Dynamical scaling of the 2D pure and dilute Baxter-Wu model

Clusters in a triangular lattice with three-spin nearest-neighbour interactions

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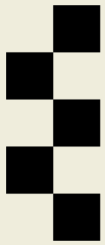

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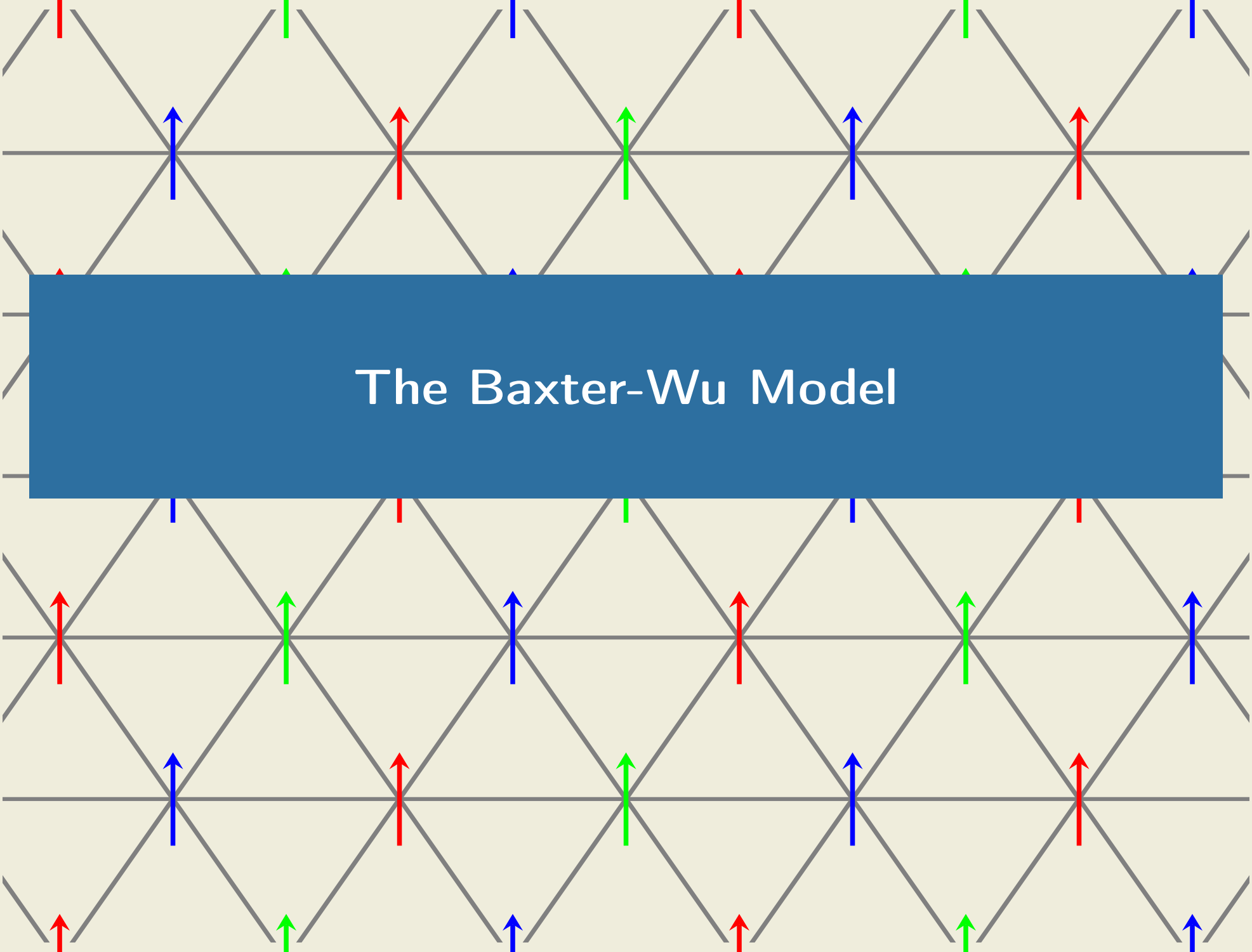
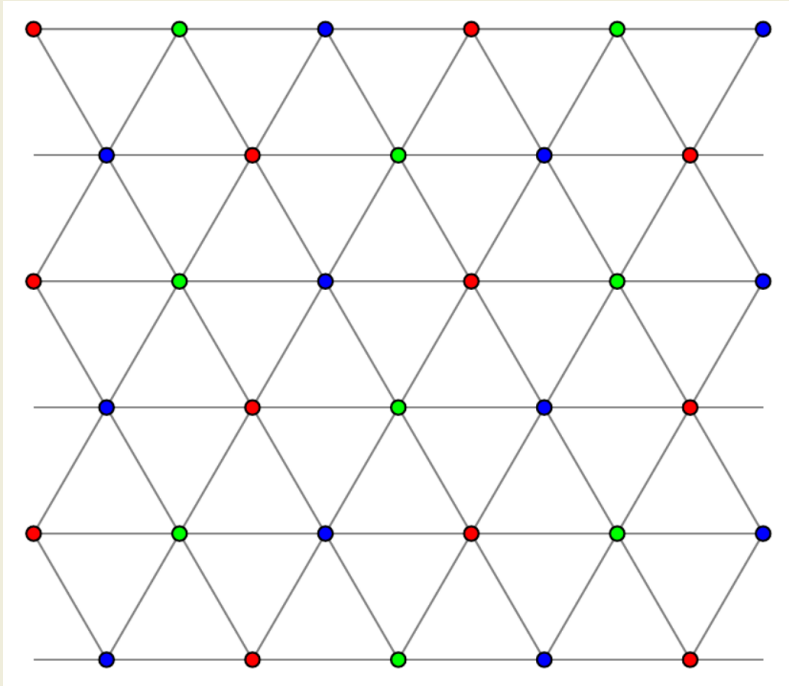
of Essex


The Baxter-Wu Model

The Spin-1/2 Baxter-Wu Model

$$H = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k$$

- 2D triangular lattice.
- Three-spin interactions.
- Three sublattices A, B, C.
- No plane reflection symmetry.
- 4-fold degenerate GS.
- $m = \sqrt{(m_{\mathrm{A}}^2 + m_{\mathrm{B}}^2 + m_{\mathrm{C}}^2)/3}.$
- 4-state Potts universality.
- $\alpha = 2/3$, $\nu = 2/3$, $\gamma = 7/6$, $\eta = 1/4$,[†] $\omega = 2$.[‡]



[†] R. J. Baxter and F. Y. Wu, Phys. Rev. Lett. 31, 1294 (1973); Aust. J. Phys. 27, 357 (1974); R. J. Baxter, ibid. 27, 369 (1974).

[‡] F. C. Alcaraz and J. C. Xavier, J. Phys. A 30, L203 (1997); A 32, 2041 (1999).

The Spin-1 Baxter-Wu Model in a Crystal Field

$$H = \underbrace{-J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k}_{E_J} + \underbrace{\Delta \sum_{i=1}^{N} \sigma_i^2}_{E_\Delta}$$

- $\sigma \in \{-1, 0, +1\}$.
- Anisotropy term (dilution).
- $\Delta \rightarrow$ density of $\sigma = 0$.
- 1^{st}, 2^{nd} order & multicriticality.

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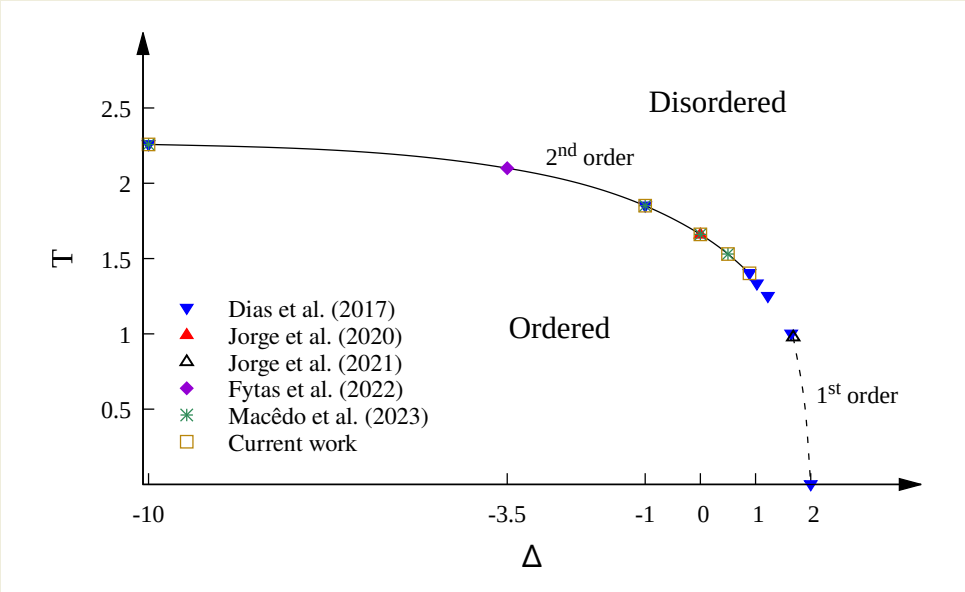
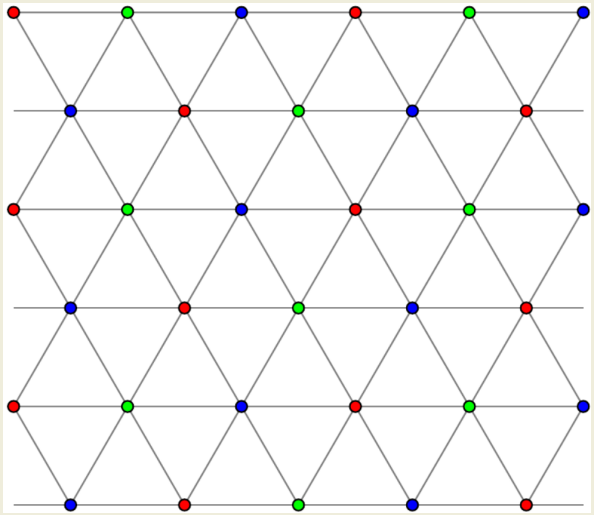


Simulation Methods

Cluster Algorithm for spin-1/2

$$E = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k$$



M.A. Novotny & H.G. Evertz (1993):

- "Freeze" a sublattice.
- Grow cluster on 2 sublattices.
- 3^{rd} sublattice $\rightarrow$ effective interactions.
- $J' \in \{-2, 0, +2\}$.
- A cluster has either ferro- or antiferro-magnetic bonds, **only**.
- No frustration.

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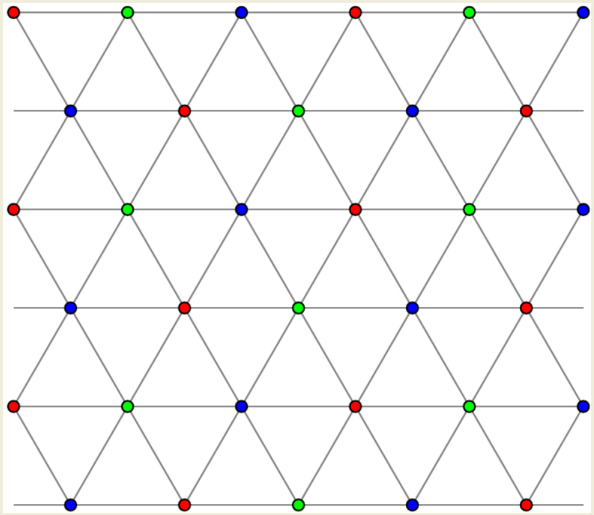

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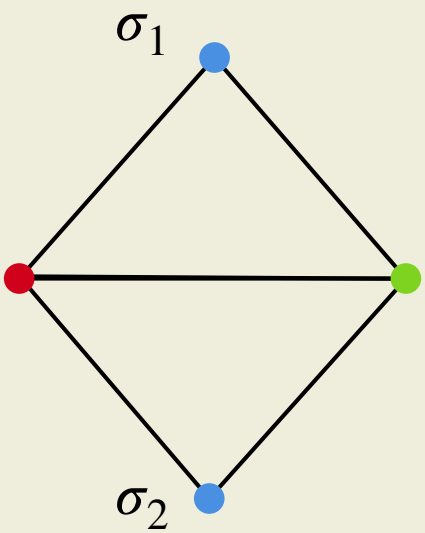

$$J' = J(\sigma_1 + \sigma_2)$$

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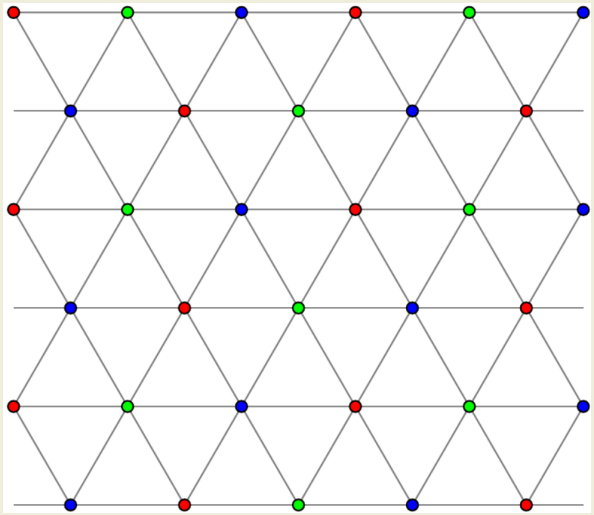

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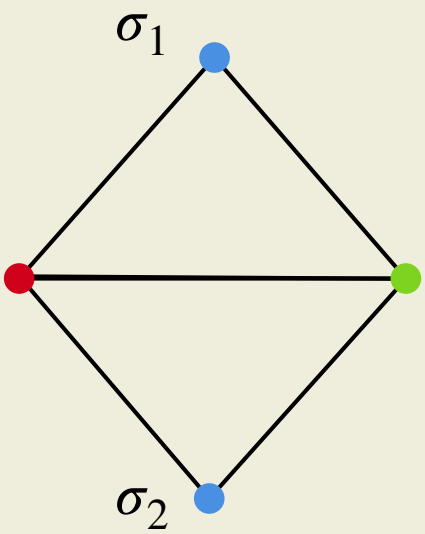

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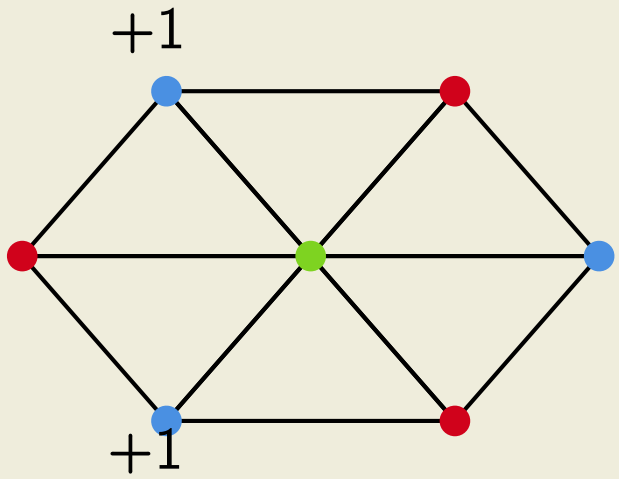
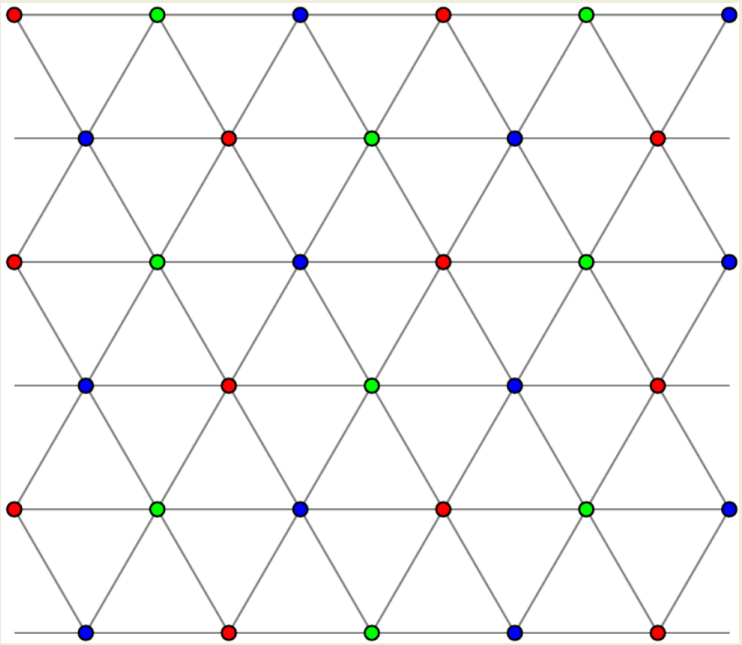
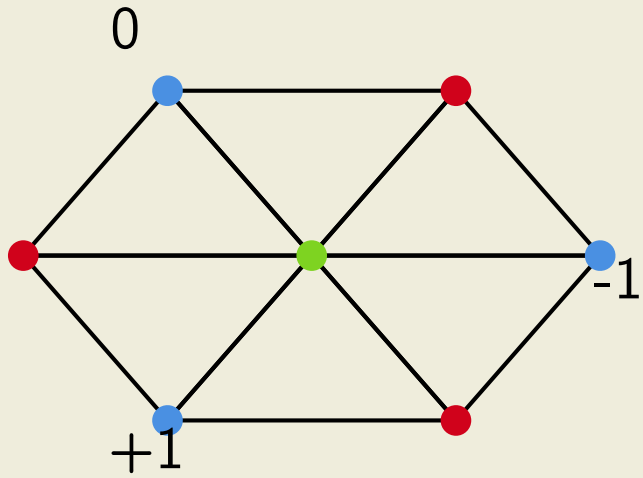


Hybrid Algorithm for spin-1

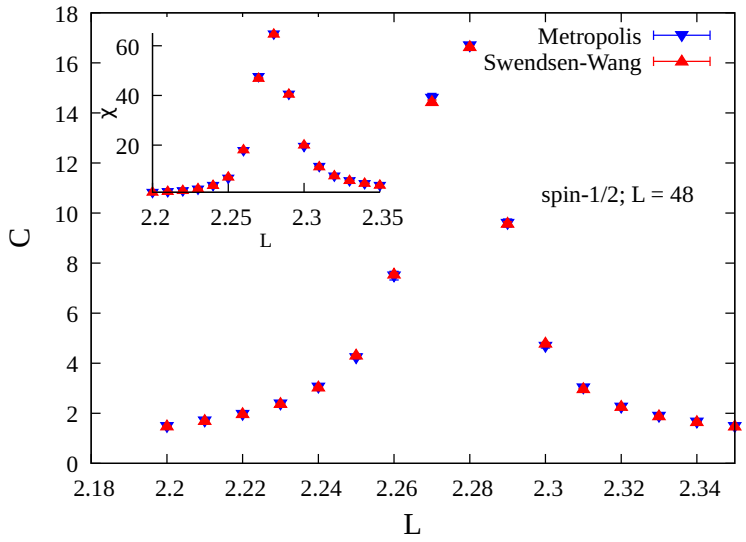
$$E = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k + \Delta \sum_i \sigma_i^2$$

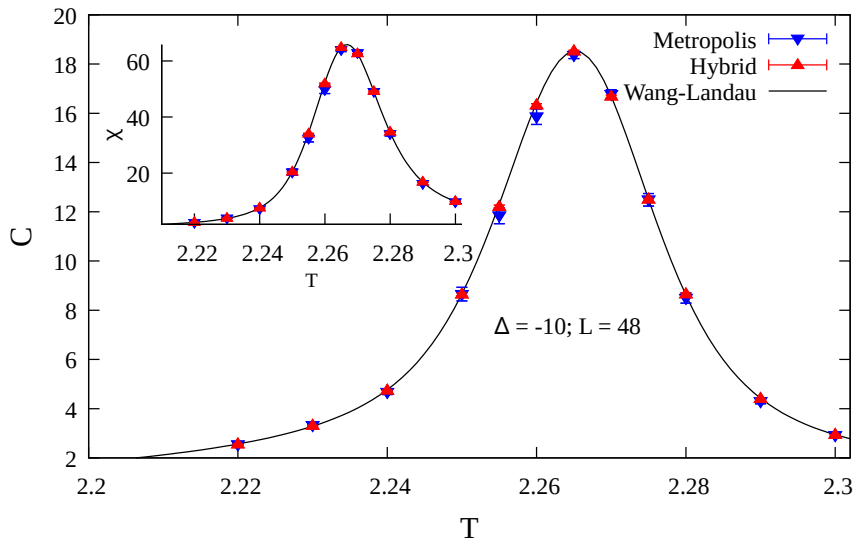
- Cluster ignores zeros.
- $\oplus$ single-spin-flip algorithm.
- One SW one sweep HB.
- $J' \in \{-2, -1, 0, +1, +2\}$.
- A cluster can have mixed interactions.

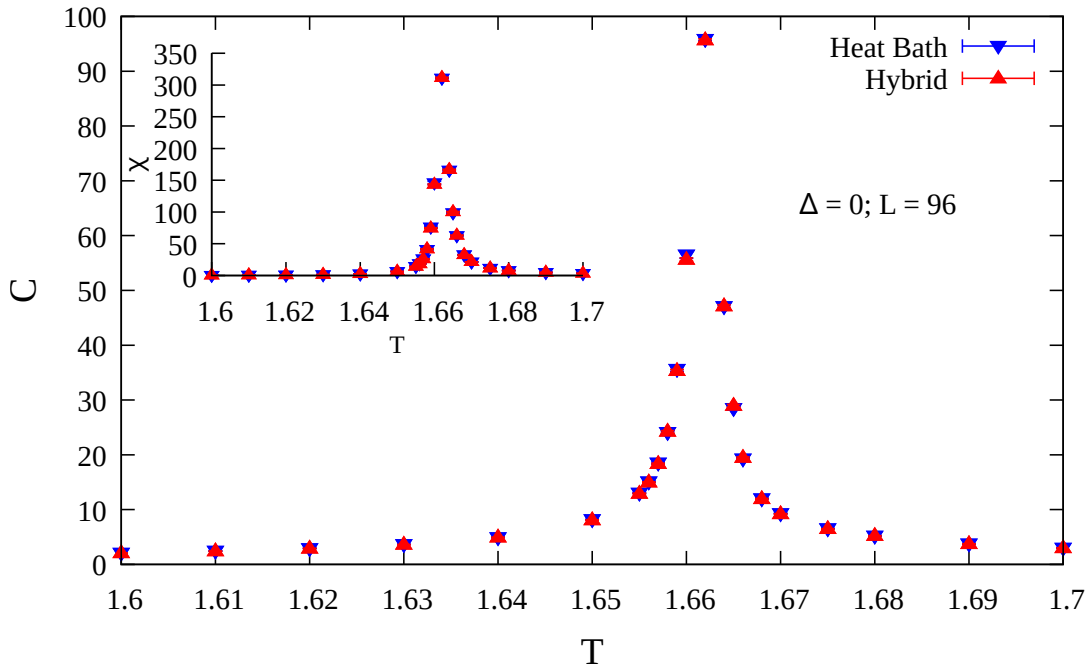


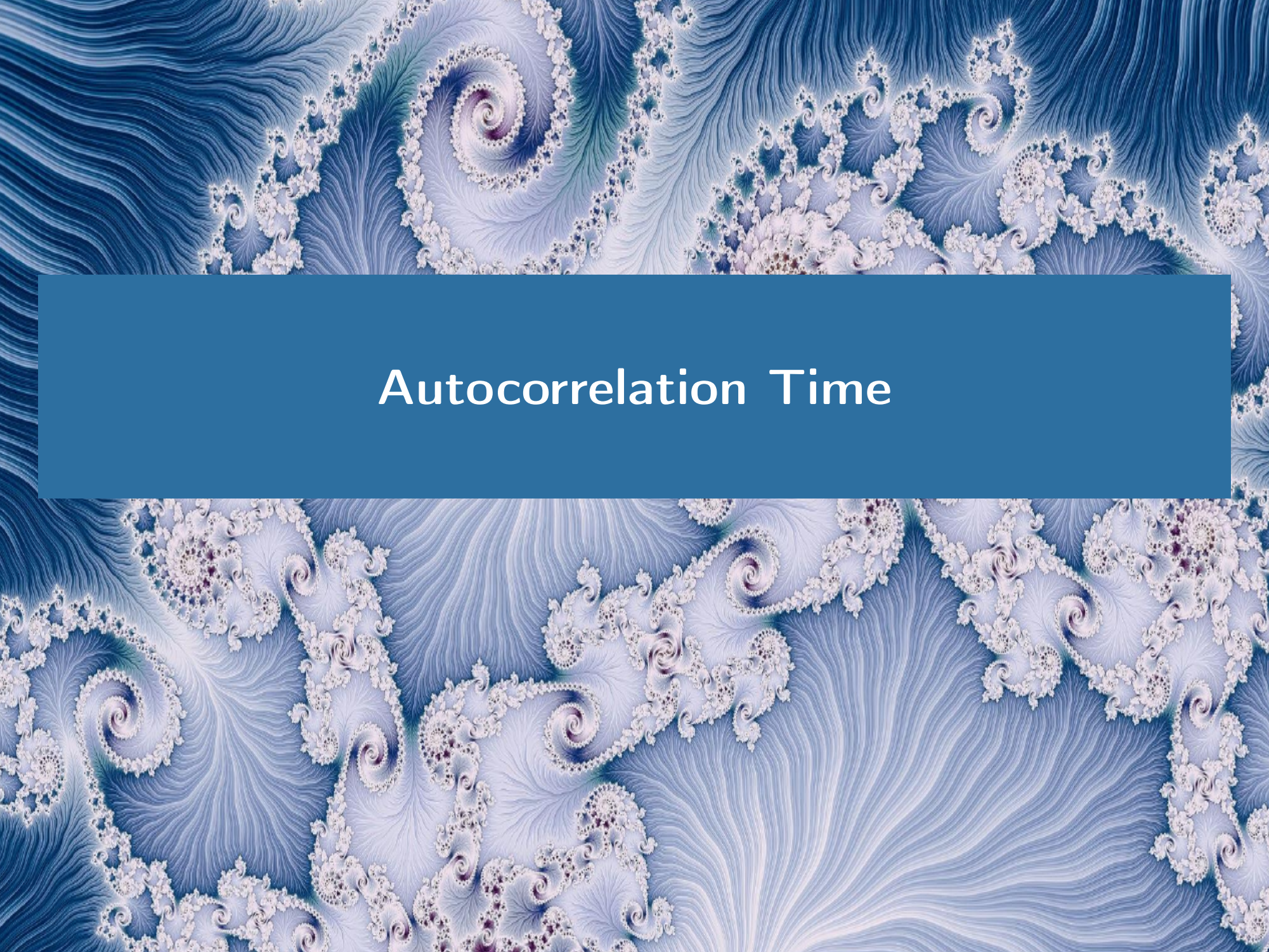


Does it work?


Metropolis
Swendsen-Wang
spin-1/2; L = 48
C
L
χ


Metropolis
Hybrid
Wang-Landau
Δ = -10; L = 48
C
T
χ


Heat Bath
Hybrid
Δ = 0; L = 96
C
T
χ


Autocorrelation Time

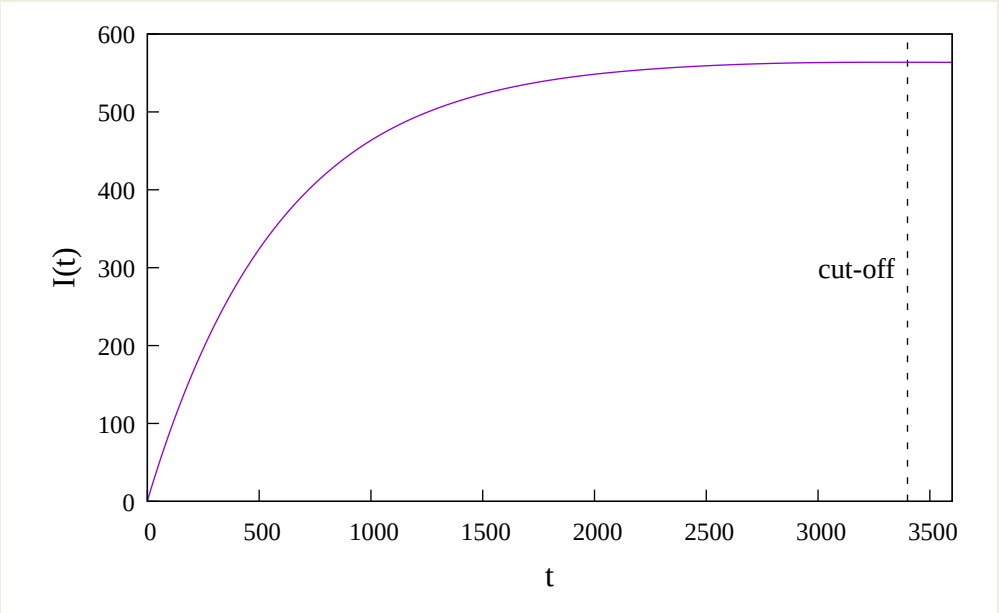
Integrated Autocorrelation Time

$$I(t) = \frac{1}{2} + \sum_{k=1}^{t} \frac{A(k)}{A(0)},$$

$$A(t) = \left\langle x_1 x_{1+t} \right\rangle - \langle x_1 \rangle \left\langle x_{1+t} \right\rangle,$$

$$\tau_{\text{int}} = I(k_{\text{max}}) \mid k_{\text{max}} > 6\tau_{\text{int}},$$

$$\tau_{\text{int}} = aL^{z}(1 + cL^{-\omega}).$$



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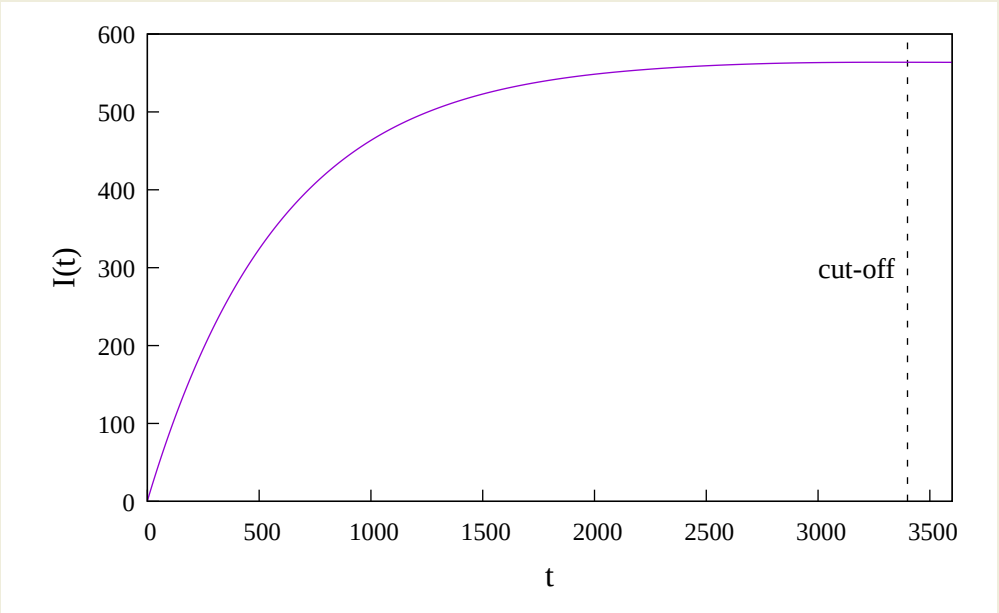
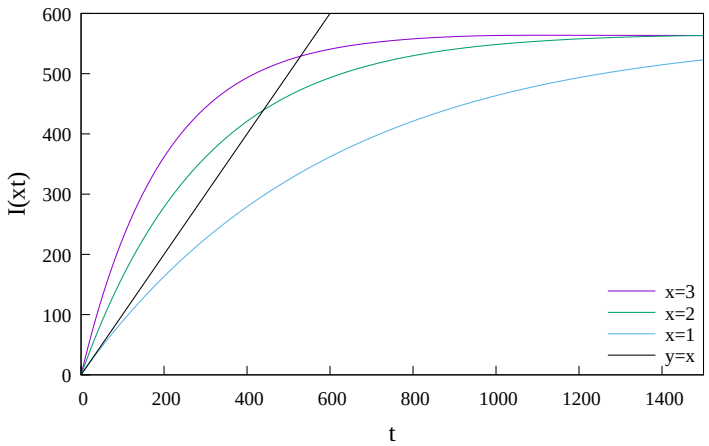
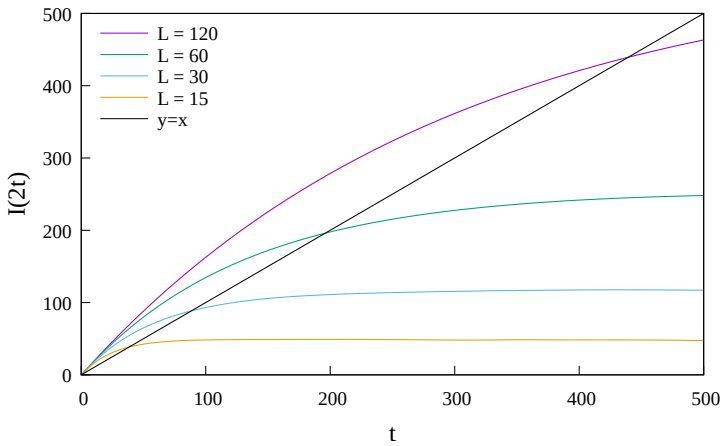


$$\tau_x = I(x\tau_x),$$

$$\tau_x \sim L^z.$$





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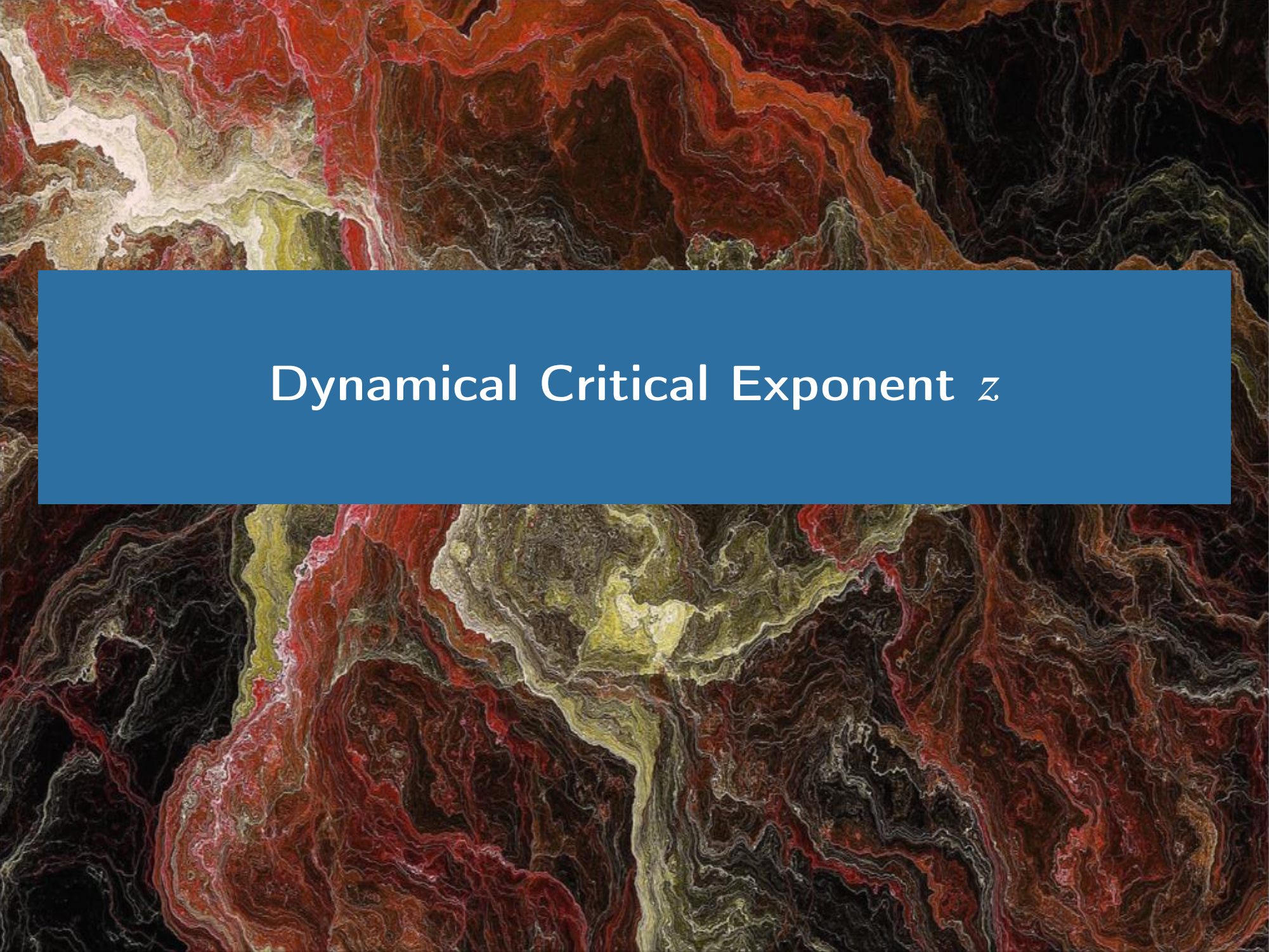
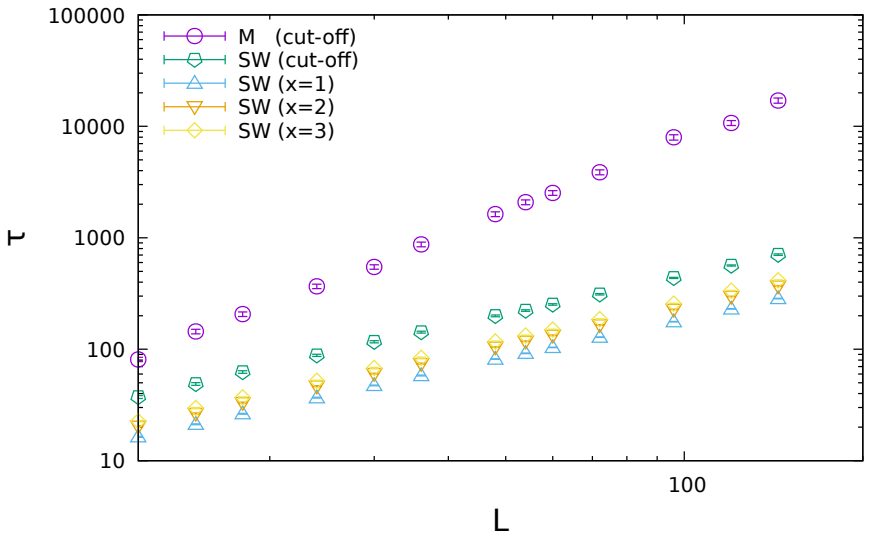
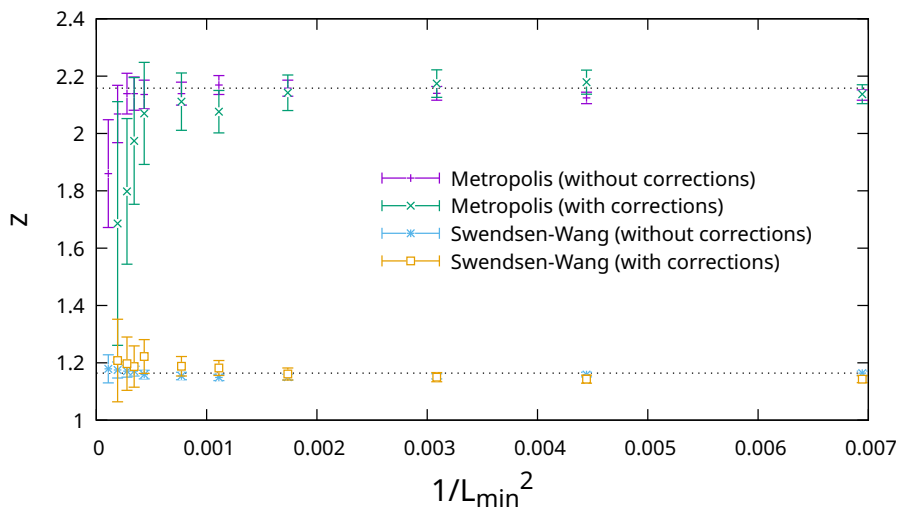

Dynamical Critical Exponent z

Spin-1/2 Baxter-Wu Model

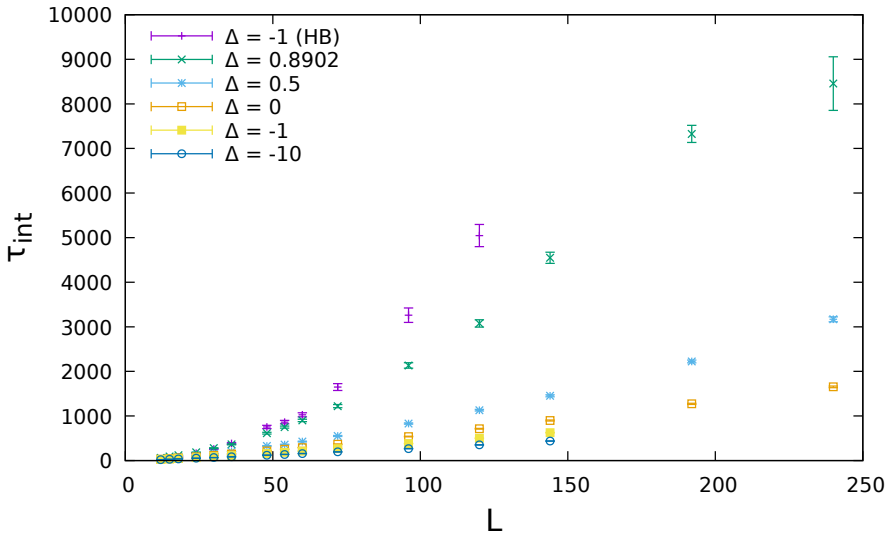


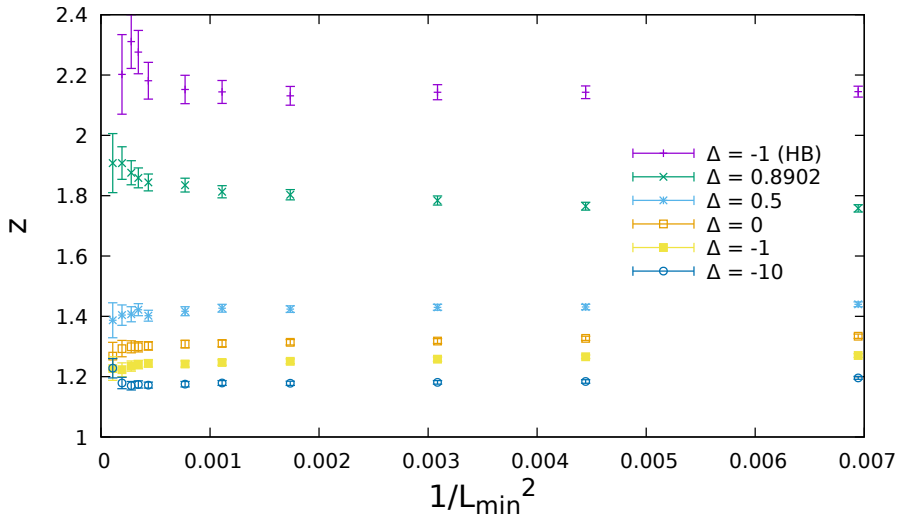


Swendsen-Wang cluster				Metropolis	
$\sim L^z$		$aL^z(1+c/L^2)$		$\sim L^z$	$aL^z(1+c/L^2)$
τ_{int}	τ_2	τ_{int}	τ_2	τ_{int}	τ_{int}
1.164(7)	1.141(4)	1.143(14)	1.142(6)	2.158(28)	2.179(42)

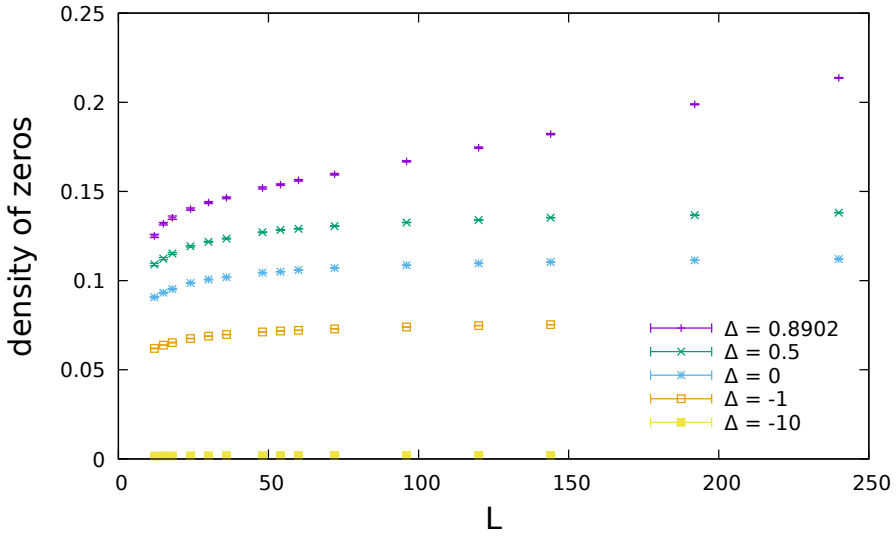
Novotny & Evertz: $z = 1.37(10)$
Velonakis & Martinos: $z = 1.272(3)$ [Physica A 390 (2011) 24–30]

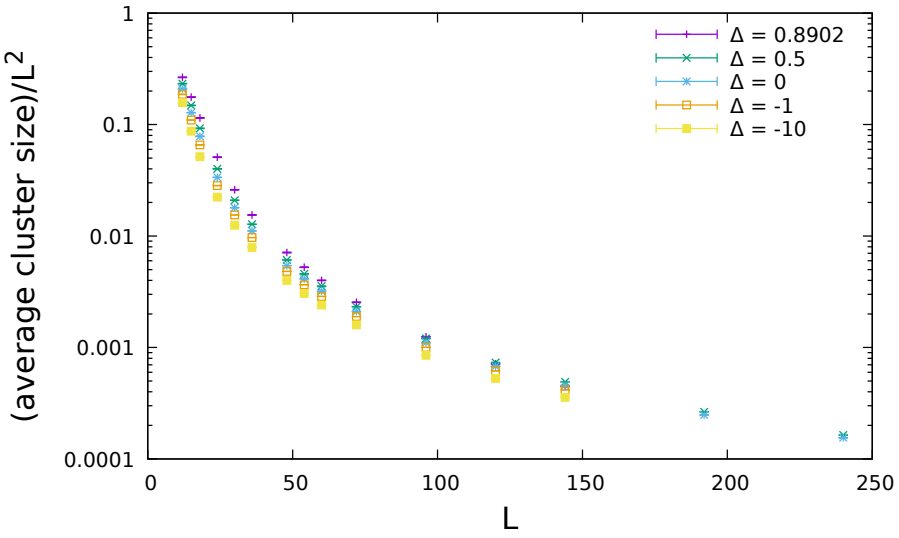
Spin-1 Baxter-Wu Model

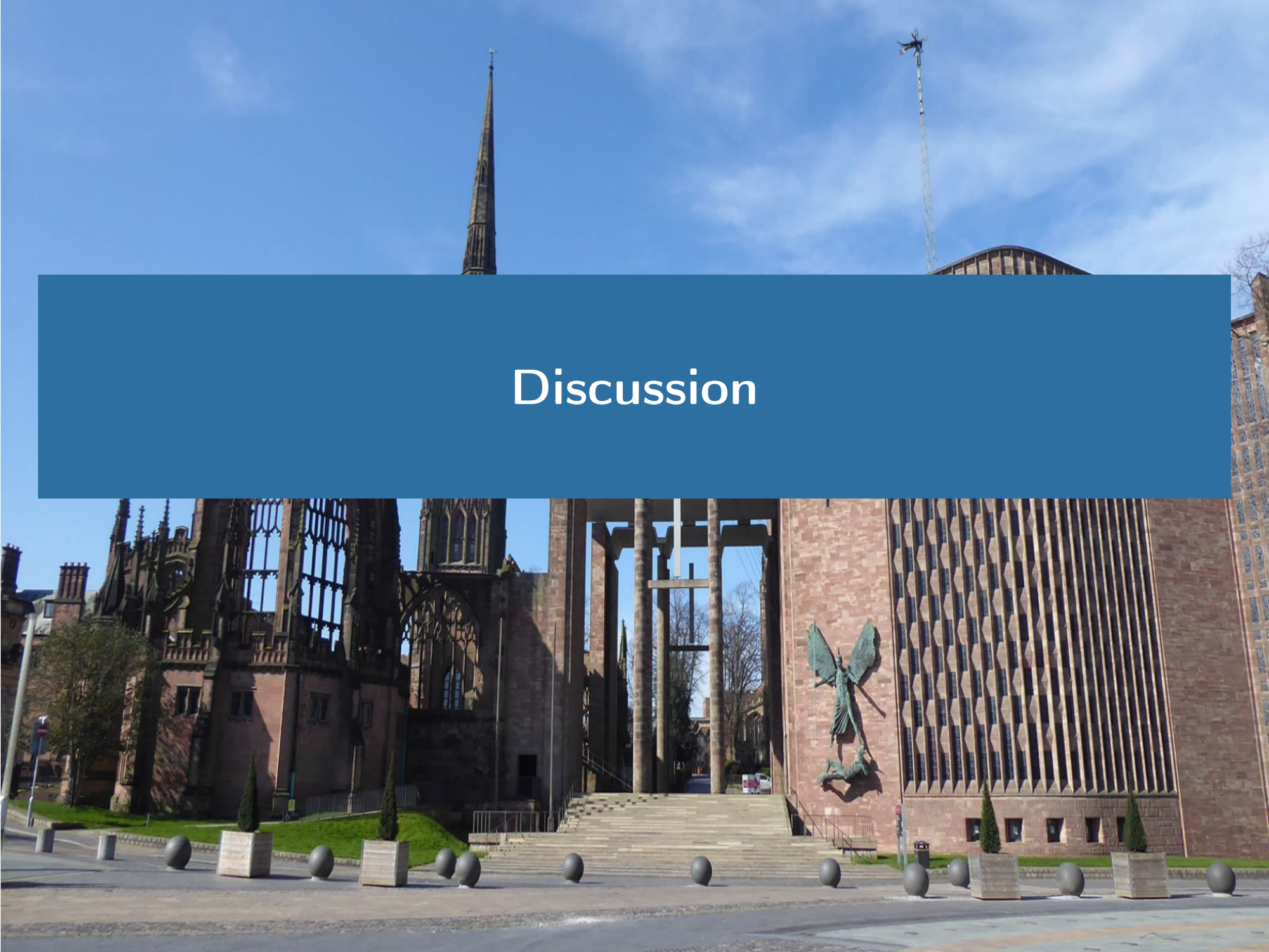




	τ_{int}		τ_2	
Δ	no corrections	corrections	no corrections	corrections
-10	1.172(10)	1.163(8)	1.171(3)	1.162(4)
-1	1.260(6)	1.234(27)	1.209(3)	1.202(7)
0	1.276(7)	1.262(9)	1.280(3)	1.273(5)
0.5	1.435(15)	1.422(10)	1.413(3)	1.421(5)
0.8902	1.786(35)	1.791(20)	1.802(12)	1.811(14)
-1 (HB)	2.145(18)	2.140(38)	-	-


density of zeros
L
0.25
0.2
0.15
0.1
0.05
0
0
50
100
150
200
250
Δ = 0.8902
Δ = 0.5
Δ = 0
Δ = -1
Δ = -10

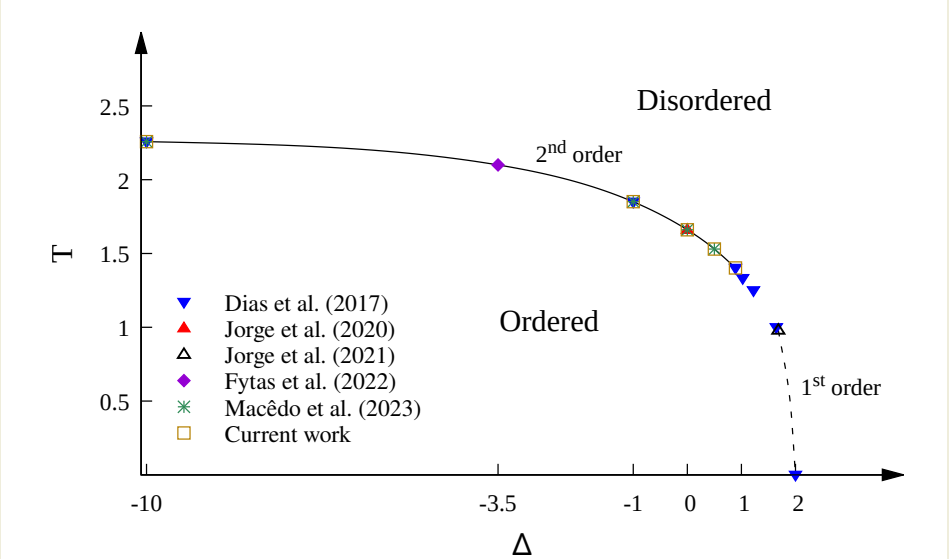

(average cluster size)/L^2
L
1
0.1
0.01
0.001
0.0001
0
50
100
150
200
250
Δ = 0.8902
Δ = 0.5
Δ = 0
Δ = -1
Δ = -10

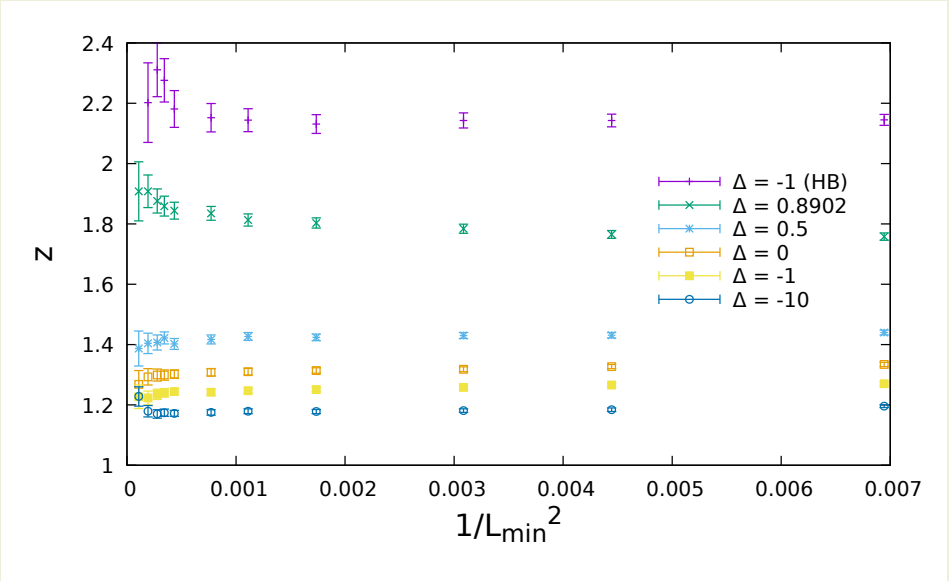


Discussion

Discussion

- Baxter-Wu:
  - spin-1/2 (SW cluster).
  - spin-1 in a crystal field (hybrid).
- Novotny & Evertz cluster.
- Dynamical critical exponent z.
- z increases with Δ. Why?
- Is the hybrid useful near $m.c.$ point?






Thank You!
Questions?