Measurments-induced quantum phase transitions

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Summary

Background: Quantum phase transition

Quantum phase transition

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Model and Measurement Protocol

The model: 1d quantum many-body system

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Growth of entanglement entropy under local projective measurements

M. Coppola, E. Tirrito, D. K. , M. Collura Phys. Rev. B 105, 094303 (2022)

Some speculations about local thermalization of nonequilibrium extended quantum systems

M. Coppola, D. K. Condensed Matter Physics, 2023, vol. 26, No. 1, 13502

Background: Quantum phase transition

Quantum phase transition

Quantum many-body system

$$H(g) = H_0 + gH_1$$

with $[H_0, H_1] \neq 0$. Turning on g the system changes from the ground state Ψ_0 to Ψ_1, crossing a quantum critical point at g_c where

$$\Delta(g) \sim |g - g_c|^{z\nu} \,, \qquad \xi(g) \sim |g - g_c|^{-\nu}$$

Quantum phase transition is a zero temperature phase transition with quantum fluctuations playing the role of thermal fluctuations.

Quantum phase transition

Typical (solvable) example: Ising quantum chain in a transverse field

$$H(g) = -J \sum_n \sigma_n^x \sigma_{n+1}^x - g \sum_n \sigma_n^z$$

Critical point at $g_c = J$ in the universality class of the 2d classical Ising model.

At $g > J$ in the ground state single phase with magnetization $\langle \sigma^x \rangle = 0$
At $g < J$ two degenerate states with finite magnetization $\langle \sigma^x \rangle = \pm m$

Entanglement Entropy

Consider a system defined on the bipartite Hilbert space $\mathfrak{H} = \mathfrak{H}_A \otimes \mathfrak{H}_B$
Suppose the total system is in a pure state $|\Psi\rangle \in \mathfrak{H}$, represented by the density operator $\rho = |\Psi\rangle\langle\Psi|$.
The entanglement entropy (EE) between the subsystem A, and the rest B is defined as

$$S_A = -\operatorname{tr}\left\{\rho_A \ln \rho_A\right\}, \qquad \rho_A = \operatorname{tr}_B\left\{\rho\right\}$$

EE is a measure of quantum entanglement (at $T = 0$)

- For disentangled state such as $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle \implies S_A = 0$.
- For entangled state $|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle \implies S_A \neq 0$.

Entanglement Entropy

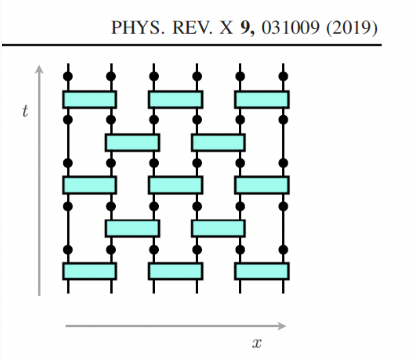
The behavior of quantum entanglement for 1 dimensional:[1] (conformally invariant) systems with $A = \ell$ a linear subsystem of size ℓ

- At the critical point
  Logarithmic Law : $S(\ell) \sim \ln \ell$
- Away from the critical point, for $\xi \ll \ell$,
  Area Law : $S(\ell) \sim \ln \xi$ independent on ℓ
- After a unitary quench from an initial product state
  $|\Psi\rangle = |\phi\rangle \otimes |\phi\rangle \otimes |\phi\rangle \cdots \otimes |\phi\rangle$
  Volume law $S \sim t$ with linear growth in time

[1] See Pasquale Calabrese et al 2009 J. Phys. A: Math. Theor. 42 500301 for a review

Measurement-Induced phase transition

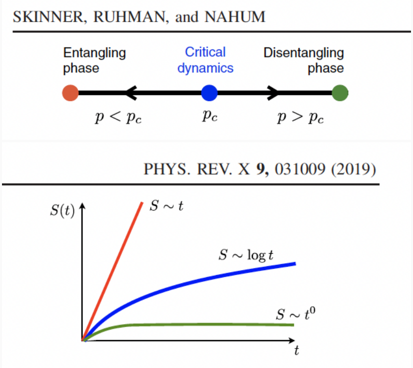
1+1 dimensional Quantum circuit + random projective measurements[2]



[2]from Brian Skinner, Jonathan Ruhman, and Adam Nahum Phys. Rev. X 9, 031009

Measurement-Induced phase transition

Measurement-Induced phase transition in the dynamics of Entanglement[3]



[3] Y. Li, Phys. Rev. B 98, 205136 (2018) ; B. Skinne et al Phys. Rev. X 9, 031009 (2019) ; A. Chan et al Phys. Rev. B 99, 224307 (2019) ; O. Alberton et al Phys Rev. Lett. 126, 170602 (2021) ; T. Müller et al Phys. Rev. Lett. 128, 010605 (2022) ; T. Minato et al Phys. Rev. Lett. 128, 010603 (2022)

Model and Measurement Protocol

The model: 1d quantum many-body system

We consider a 1-d extended quantum system with local interactions: simple hopping fermions on a 1-d lattice[4]

$$H = -\frac{1}{2}\sum_n \left(c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n\right) = \sum_q \epsilon_q \eta_q^\dagger \eta_q \ .$$

- The Hamiltonian H generates the unitary dynamics $U = e^{-iHt}$.
- Gaussian states (Wick theorem applies) are fully characterized by the two-point function $C_{ij} = \langle c_i^\dagger c_j \rangle$.
- If $\rho(0)$ is gaussian, then $\rho(t) = e^{iHt}\rho(0)e^{-iHt}$ remains gaussian and fully characterized by $C_{ij}(t) = \langle c_i^\dagger(t) c_j(t)\rangle$.
- For Gaussian states the EE is obtained from the eigenvalues $\{\lambda_k\}$ of the $\ell \times \ell$ reduced correlation matrix C_ℓ:

$$S(\ell) = -\sum_k \lambda_k \ln \lambda_k + (1-\lambda_k)\ln(1-\lambda_k) \ .$$

[4] Under Jordan-Wigner transformation quivalent to XX spin 1/2 quantum chain

$$H_{XX} = \sum_n \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right)$$

Non-unitary dynamics under local projective measurements

The unitary dynamics is perturbed by random projective measurements of a local observable

$$Q_\Omega = \sum_p q_p P_\Omega^{(p)} \,, \qquad \sum_p P_\Omega^{(p)} = \mathbb{I}_\Omega \,.$$

Just after a measurement of the local observable Q_Ω with an outcome q_k the state is projected according to the Born rule

$$|\Psi\rangle \longrightarrow \frac{P_\Omega^k |\Psi\rangle}{\langle\Psi| P_\Omega^k |\Psi\rangle} \,.$$

We consider local occupation numbers $\hat{n}_j = c_j^\dagger c_j$.

Non-unitary dynamics under local projective measurements

Within a time step dt, a site k is choosen at random and a measurement of $\hat{n}_k$ is performed with a probability dt/τ where
$1/\tau$ is the rate of measurements.

- If $\hat{n}_k$ is measured then the state is projected according to the outcome:

$$\text{if } n_k = 1 \qquad |\Psi(t)\rangle \longrightarrow \frac{\hat{n}_k|\Psi(t)\rangle}{\langle\Psi(t)|\hat{n}_k|\Psi(t)\rangle}$$

$$\text{if } n_k = 0 \qquad |\Psi(t)\rangle \longrightarrow \frac{(1-\hat{n}_k)|\Psi(t)\rangle}{\langle\Psi(t)|(1-\hat{n}_k)|\Psi(t)\rangle}$$

- If there is no measurement the evolution is unitary with $U = e^{-iHdt}$.

Non-unitary dynamics under local projective measurements

- At time $t = 0$ we start with the Néel state (Gaussian state) :

$$|NS\rangle = |0\rangle \otimes |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes \cdots \otimes |0\rangle \otimes |1\rangle \dots$$

- Under the measurement of n_k the state remains gaussian
- The projection rules for the two-point functions

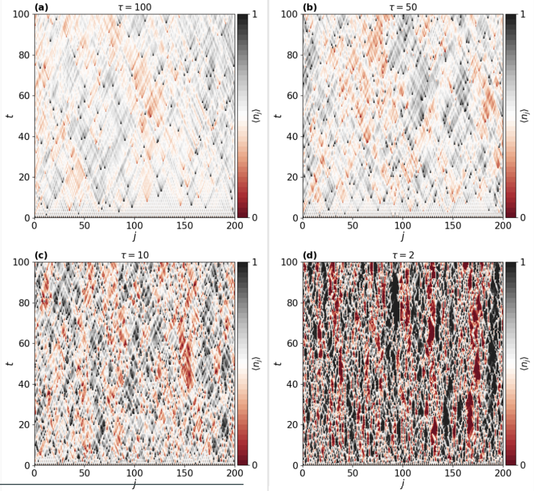
$$\text{if } n_k = 1 \qquad C_{ij}(t) \longrightarrow C_{ij}(t) + \delta_{ik}\delta_{jk} - \frac{C_{ik}(t)C_{kj}(t)}{C_{kk}(t)}$$

$$\text{if } n_k = 0 \qquad C_{ij}(t) \longrightarrow C_{ij}(t) - \delta_{ik}\delta_{jk} + \frac{(\delta_{ik} - C_{ik}(t))(\delta_{jk} - C_{kj}(t))}{1 - C_{kk}(t)}$$

Quantum trajectories

Quantum trajectories

Particle density trajectory after a quench from a Néel state[5]



[5]M. Coppola, E. Tirrito, D. K. , M. Collura Phys. Rev. B 105, 094303 (2022)

Entanglement Entropy Dynamics

We generate many quantum trajectories $\mathcal{T}_i$ which are characterized at a given time t by the density operator $\rho^i(t)$.

Average EE over the ensemble of random quantum trajectories

$$\overline{S}(t,\ell) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} S_{\rho^i(t)}(\ell) \, .$$

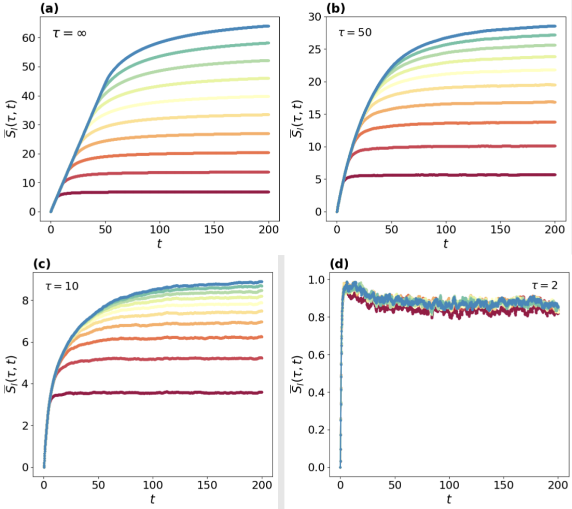
Some observations[6]

- If $\overline{\rho} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \rho^i$, the associated entropy $S_{\overline{\rho}} \neq \overline{S}$.
- In the continuous limit ($dt \to 0$), $\overline{\rho}$ is governed by a Lindblad evolution:

$$\frac{d\overline{\rho}}{dt} = -i[H,\overline{\rho}] - \frac{1}{\tau} \sum_j [\hat{n}_j, [\hat{n}_j, \overline{\rho}]]$$

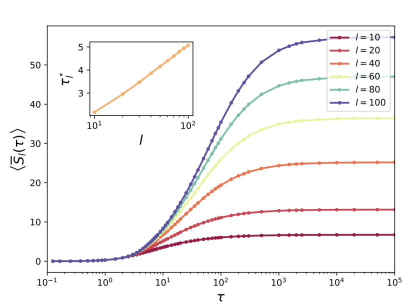
[6] X. Cao, A. Tilloy, and A. De Luca, SciPost Phys. 7, 024 (2019).

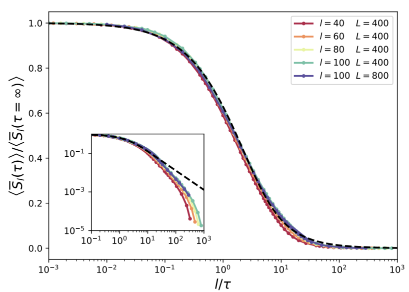
Entanglement Entropy Dynamics



- For $\tau = \infty$ the entropy grows linearly: volume law
- Decreasing τ, logarithmic growth $\sim \ln t$ which eventually saturates at large time $t \gg \ell$.
- For very small τ the EE shows a rapid saturation to a plateau which is independent on the subsystem size ℓ.

Stationary Entanglement





At small τ (very high measurement rates $1/\tau$) the steady EE is independent on ℓ. For $\tau > \tau^* \sim \ln \ell$ the steady EE depends on ℓ.

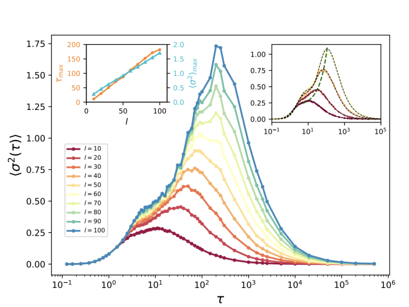
Measurements introduce a typical correlation length scale $\xi(\tau) \sim e^{\alpha\tau}$.

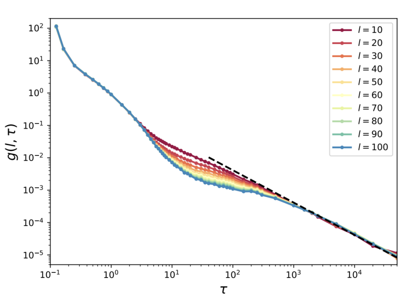
For $\ell \gg \xi(\tau)$ only sites close to the border of the subsystem are correlated with the rest of the system $\implies$ the EE is ℓ-independent Area Law.

As $\xi(\tau)$ gets larger and larger, more and more sites are involved in generating correlation with the rest of the chain. For $\xi(\tau) > \ell$ the entire subsystem contributes to the EE and this essentially results in a volume-law behaviour.

Analytical arguments quasi-particles picture

Fluctuations of the Stationary Entanglement





- Fluctuations of the stationary EE as a function of the parameter τ.
- The absolute maximum point $\langle \sigma^2 \rangle$ and its position τ_{max} increase linearly with the subsystem size ℓ.
- Double-peak structure of the variance of the EE.
- Good rescaling with two different regimes.

Summary

Summary

- Initial logarithmic time growth of the EE
- The stationary EE manifests a volume - area law transition
- The transition toward the area law occurs at infinitesimal measurement rate for $\ell \to \infty$
- Sensitive to the unravelling