Critical dynamics of the $\pm J$ Ising model

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Outline

Introduction

Hamiltonian for the $\pm J$ *Ising* model is

$$H = -\sum_{\langle ij \rangle} J_{ij} S_i S_j \ ,$$

where $S_i = \pm 1$ are Ising spins, placed at each site i of the lattice, and the exchange couplings J_{ij} are quenched random variables with

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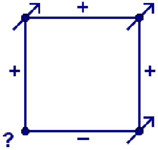

Frustration/disorder in a magnetic system:

- low-temperature ordered phase is destroyed?
- critical properties are changed?
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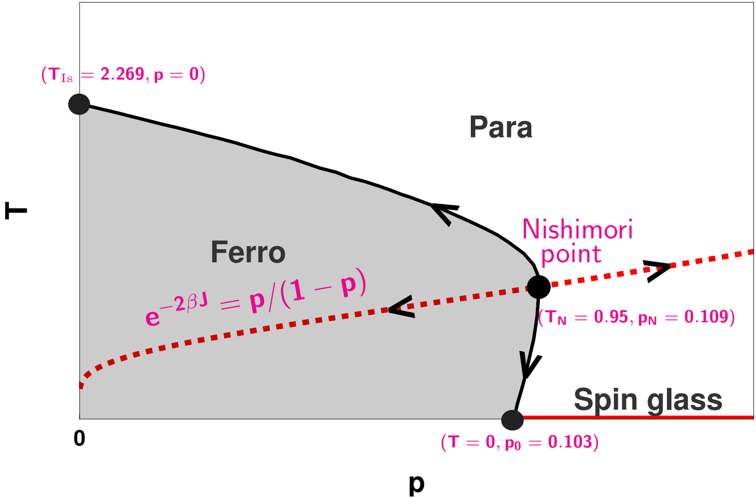
Two-dimensional $\pm J$ Ising model:

- As disorder increases, T_c decreases[1].
- With disorder, new fixed points emerge[2].
- spin glass phase[3]: Yes, but at zero T and large p ($p > 0.103$).

[1] V. S. Dotsenko and V. S. Dotsenko. J. Phys. C: Solid State Phys. **15**, 495, (1982).
[2] W. L. McMillan, Phys. Rev. B **29**, 4026 (1984).
[2] M. Picco, A. Honecker, and P. Pujol. J. Stat. Mech., P09006, (2006).
[2] M. Hasenbusch, F. P. Toldin, A. Pelissetto, and E. Vicari. Phys. Rev. E **77**, 051115 (2008).
[3] C. Amoruso and A. K. Hartmann, Phys. Rev. B **70**, 134425 (2004).



Phase diagram of the $2d \pm J$ Ising model

Key concerns

3 non-trivial fixed points: 3 different universality classes

— unique classification of universality classes
(different conformal field theories)

— universality of dynamical critical exponents in same class?

Y. Ozeki, S. Yotsuyanagi, and N. Ito, J. Phys. Soc. Japan **81**, 074602 (2012).

— critical percolation for quenches below/at T_c from paramagnetic state?

T. Blanchard, F. Corberi, L. F. Cugliandolo, and M. Picco, EPL **106** 66001 (2014).

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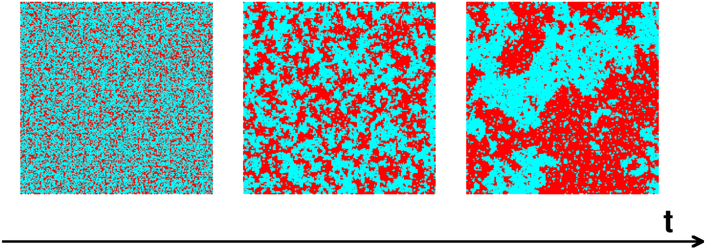
Critical dynamics of the model via (single-flip) Monte Carlo simulations

Critical dynamics after a quench from $T = \infty$:



- Correlation length, $\xi(\mathbf{t}) \sim \mathbf{t}^{1/z_c}$.
- Dynamical scaling:

$$C(r,t) = C_{\rm eq}(r) F\left(\frac{r}{\xi(t)}\right), \quad \text{with} \quad C_{\rm eq}(r) \sim \frac{1}{r^\eta} \; .$$

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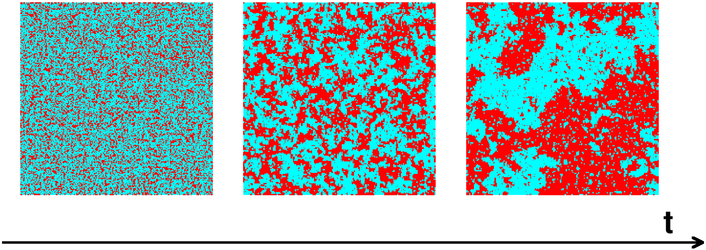

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- We calculated the values of exponent z_c from the critical quenches to phase boundary.

Calculation of exponent z_c :

- $z_c = \lim_{t\to\infty} z_{\mathrm{eff}}(t)$,

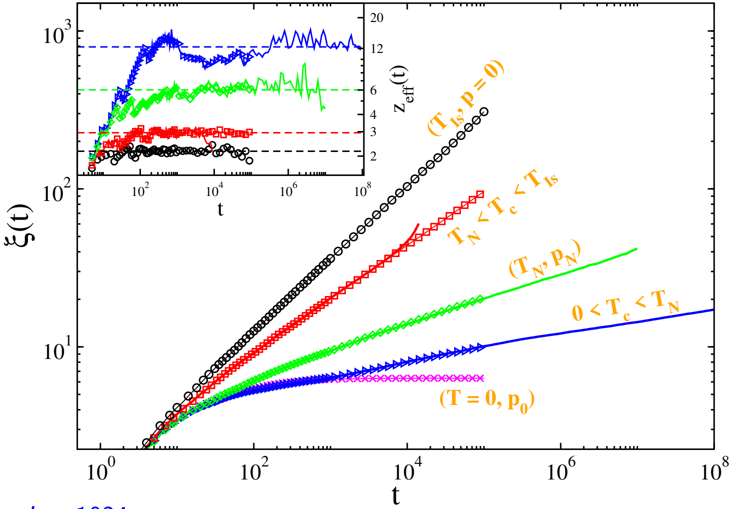
$$\frac{1}{z_{\mathrm{eff}}(t)} = \frac{d \ln \xi(t)}{d \ln t} \; .$$

- Short-time-critical dynamics*: for the ordered initial condition, magnetization density,

$$\lim_{L\to\infty} M(t) \sim t^{-\beta/\nu z_c} \; .$$

* H. Janssen, B. Schaub, and B. Schmittmann, Z. Phys. B: Cond. Matt. **73**, 539 (1989); B. Zheng, Int. J. Mod. Phys. B **12**, 1419 (1998).

Simulation results (quench from $T = \infty$ to PF line):



symbols: $L = 1024$

lines: $L = 128$

Correlation length $\xi(t)$ vs time t.

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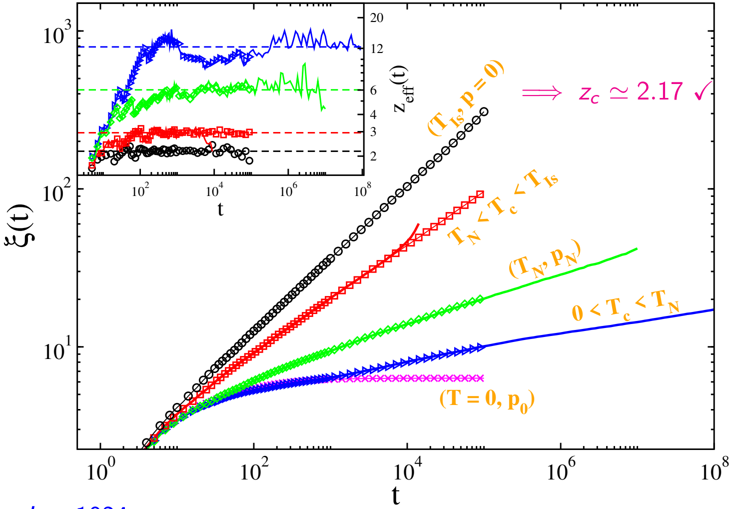

$\Longrightarrow$ $z_c \simeq 2.17$ ✓

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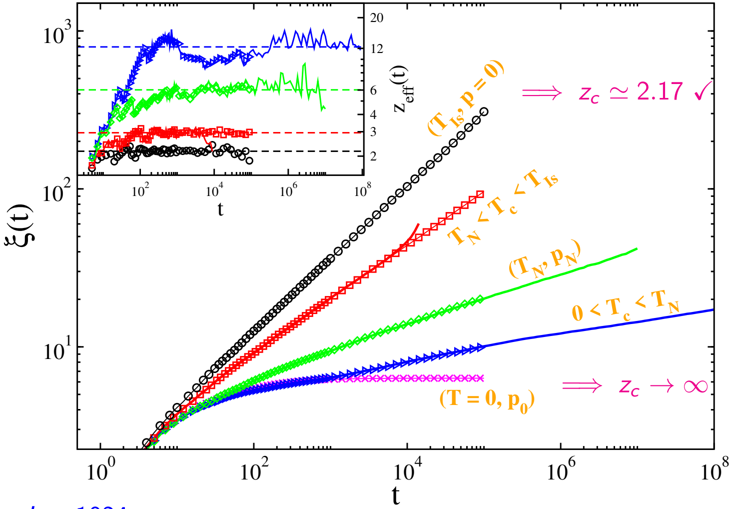

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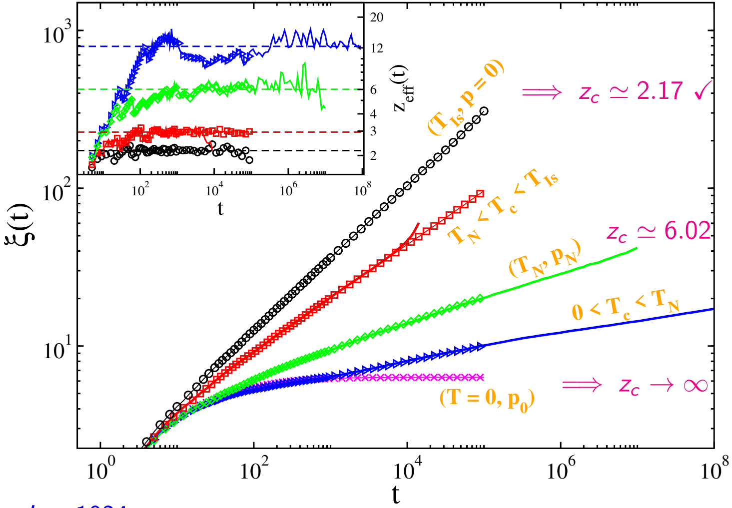

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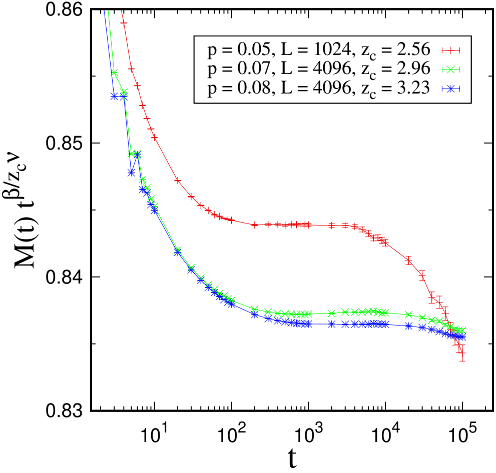
Quench from ordered initial state: $M(t) \sim t^{-\beta/z_c\nu}$ $(T_c > T_N)$



M(t)$t^{\beta/z_c\nu}$ vs t in log-linear scale.

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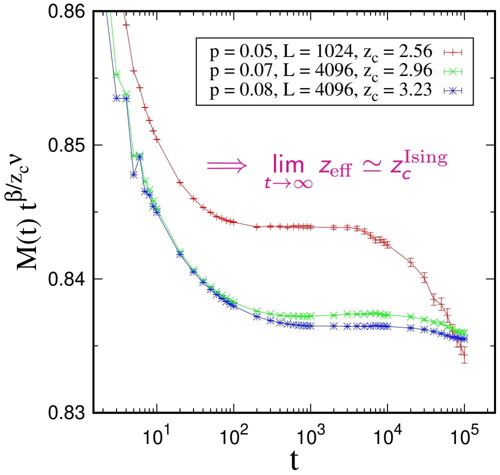
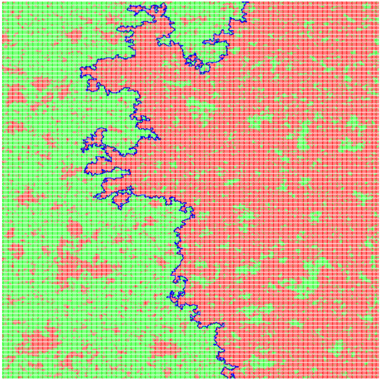


M(t)t$^{\beta/z_c\nu}$ vs t in log-linear scale.

Classification of universality: geometry of interfaces

- Stochastic Loewner evolution (SLE_κ) provides a unique way to relate a $2d$ critical system to a CFT.
- κ is a diffusion parameter associated to Brownian drive in SLE_κ.
- $\forall \kappa > 0$, there is a distinct universality class.



(An inteface at Ising critical point, in the continuum limit, is described by $\mathrm{SLE}_{\kappa=3}$)

S. Smirnov, Ann. Math. **172**, 1435 (2010).

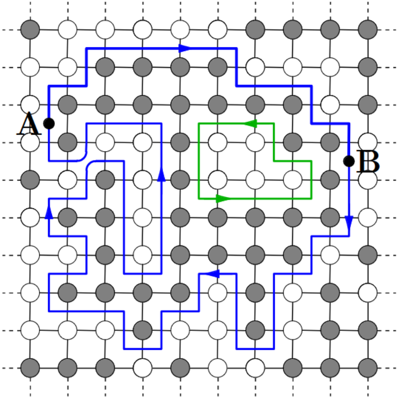
Calculation of parameter κ :

Variance of winding angle,

$$\langle\theta^2(r)\rangle^* = a + \frac{4\kappa}{8+\kappa}\ln\left(\frac{r}{r_0}\right) .$$

During nonequilibrium dynamics,

$$\langle\theta^2(r,t)\rangle^\dagger \sim \frac{4\kappa}{8+\kappa}\ln\left(\frac{r}{[\xi(t)]^{d_l}}\right) .$$



*B. Duplantier and H. Saleur, Phys. Rev. Lett. **60**, 2343 (1988).

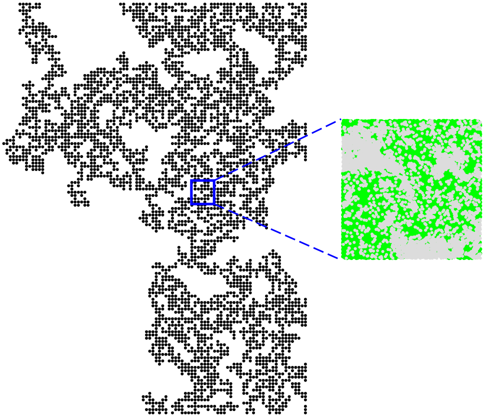
*O. Schramm, Isr. J. Math. **118**, 221-288 (2000)

†T. Blanchard, L. F. Cugliandolo, and M. Picco, J. Stat. Mech. **2012**, P05026 (2012).

Quench from $T = \infty$ to $T_{\rm Is}$:

$$\kappa \sim \begin{cases} 3, & r < \xi(t) \quad \text{(critical Ising class)} \\ 6, & r > \xi(t) \quad \text{(critical percolation)} \end{cases}$$

[Papers by Blanchard and co-workers]



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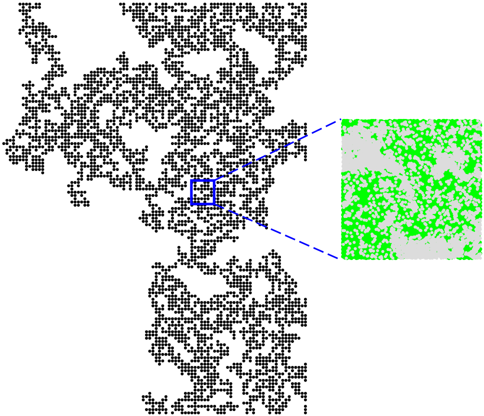

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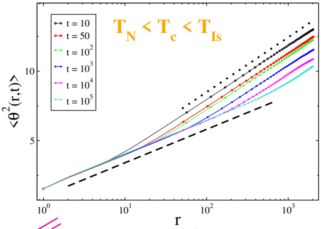
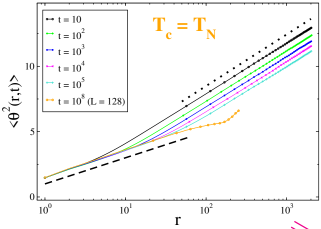
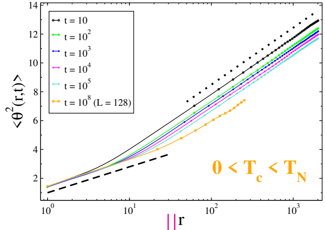


$\kappa^{(s)} \simeq 2.97$



$\kappa^{(s)} \simeq 2.2$



$\kappa^{(s)} \simeq 1.9?$

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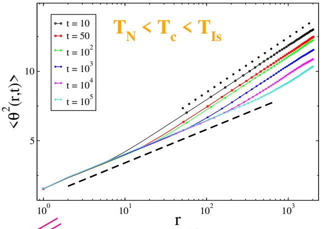

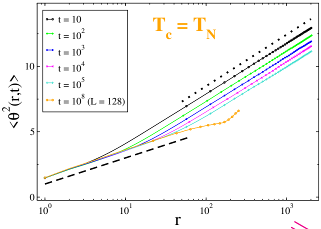

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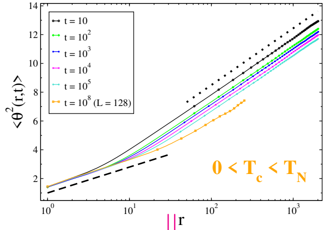
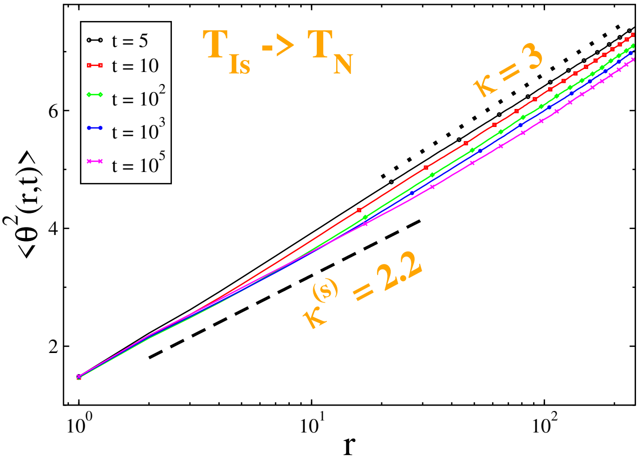


$\kappa \simeq 6; \quad r > \xi(t)$

for all $T_c \in$ PF line

$\kappa^{(s)} \simeq 1.9?$

Quench from T_{Is} to T_{N}:



Conclusion

Universality class	$\mathbf{z_c}^{\dagger}$	κ for $r < \xi(t)$	κ for $r > \xi(t)$
Ising	2.17	3	6
Nishimori	6.02	2.2	6
Strong-disorder	∞	1.9	6

† asymptotic value

R. Agrawal, L. F. Cugliandolo, L. Faoro, L. B. Ioffe, and M. Picco. (Accepted in PRE)

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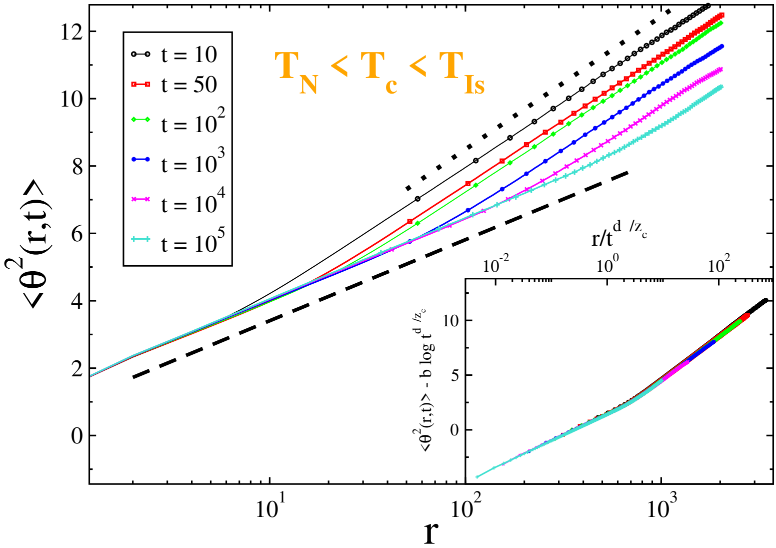
Universality of dynamical exponents in Nishimori class**

$\mathbf{z_c}$	$\lambda_\mathbf{c}$	$\theta_\mathbf{c}$
6.02	1.32	0.08

** In preparation

Thank You!



t = 10
t = 50
t = 10^2
t = 10^3
t = 10^4
t = 10^5
T_N < T_c < T_Is
<θ^2(r,t)>
r
r/t^d /z_c
<θ^2(r,t)> - b log t^d /z_c