HU Berlin

A finite temperature investigation of dual superconductivity in the modified SO(3) LGT

In collaboration with:
A. Barresi, M. D'Elia and M. Müller-Preussker

LEILAT 04
Leipzig June 5th 2004

- Motivations
- Confinement, topology and order parameters
- Pure Adjoint YM, Twist and ergodicity
- Finite temperature transition in the trivial Twist sector
- Outlook & Conclusions

Motivations

Is confinement a characteristic of pure" Yang-Mills theories?
Natural questions:

- How is $\mathbb{Z}_N$ symmetry breaking linked to the relevant DOF in the continuum limit and to fermions and χSB?
- What are the differences between fundamental and adjoint $\simeq SU(N)/\mathbb{Z}_N$ discretizations for pure YM?
- Are the relevant degrees of freedom topological and where they arise from?
- Goal: write an effective theory for them...

"Playground": $SU(2)$ and $SO(3) \simeq SU(2)/\mathbb{Z}_2$

SU(N) & deconfinement phase transition

Natural gauge inv. non-pert. UV and IR cutoff

$$S = \sum_P \sum_\nu \beta_\nu \Big(1 - \chi_\nu(U_P)\Big); \frac{1}{g^2} = \sum_\nu \beta_\nu c_\nu$$

$$U_P = \prod_{(x,x+\hat{\mu})\in\gamma} U_\mu(x) \quad U_\mu(x) \in SU(N)$$

In the fundamental representation

$$S = \beta_F \sum_P \Big(1 - \frac{1}{N} Re(Tr_F U_P)\Big)$$

- spontaneous symmetry breaking linked to the center $\mathbb{Z}_N$ of the group
- confinement at $T = 0$ and finite temperature deconfinement phase transition (order depends on N)

i.e. the Polyakov line $Tr_F(\prod_t U_4(\vec{x}, t))$ is not invariant under a global $\mathbb{Z}_N$ flip that leaves S invariant

Confinement and topology

G group classifying gauge transformations, spontaneously broken to G_1 by some mechanism, i.e. Higgs. S^n compactified space on which gauge fields are living

$$\Omega : \quad S^n \to G/G_1$$

and

$$G_2 = \pi_n(G/G_1)$$

is the group classifying the solutions quantum numbers
't Hooft - Polyakov Monopoles: Can be defined in the Hamiltonian picture, "naturally" survive quantization
Georgi-Glashow model; Seiberg-Witten N=2 SUSY Yang-Mills

A "disorder parameter" for monopole condensation proposed and tested numerically (Di Giacomo et al., Fröhlich-Marchetti)

Introduce a non-local scalar "boson" related to the gauge fixing and to a lattice discretization of the transverse monopole magnetic field

$$\Phi_i(\vec{x},\vec{y}) = G\, e^{i\tau_3 b_i(\vec{x}-\hat{\imath},\vec{y})}\, G^\dagger\,; \quad \vec{\nabla}\cdot\vec{b} = 0$$

acting at a fixed time slice t_0 on every link in the time direction

$$U_4(\vec{x}+\hat{\imath},t_0) \;\rightarrow\; \Phi_i(\vec{x}+\hat{\imath},\vec{y})\, U_4(\vec{x}+\hat{\imath},t_0)$$

Only the time-like plaquettes $U_{i4}(\vec{x},t_0)$ are affected, leading to a modified "monopole" action S_M, the disorder parameter being:

$$\langle\mu(t)\rangle = \frac{\int(DU)e^{-S_M(t)}}{\int(DU)e^{-S}}; \;\; \rho = \frac{\partial}{\partial\beta}\log\langle\mu\rangle = \langle S\rangle - \langle S_M\rangle_M$$

Representation "independant"...

What about “Vortices”?

Original ’t Hooft proposal: Topological classification directly related to the properties of gauge transformations at finite temperature along the compactified time direction.

Natural YM gauge group is $SU(N)/\mathbb{Z}_N$

$$\pi_1(SU(N)/\mathbb{Z}_N) \sim \mathbb{Z}_N$$

Non perturbatively can be defined in any representation if $g(\vec{x}, 0) \neq g(\vec{x}, 1/T)$ (allowed by pure YM boundary conditions)
Alternatively see Fröhlich-Marchetti

How do they couple to fermions?
Exceptional groups G_2 (Wiese et al.)?

't Hooft construction in SU(N) (de Forcrand et al.)

Temporal twists $\rightarrow$ maximal 't Hooft loop, dual to Wilson loop
Incorporate in $SU(N)$ all (temporal) twisted b.c.; vortex free energy:

$$Z_e(\vec{e}) = e^{-\frac{1}{T}F(\vec{e};L,T)} = \frac{\sum_{k_i=0}^{N-1} e^{-2\pi i \vec{e}\cdot\vec{k}/N} Z_k(\vec{k})}{\sum_{k_i=0}^{N-1} Z_k(\vec{k})},$$

$$P(\vec{x})P^\dagger(\vec{x}+L\vec{e}) = e^{-2\pi i \vec{e}\cdot\vec{k}/N}\mathbb{I}$$

$$Z_e(\vec{e}) = e^{-\frac{1}{T}F_e(\vec{e};L,T)} = \langle P(\vec{x})P^\dagger(\vec{x}+L\vec{e})\rangle_{L,T}$$

Within the Villain formulation one can prove that

$$\sum_{\text{t.s.}} Z_{SU(2)} = A \sum_{\sigma_P=\pm 1} \int (DU) e^{\beta_V \sum_P \sigma_P \mathrm{Tr}_F U_P} \cdot \prod_c \delta(\sigma_c - 1).$$

HU Berlin

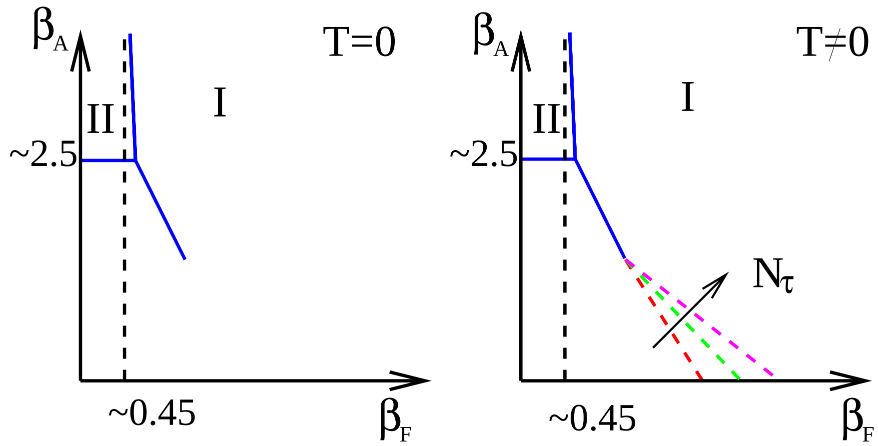
Fundamental-adjoint action with chemical potential

Bhanot-Creutz (1981)

$$S = \beta_A \sum_P \left(1 - \frac{1}{3} Tr_A U_P\right) + \beta_F \sum_P \left(1 - \frac{1}{2} Tr_F U_P\right)$$

$$\frac{1}{g^2} = \frac{1}{4}\beta_F + \frac{2}{3}\beta_A$$

Phase diagram



similar phase diagram shared by $SU(N)$ theories with $N \geq 3$

Halliday-Schwimmer (1981); σ_P auxiliary $\mathbb{Z}_2$ plaquette variable

$$S = \beta_V \sum_P \left(1 - \frac{1}{2}\sigma_P Tr_F U_P\right) + \beta_F \sum_P \left(1 - \frac{1}{2} Tr_F U_P\right)$$

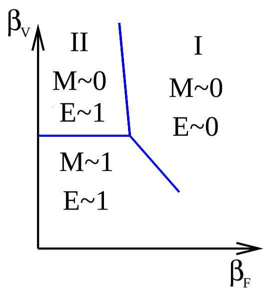
$$M = 1 - \langle \frac{1}{N_c} \sum_c \sigma_c \rangle \qquad \sigma_c = \prod_{P \epsilon \partial c} \sigma_P$$

$$E = 1 - \langle \frac{1}{N_l} \sum_l \sigma_l \rangle \qquad \sigma_l = \prod_{P \epsilon \hat{\partial} l} \sigma_P$$

1st order transitions caused by $\mathbb{Z}_2$ monopoles and vortices

"deconfining" one overshadowed; remove lattice artifacts adding

$$\lambda \sum_c (1 - \sigma_c) \qquad \gamma \sum_l (1 - \sigma_l)$$



Gavai and collaborators (1999):

$\beta_V - \beta_F$ phase diagram with suppression terms

Lines of 2^{nd} order $T \neq 0$ transitions for $\lambda \geq 1$ and $\gamma \geq 5$

$\beta_F = 0, \gamma = 0$ no simmetry breaking/order parameter

Thermodynamic observables very "expensive" (only $N_\tau = 2$)

Behaviour of $\langle L_A \rangle$ through the bulk transition

$\beta < \beta_c$	$\beta > \beta_c$
$\langle L_A \rangle \simeq 0$	$\langle L_A \rangle \nearrow 1$, $\langle L_A \rangle \searrow -\frac{1}{3}$

Linked to twists (de Forcrand and Jahn)! Ergodicity problems...

Adjoint YM at finite T

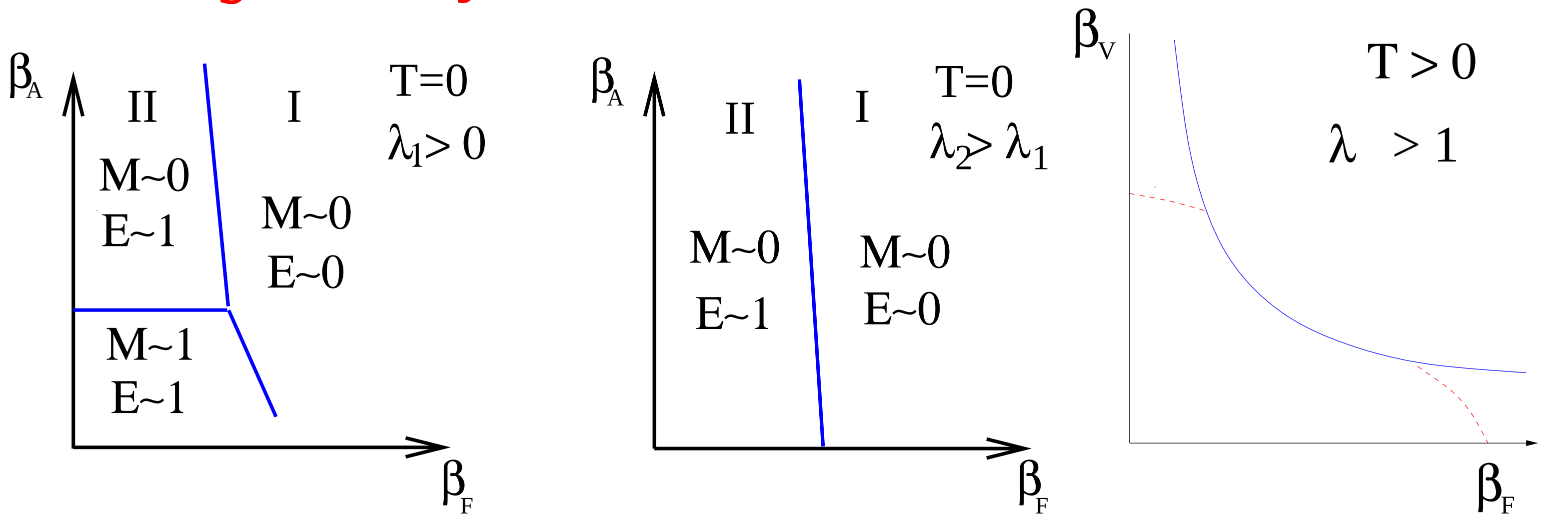
(Barresi, Burgio, Müller-Preussker, PRD, YALELAT 03)

- Adjoint Wilson action with chemical potential.

$$S = \beta_A \sum_P \left(1 - \frac{1}{3}(4Tr_F^2 U_P - 1)\right) + \lambda \sum_c (1 - \sigma_c)$$

$$U_\mu(x) \to -U_\mu(x) \Rightarrow \sigma_c \to \sigma_c;\ \sigma = sgn(Tr_F U_P)$$

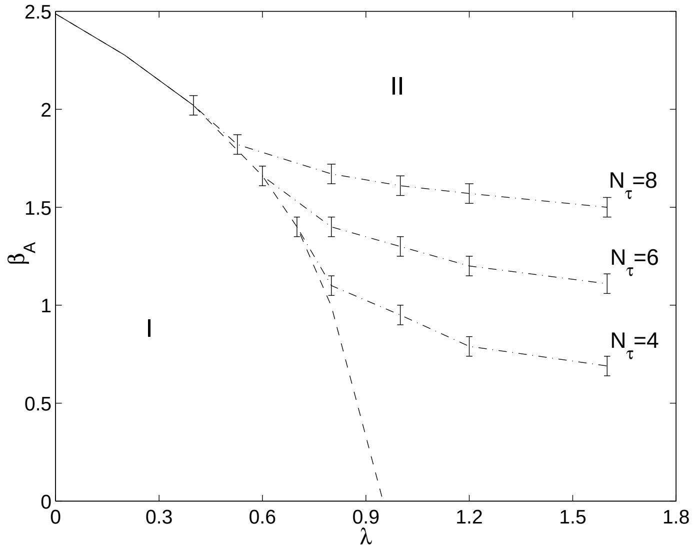
- suppression $\mathbb{Z}_2$ monopoles enhancement of phase **II** ($\lambda > 0$)
- Phase diagram very different from Villain!

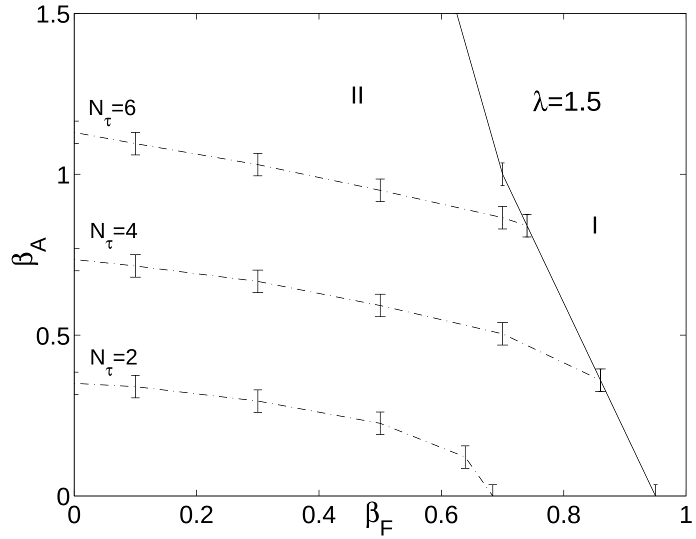


In the $\lambda - \beta_A$ plane:

- Phase I $\rightarrow$ superposition of all Twist sectors
- Phase II $\rightarrow$ Twist sectors "freeze"
- Twist susceptibility scales as 4d-Ising $\lambda > \lambda_c$
- λ_c scales back with the volume! 1st order finite volume effect!
- at the 2nd order bulk standard metropolis still ergodic!
- Relationship between twist and $\langle L_A \rangle$ lost. Always 0 for low β.
- Deep in phase II twist sectors become rigid again. Need for a better algorithm to study full partition function. Order param.?
- Study at finite T and fixed Twist possible by monitoring *spatial* L_F distribution

E.g. in the trivial Twist sector:





L_F distribution changes from Haar measure ($\langle L_A \rangle = 0$) to double peaked at ± 1 ($\langle L_A \rangle \to 1$). Analysis of its moments gives upper bound for $\beta_A^c(N_\tau)$. Drawback: no order param., no critical exps! Non trivial Twist complicates life. At intermediate β $\langle L_A \rangle > 0$. $\langle L_A \rangle \to -\frac{1}{3}$ only for $\beta_A \to \infty$ (L_F peaked at 0). Coexistence of two phases for L_F. Domain walls? Continuum limit?

Pisa (dis)order parameter in SO(3)

$$S_M \;=\; \frac{4}{3}\beta_A \sum_{\mu,\nu,x}\left(1-\frac{Tr_F^2\widetilde{U}_{\mu\nu}(x)}{4}\right)+\lambda\sum_c(1-\sigma_c)$$

$\widetilde{U}_{\mu\nu}(x)$ includes modified plaquette at time t.

By Cluster property:

$$D(T)=\langle\bar{\mu}(\vec{y},t+T)\mu(\vec{y},t)\rangle\,;\;\; D(T)\simeq_{T\to\infty} A\,\mathrm{exp}(-MT)+\langle\mu\rangle^2$$

$\langle\mu\rangle \neq 0 \rightarrow$ spontaneous breaking of $U(1)$ magnetic symmetry (dual superconductivity). In the thermodynamic limit:

$$\langle\mu\rangle\begin{cases}\neq 0 & T<T_c\\ = 0 & T>T_c\end{cases}$$

if deconfinement is associated with the transition from a dual superconductor to normal state.

At $T \sim T_c$ temporal extent aN_τ comparable to correlation length $\rightarrow$ no way of putting a monopole-antimonopole pair at large distance along t

Measure directly $\langle \mu \rangle$, but C^*-PBC needed along t for e^{-S_M} to ensure magnetic charge conservation:

$$U_i(\vec{x}, t = N_\tau) = U_i^*(\vec{x}, t = 0)$$

For adjoint links:

$$U_i(\vec{x}, t = N_\tau) = U_i^C(\vec{x}, t = 0) = (\mathbb{I}_3 + 2T_2^2)U_i(\vec{x}, t = 0)(\mathbb{I}_3 + 2T_2^2)$$

Charge conjugation realized in both representations through rotations of a π angle around the color 2-axis, expressed in the fundamental representation by $i\sigma_2$ and in the adjoint by $\mathbb{I}_3 + 2T_2^2$.

$\langle\mu\rangle$ has huge fluctuations! Way out: measure

$$\rho = \frac{d}{d\beta}\log\langle\mu\rangle = \langle S\rangle_S - \langle S_M\rangle_{S_M}\,, \tag{1}$$

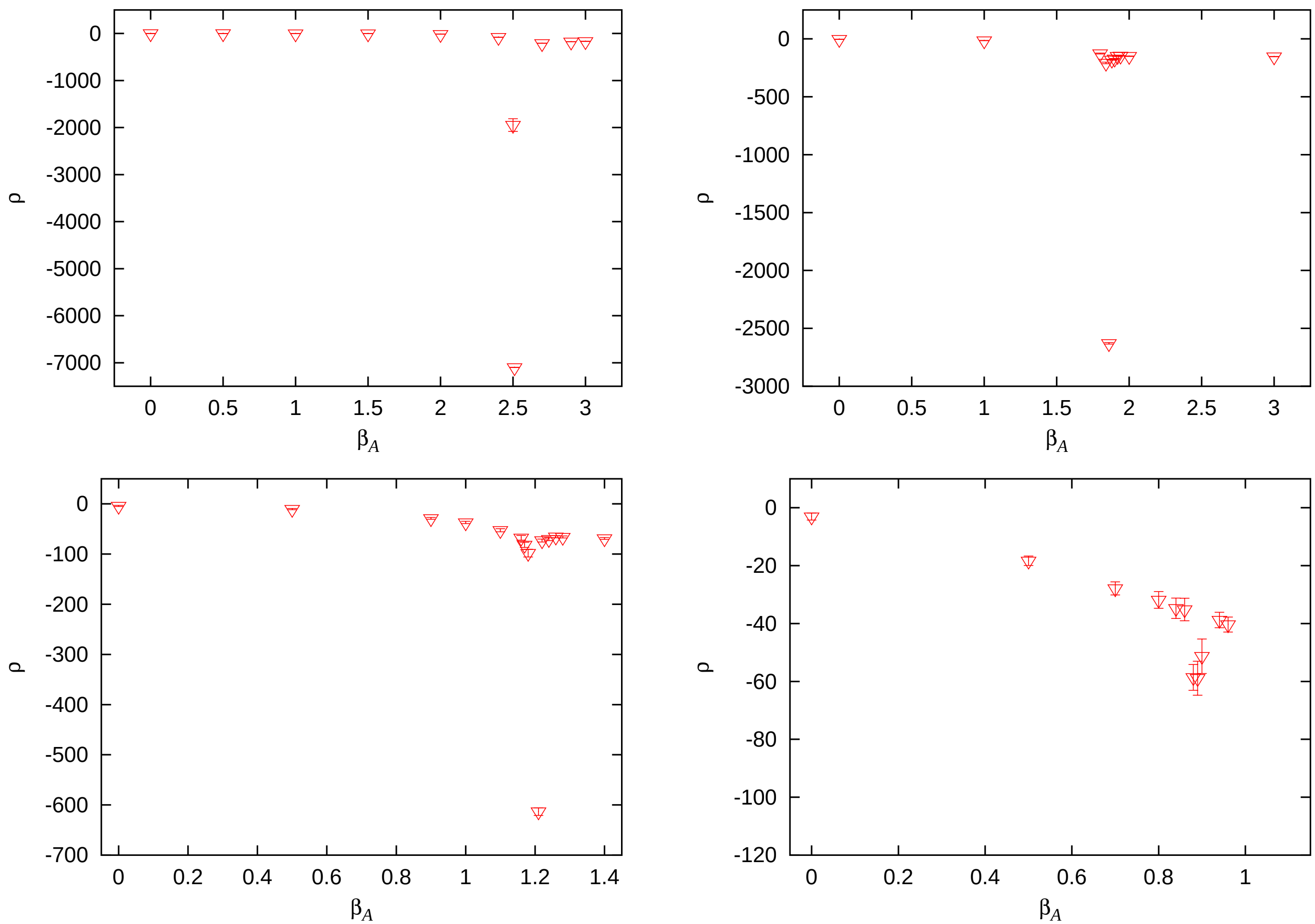
Inverse relation given by

$$\langle\mu\rangle = \exp\left(\int_0^\beta \rho(\beta')d\beta'\right)\,. \tag{2}$$

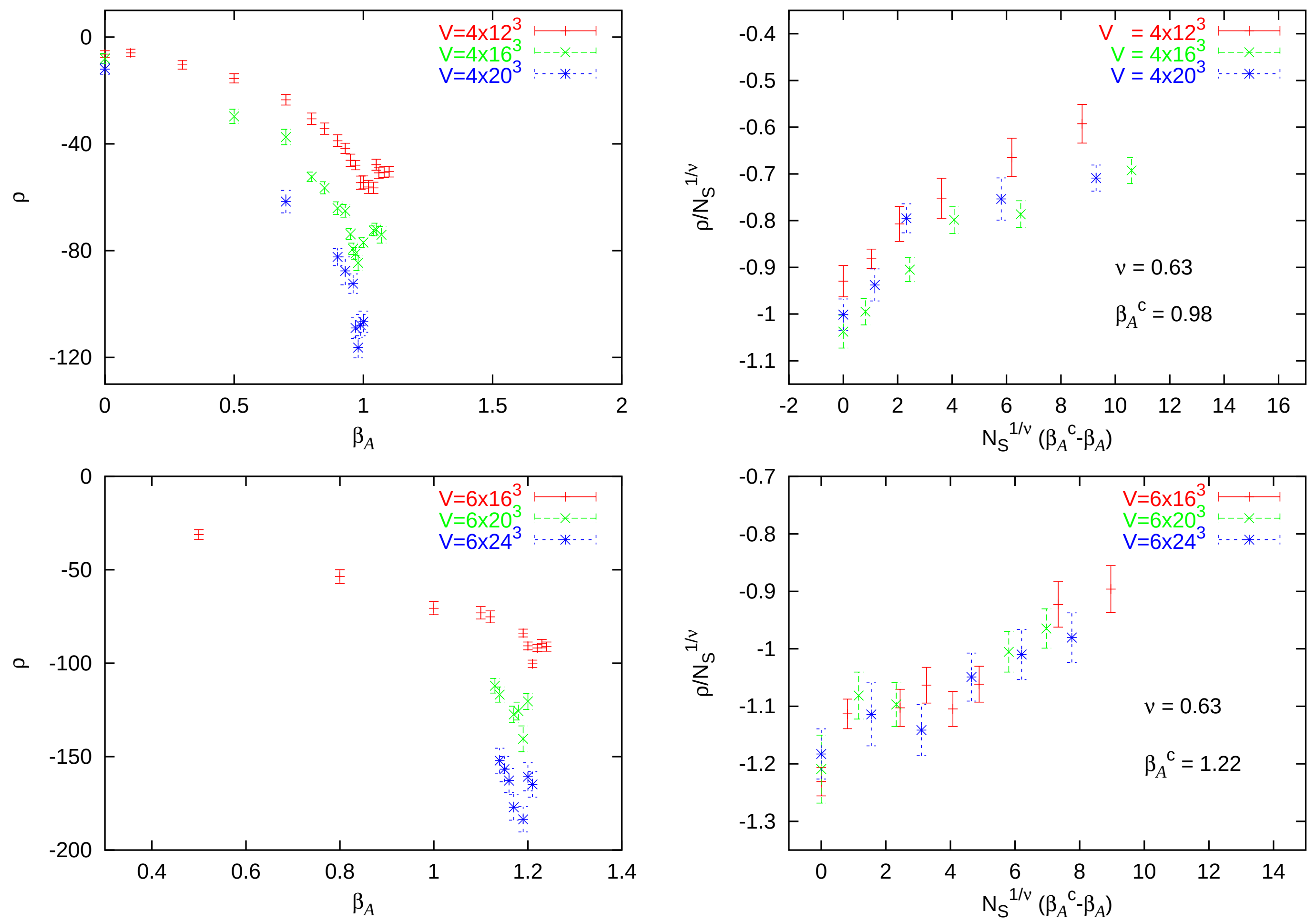
• ρ should stay finite in the thermodynamical limit for $\beta < \beta_c$ if $\langle\mu\rangle \neq 0$ in the confined phase
• a sharp negative peak for ρ diverging in the thermodynamical limit signals a phase transition associated to the breaking of a dual magnetic symmetry.

HU
Berlin

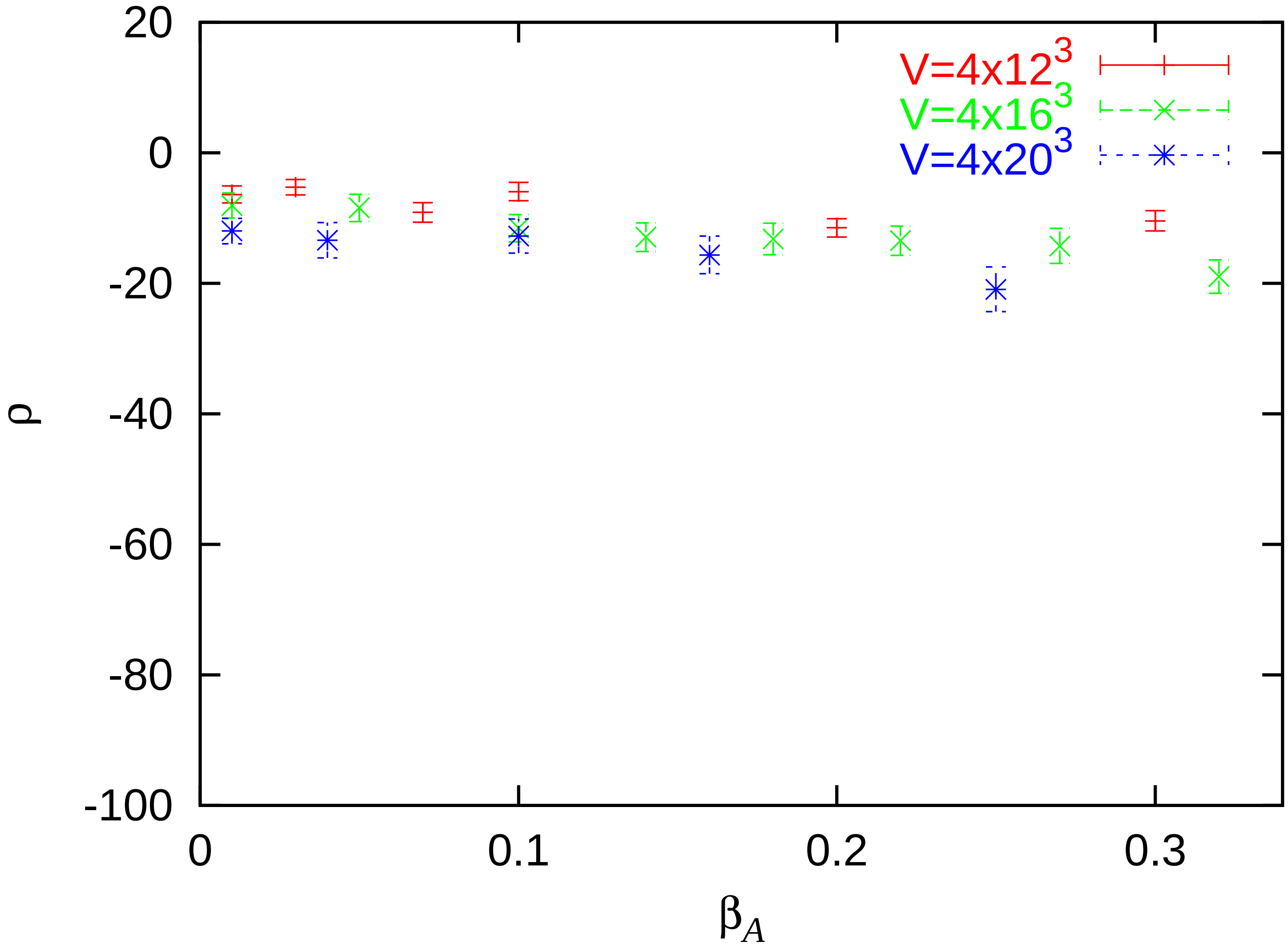
Good order parameter for the bulk transition... (λ = 0 - 0.8)



...but also for the physical one! Good scaling with $N_\tau = 4{,}6$ and FSS in agreement with Ising 3d critical exponents



Morever ρ remains correctly finite at low β...



Outlook & conclusions

- Finite T phase transition for SO(3) for trivial twist.
- Prove condensation of monopoles to see critical exponents!
- Works also for non-trivial twist.
- Sum over all twists for ergodicity. Parallel tempering?
- Pisa parameter only way also with ergodic algorithm?
- Non trivial twist has peculiar properties. Physical meaning? Morita duality and non-commutative YM...