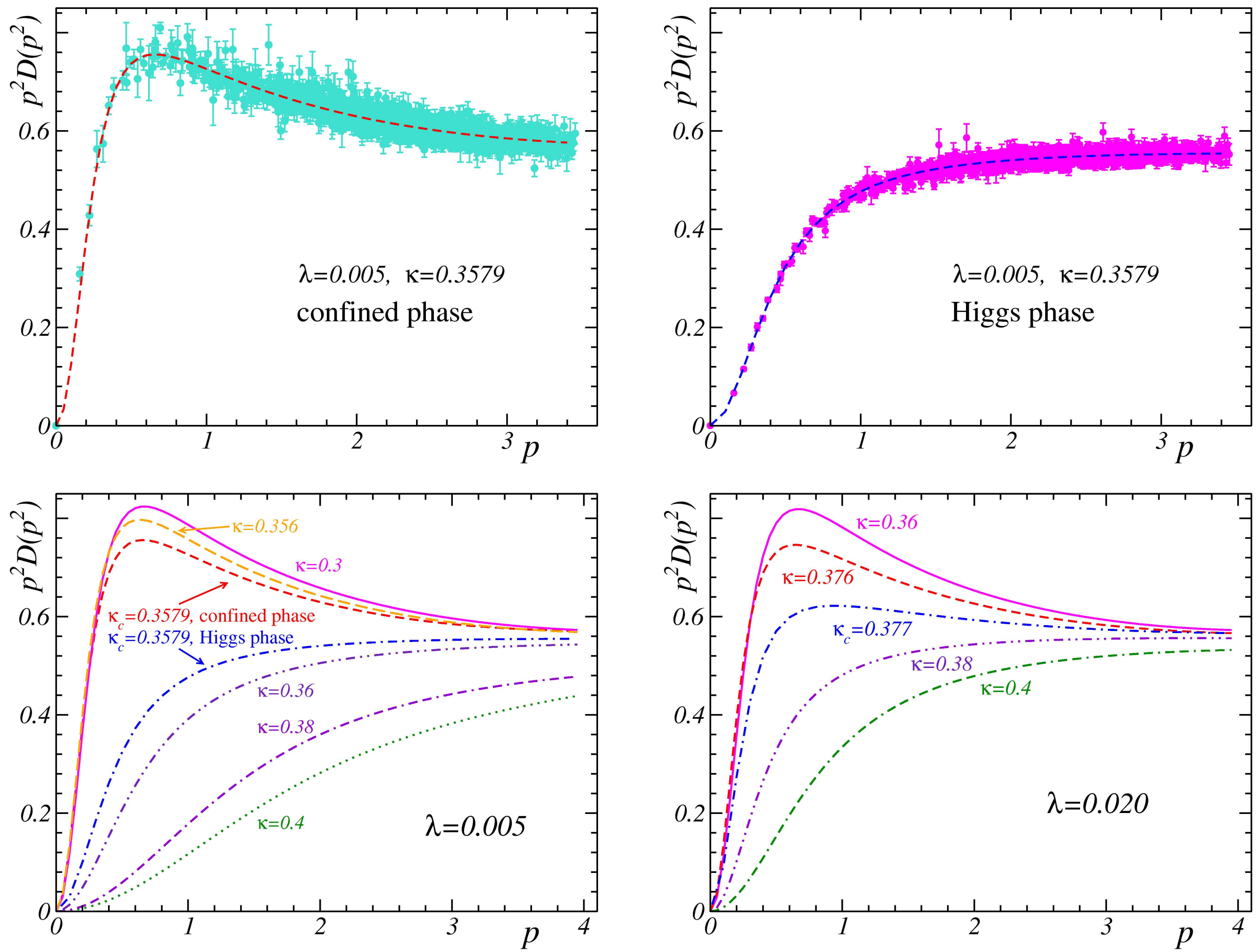
$$D_{\mu\nu}(\vec{p}) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2) + \frac{p_\mu p_\nu}{p^2} \frac{F(p^2)}{p^2}$$

- Landau gauge fulfilled $\rightarrow F(p^2)$ vanishes
- structure function $D(p^2)$ obtained via projection
- ansatz for fitting:

$$D(p^2) = \frac{Z m^{2\alpha}}{\beta(p^{2(1+\alpha)} + m^{2(1+\alpha)})} + C$$

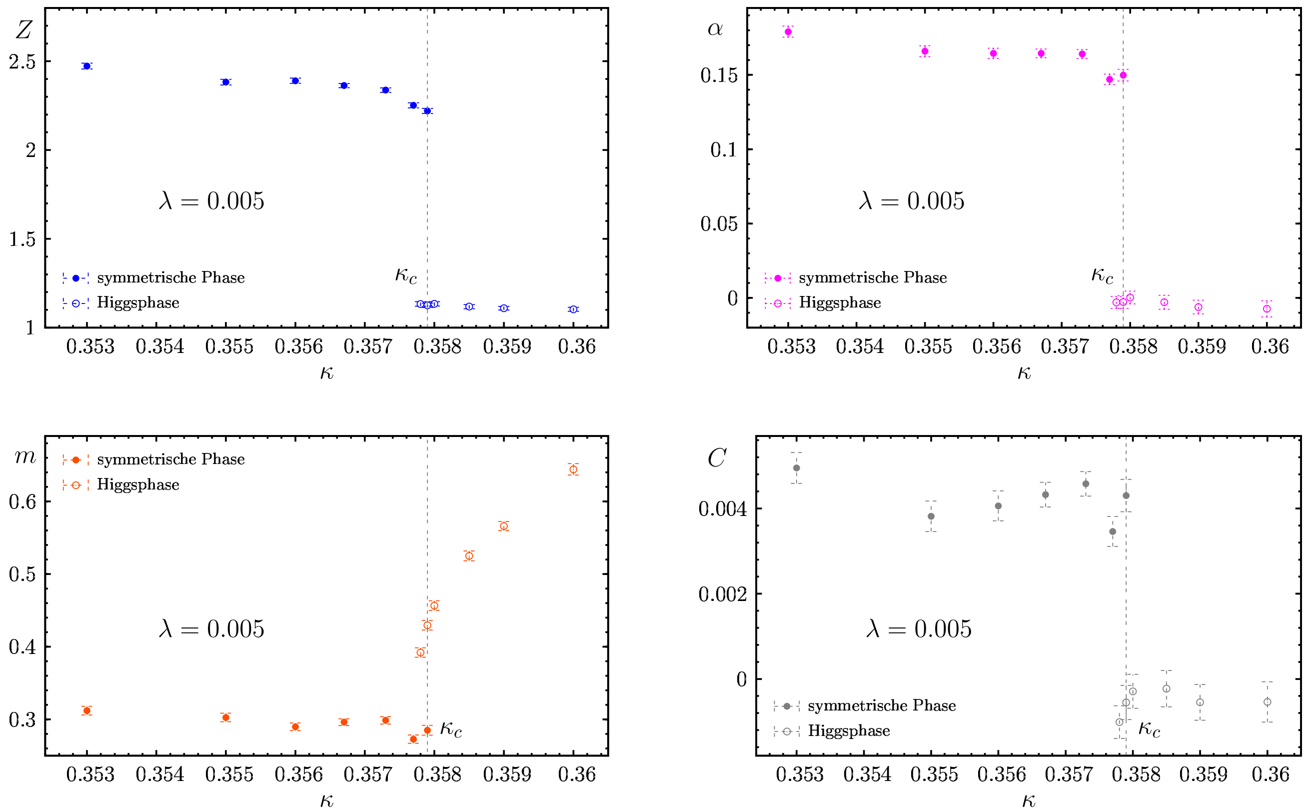
Photon propagator

Momentum dependence of $p^2D(p^2)$



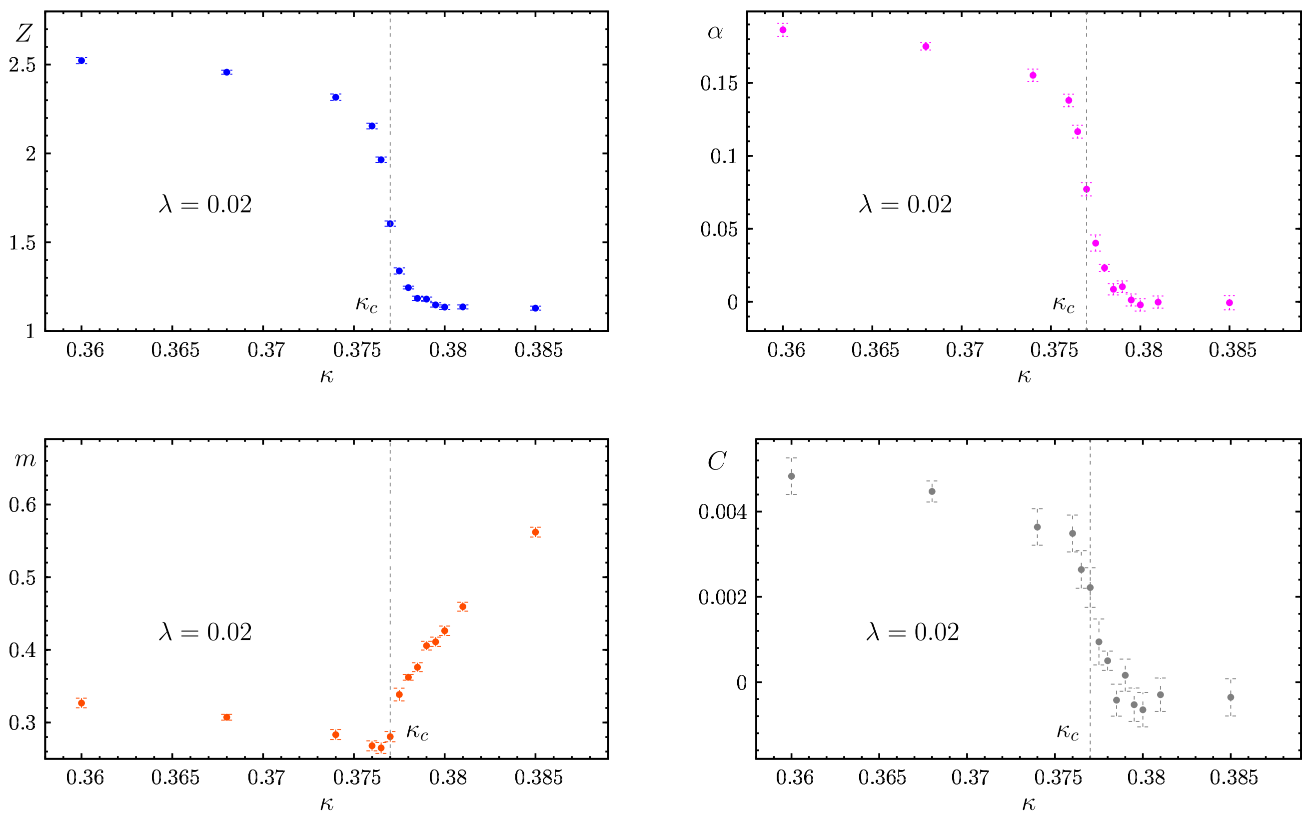
Photon propagator

Fit parameters in the 1^{st} order transition region



Photon propagator

Fit parameters in the continuous transition region



Summary

- phase structure of the Abelian Higgs model in the fundamental representation confirmed
- improvement in the gauge fixing algorithm using preselection and discrete subgroups
- propagator well described by 4 parameters: mass, anomalous dimension, renormalization constant, contact term
  - positive anomalous dimension α in confined phase
  - trivial massive gauge field propagator in the Higgs region
  - α strongly sensitive to successful gauge fixing, mass less sensitive
  - propagator reflects discontinuous change for small λ
  - continuous region: behaviour similar to London Limit $\rightarrow$ compact phase of the Higgs field is the main ingredient which influences the propagator

Outlook

- fixing the end point more precisely
- in progress: study of cAHM$_3$ with $Q = 2$;

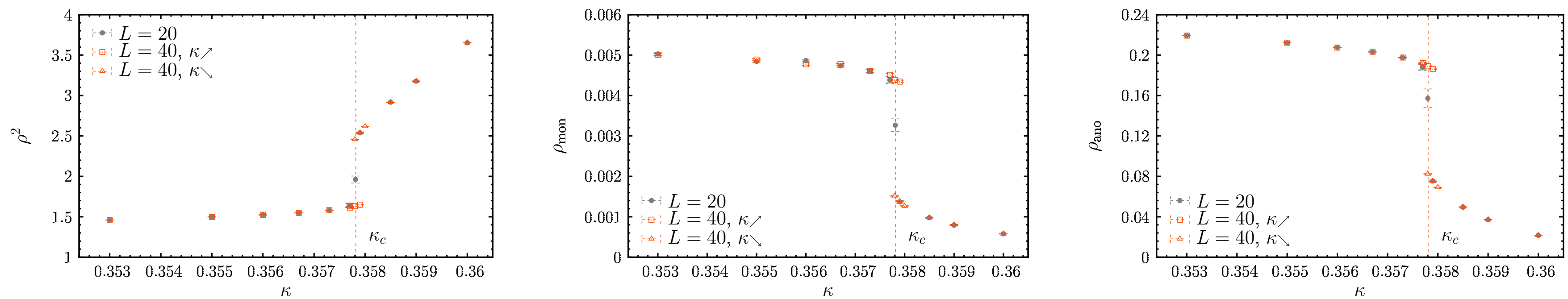
  new features:

  - expected transition from first to second order (known to exist in London Limit)
  - test charges with $q = 2$ not confined in both phases
  - monopoles can form long chains due to their double ”valence” (ANO strings)
  - negative anomalous dimension in vicinity of second order phase transition

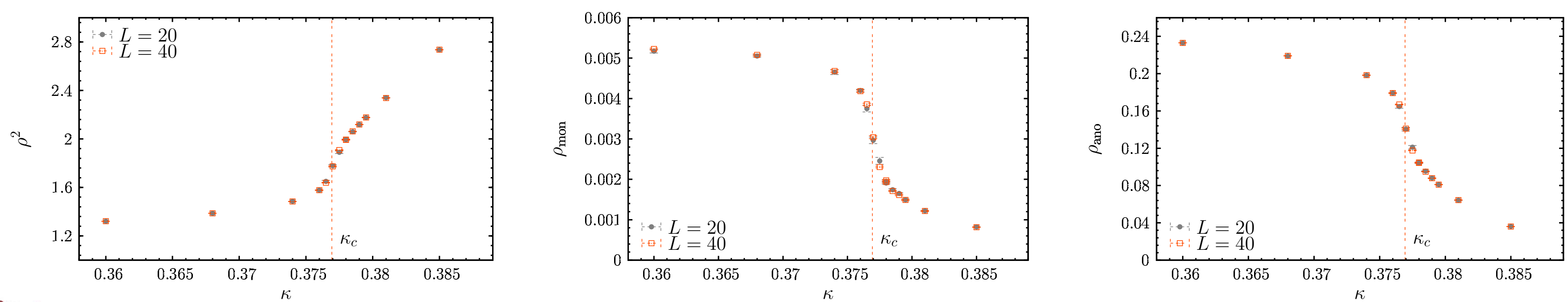
$Q = 1$ phase structure

for propagator measurements $\rightarrow$ determine regions of 1^{st} order or cont. transition, respectively

$\lambda = 0.005$



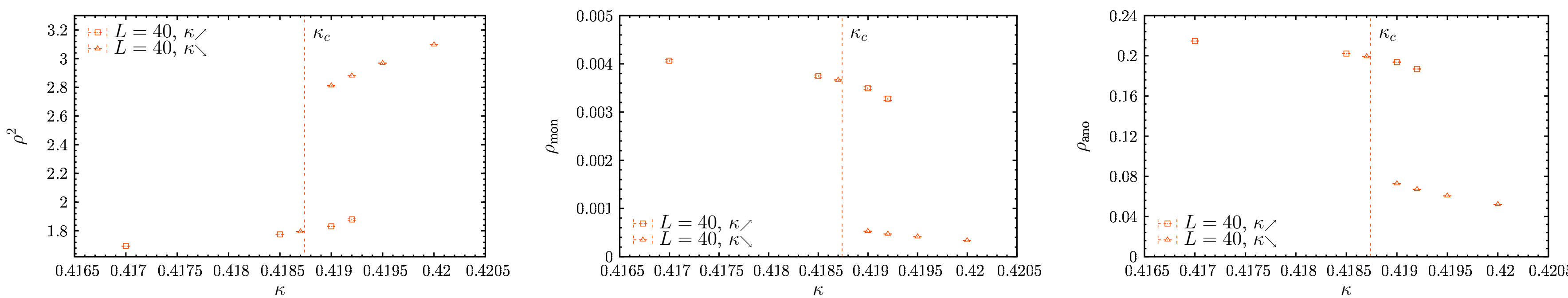
$\lambda = 0.02$



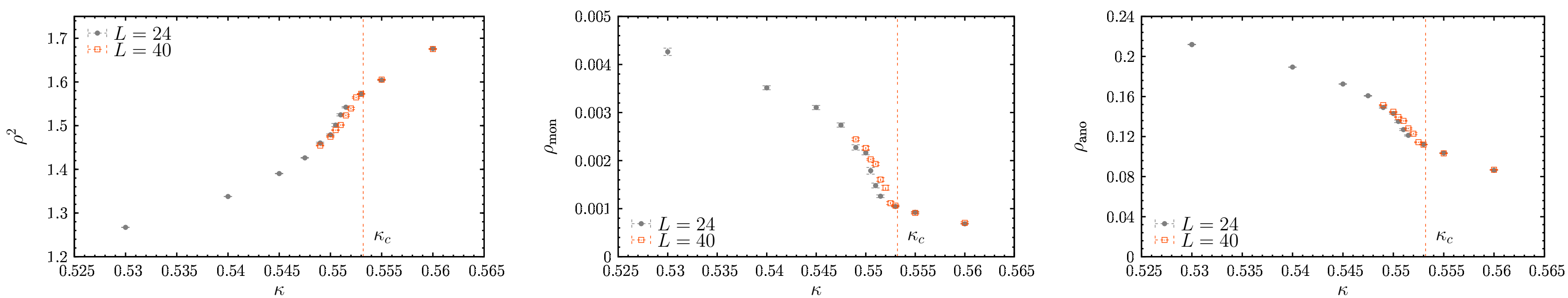
$Q = 2$ phase structure

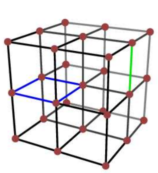
$Q = 2 \rightarrow$ determine regions of 1^{st} order or 2^{nd} order, respectively

$\lambda = 0.02$



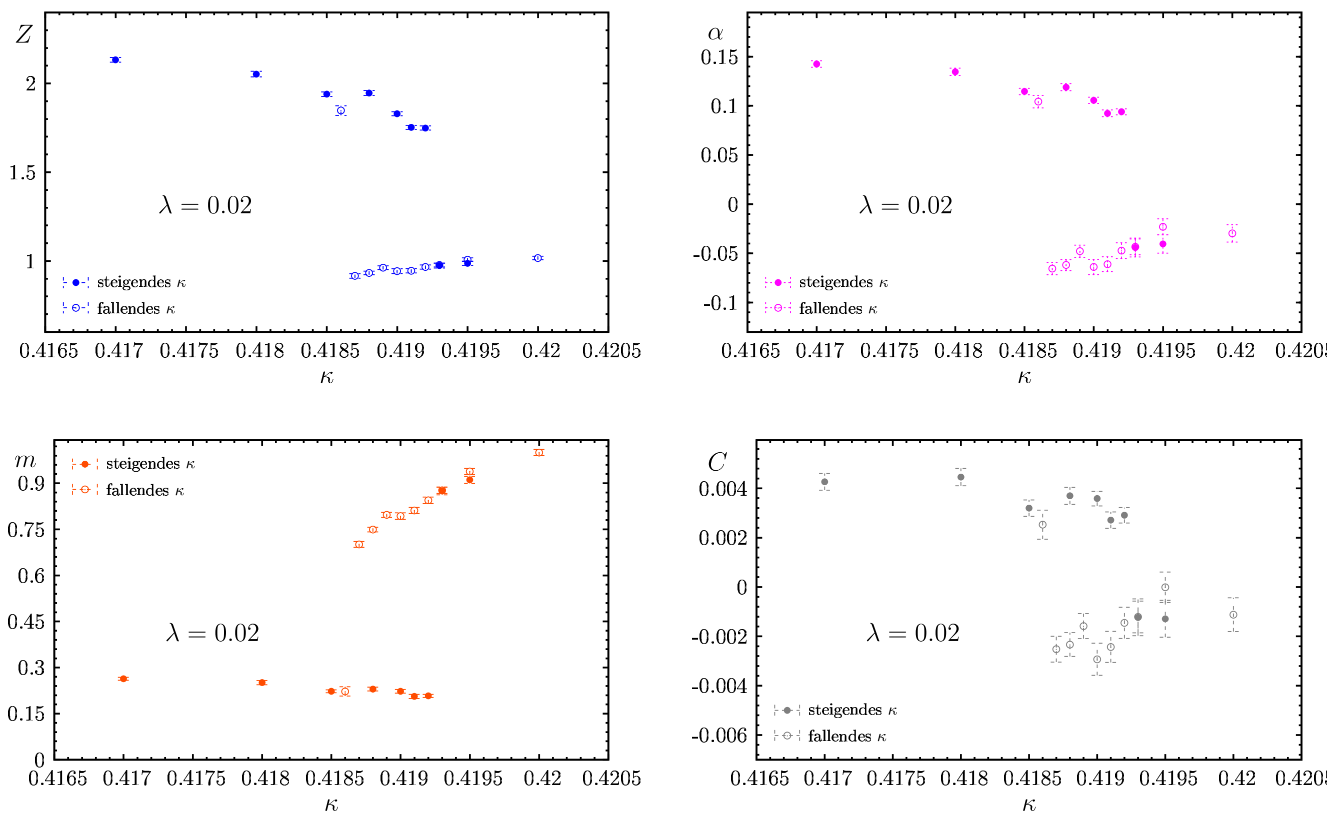
$\lambda = 0.2$





$Q = 2$ photon propagator

Fit parameters in the 1^{st} order transition region ($Q = 2$)



$Q = 2$ photon propagator

Fit parameters in the 2^{nd} order region ($Q = 2$)