

Dynamical scaling of the 2D pure and dilute Baxter-Wu model

Clusters in a triangular lattice with three-spin nearest-neighbour interactions

Alexandros Vasilopoulos
Michail Akritidis
Nikolaos G. Fytas
Martin Weigel



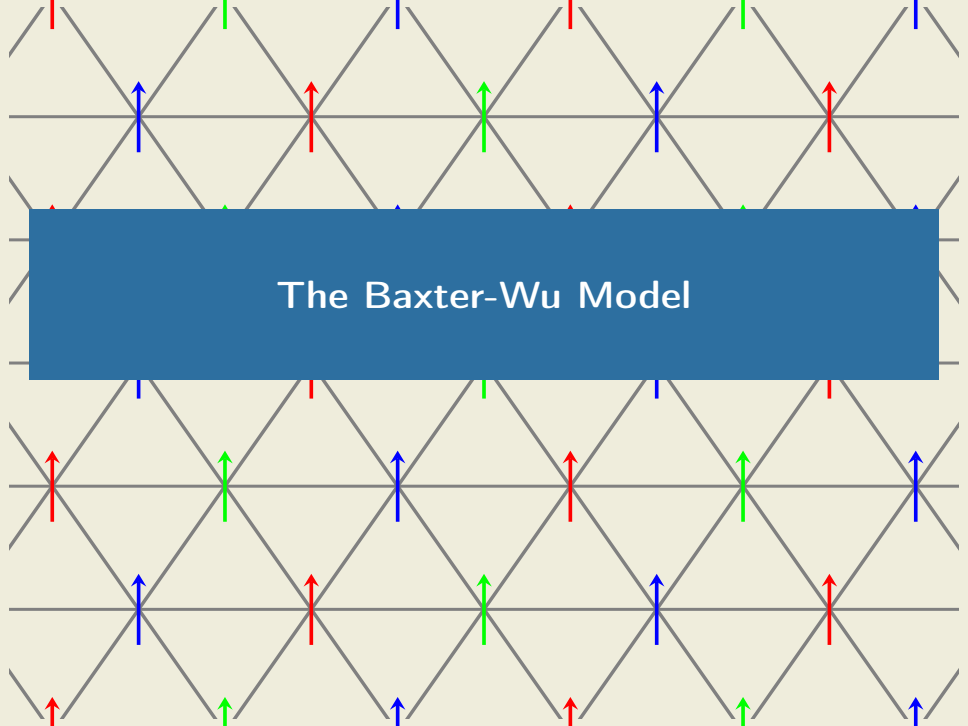
TECHNISCHE UNIVERSITÄT
CHEMNITZ



Coventry
University



University
of Essex

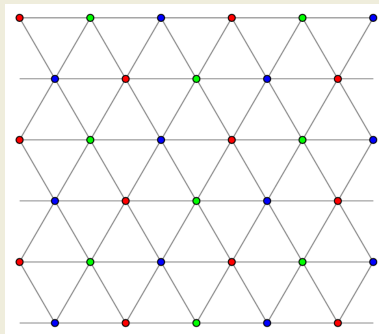
The background of the slide is a light beige color with a repeating pattern of gray lines forming a triangular lattice. At each vertex of this lattice, there is a small arrow pointing upwards. The arrows are colored in a repeating sequence: red, green, and blue. A solid blue rectangular box is centered horizontally and vertically on the slide, containing the title text in white.

The Baxter-Wu Model

The Spin-1/2 Baxter-Wu Model

- 2D triangular lattice.
- Three-spin interactions.
- Three sublattices A, B, C.
- No plane reflection symmetry.
- 4-fold degenerate GS.
- $m = \sqrt{(m_A^2 + m_B^2 + m_C^2)/3}$.
- 4-state Potts universality.
- $\alpha = 2/3$, $\nu = 2/3$, $\gamma = 7/6$,
 $\eta = 1/4$,[†] $\omega = 2$.[‡]

$$H = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k$$



[†]R. J. Baxter and F. Y. Wu, Phys. Rev. Lett. 31, 1294 (1973); Aust. J. Phys. 27, 357 (1974); R. J. Baxter, ibid. 27, 369 (1974).

[‡]F. C. Alcaraz and J. C. Xavier, J. Phys. A 30, L203 (1997); A 32, 2041 (1999).

The Spin-1 Baxter-Wu Model in a Crystal Field

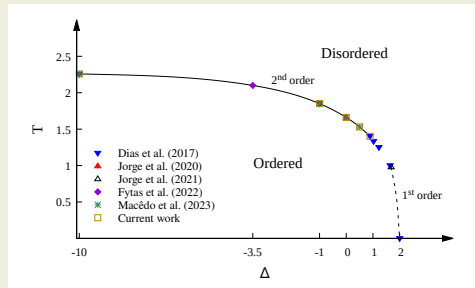
$$H = \underbrace{-J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k}_{E_J} + \Delta \underbrace{\sum_{i=1}^N \sigma_i^2}_{E_\Delta}$$

- $\sigma \in \{-1, 0, +1\}$.
- Anisotropy term (dilution).
- $\Delta \rightarrow$ density of $\sigma = 0$.
- 1st, 2nd order & multicriticality.

The Spin-1 Baxter-Wu Model in a Crystal Field

$$H = \underbrace{-J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k}_{E_J} + \underbrace{\Delta \sum_{i=1}^N \sigma_i^2}_{E_\Delta}$$

- $\sigma \in \{-1, 0, +1\}$.
- Anisotropy term (dilution).
- $\Delta \rightarrow$ density of $\sigma = 0$.
- 1st, 2nd order & multicriticality.

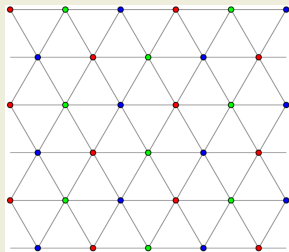




Simulation Methods

Cluster Algorithm for spin-1/2

$$E = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k$$

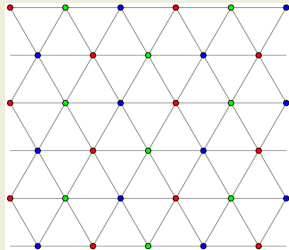


M.A. Novotny & H.G. Evertz (1993):

- “Freeze” a sublattice.
- Grow cluster on 2 sublattices.
- 3rd sublattice → effective interactions.
- $J' \in \{-2, 0, +2\}$.
- A cluster has either ferro- or antiferro-magnetic bonds, **only**.
- No frustration.

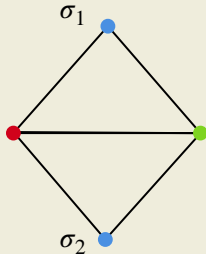
Cluster Algorithm for spin-1/2

$$E = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k$$



M.A. Novotny & H.G. Evertz (1993):

- “Freeze” a sublattice.
- Grow cluster on 2 sublattices.
- 3rd sublattice → effective interactions.
- $J' \in \{-2, 0, +2\}$.
- A cluster has either ferro- or antiferro-magnetic bonds, **only**.
- No frustration.

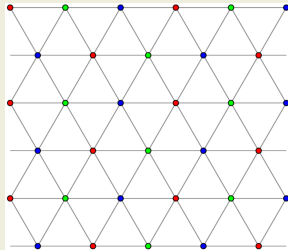


$$J' = J(\sigma_1 + \sigma_2)$$



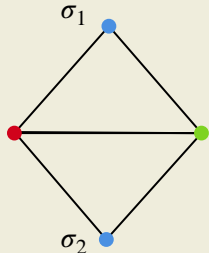
Cluster Algorithm for spin-1/2

$$E = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k$$

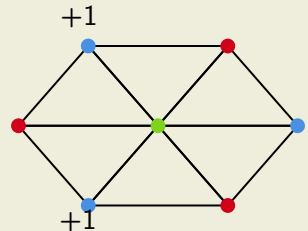


M.A. Novotny & H.G. Evertz (1993):

- “Freeze” a sublattice.
- Grow cluster on 2 sublattices.
- 3rd sublattice → effective interactions.
- $J' \in \{-2, 0, +2\}$.
- A cluster has either ferro- or antiferro-magnetic bonds, **only**.
- No frustration.



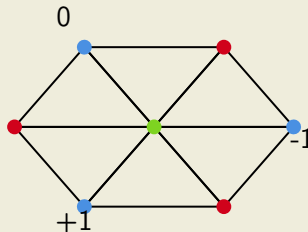
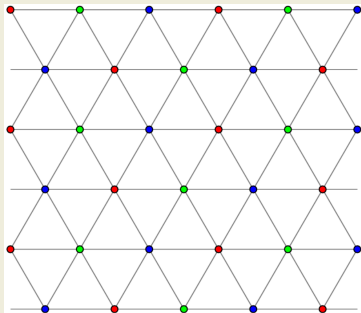
$$J' = J(\sigma_1 + \sigma_2)$$



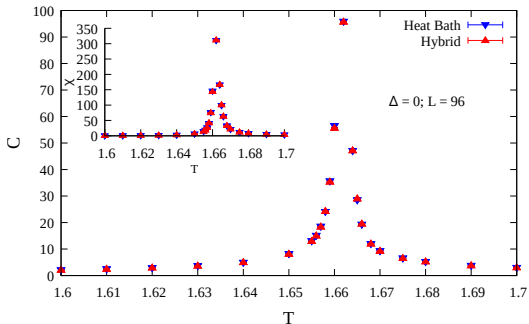
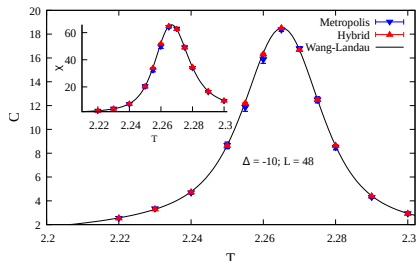
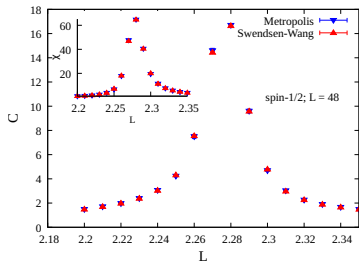
Hybrid Algorithm for spin-1

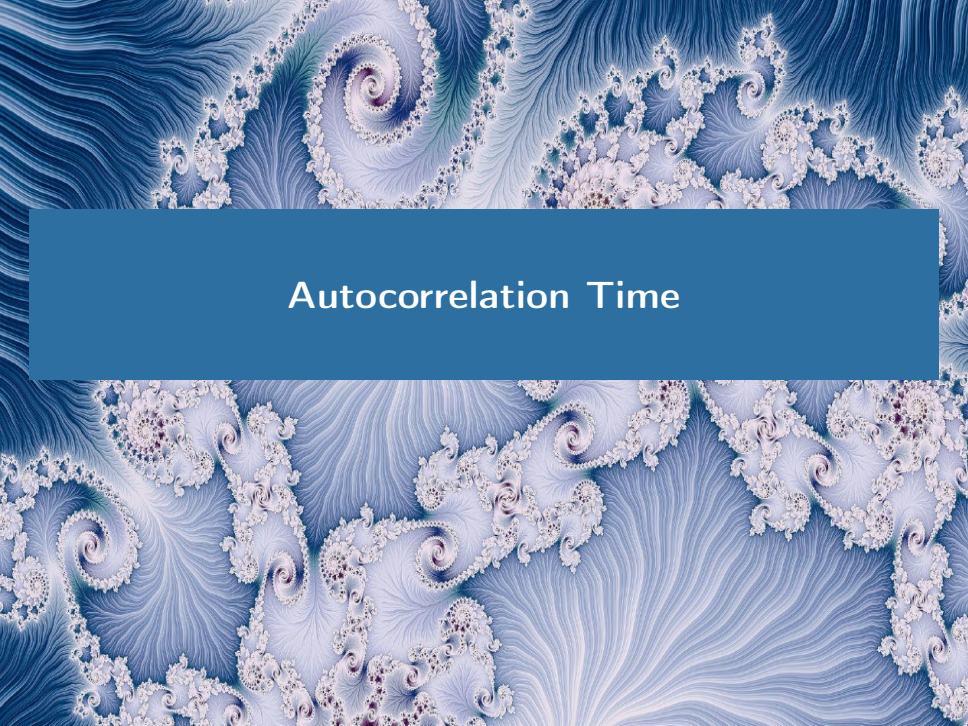
$$E = -J \sum_{\langle ijk \rangle} \sigma_i \sigma_j \sigma_k + \Delta \sum_i \sigma_i^2$$

- Cluster ignores zeros.
- $\oplus$ single-spin-flip algorithm.
- One SW one sweep HB.
- $J' \in \{-2, -1, 0, +1, +2\}$.
- A cluster can have mixed interactions.



Does it work?





Autocorrelation Time

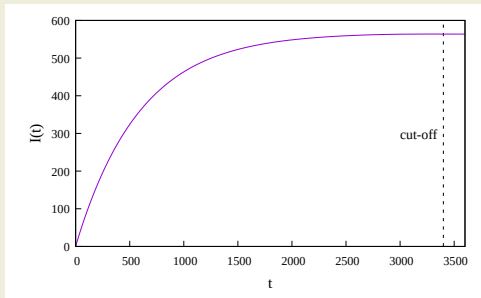
Integrated Autocorrelation Time

$$I(t) = \frac{1}{2} + \sum_{k=1}^t \frac{A(k)}{A(0)},$$

$$A(t) = \langle x_1 x_{1+t} \rangle - \langle x_1 \rangle \langle x_{1+t} \rangle,$$

$$\tau_{\text{int}} = I(k_{\text{max}}) \mid k_{\text{max}} > 6\tau_{\text{int}},$$

$$\tau_{\text{int}} = aL^z(1 + cL^{-\omega}).$$



[†]Martin Hasenbusch et al J. Stat. Mech. (2007) P11009.

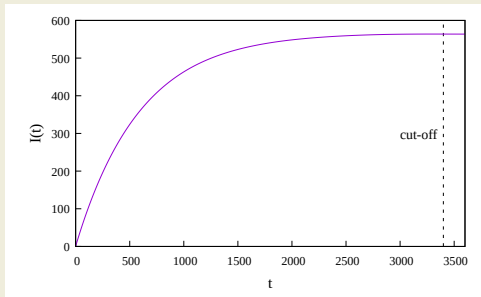
Integrated Autocorrelation Time

$$I(t) = \frac{1}{2} + \sum_{k=1}^t \frac{A(k)}{A(0)},$$

$$A(t) = \langle x_1 x_{1+t} \rangle - \langle x_1 \rangle \langle x_{1+t} \rangle,$$

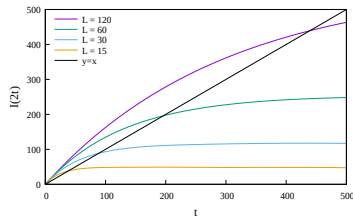
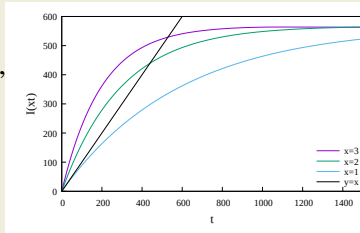
$$\tau_{\text{int}} = I(k_{\text{max}}) \mid k_{\text{max}} > 6\tau_{\text{int}},$$

$$\tau_{\text{int}} = aL^z(1 + cL^{-\omega}).$$

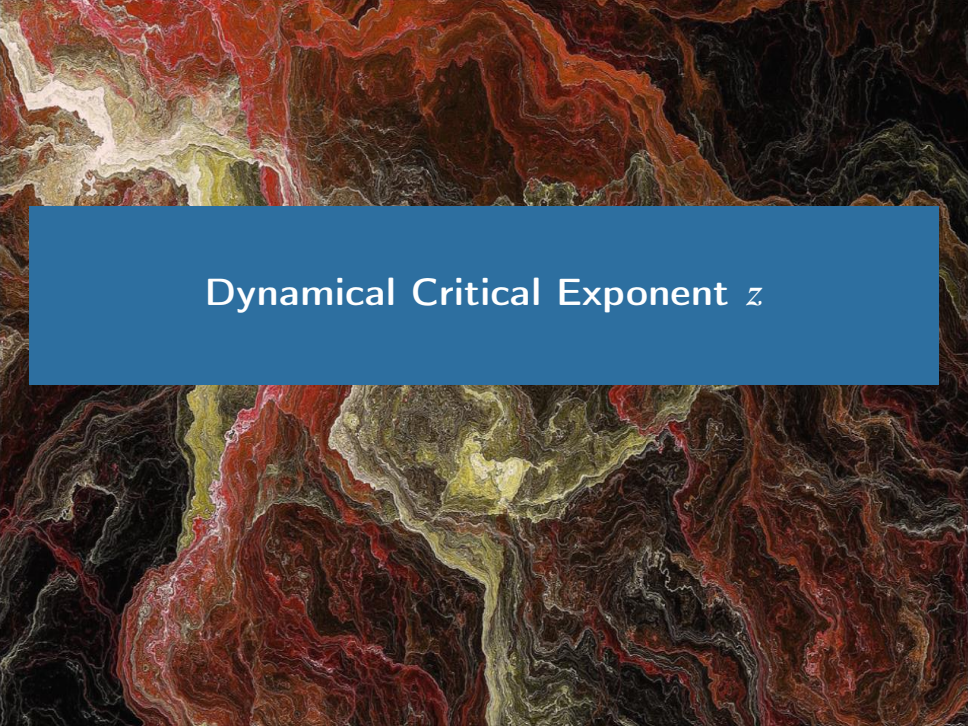


$$\tau_x = I(x\tau_x),$$

$$\tau_x \sim L^z.$$

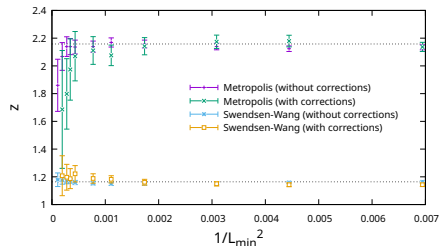
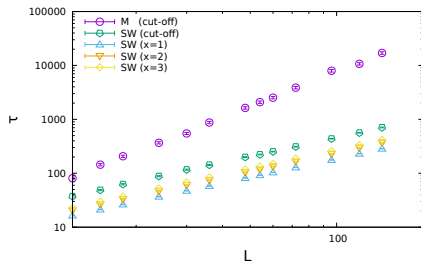


[†] Martin Hasenbusch et al J. Stat. Mech. (2007) P11009.

The background of the slide is a complex marbled paper pattern. It features swirling, organic shapes in a rich palette of deep reds, dark browns, and blacks, interspersed with lighter, creamy yellow and off-white veins. The overall effect is dense and textured, resembling traditional stone or shell marbling. A solid blue horizontal bar is positioned across the middle of the image, serving as a background for the title text.

Dynamical Critical Exponent z

Spin-1/2 Baxter-Wu Model

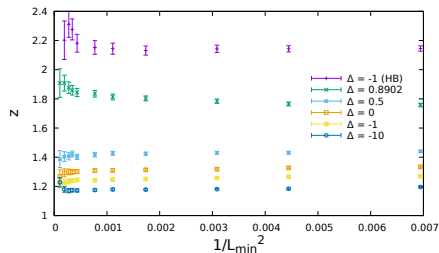
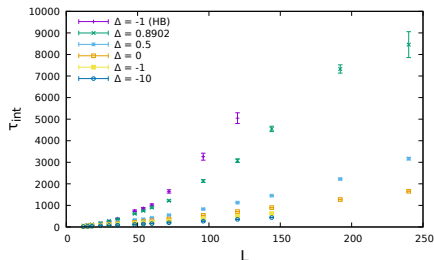


Swendsen-Wang cluster				Metropolis	
$\sim L^z$		$aL^z(1 + c/L^2)$		$\sim L^z$	$aL^z(1 + c/L^2)$
τ_{int}	τ_2	τ_{int}	τ_2	τ_{int}	τ_{int}
1.164(7)	1.141(4)	1.143(14)	1.142(6)	2.158(28)	2.179(42)

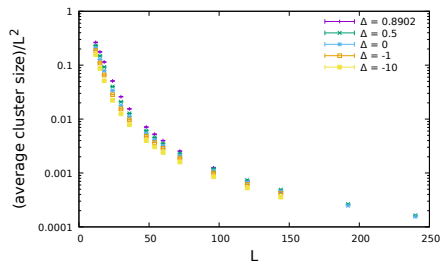
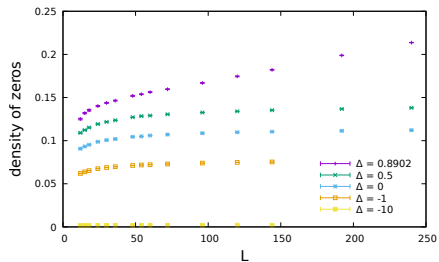
Novotny & Evertz: $z = 1.37(10)$

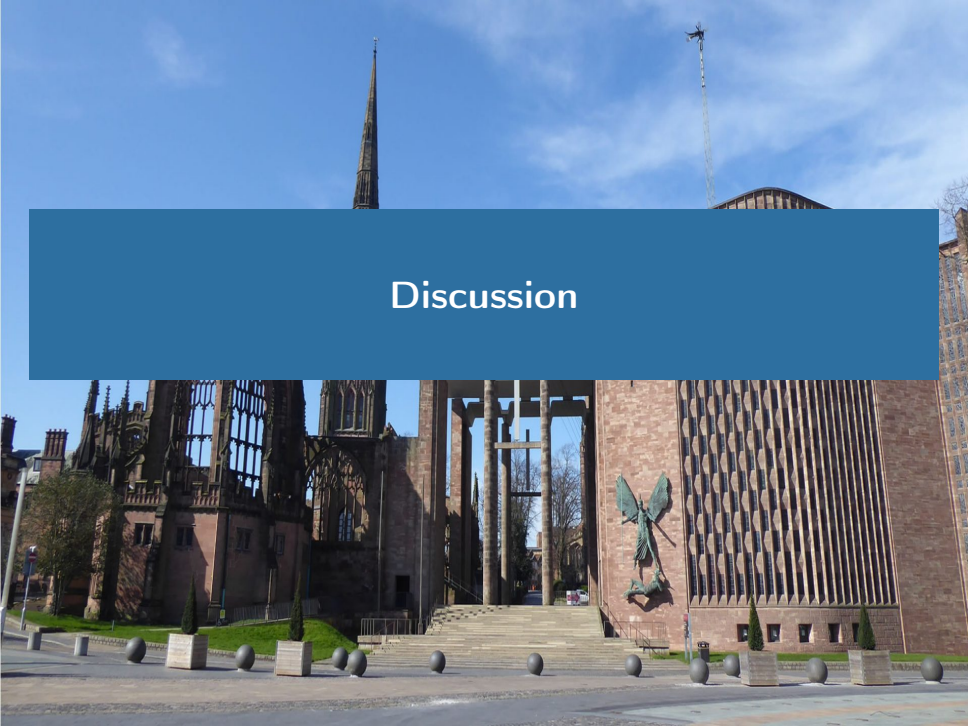
Velonakis & Martinos: $z = 1.272(3)$ [Physica A 390 (2011) 24–30]

Spin-1 Baxter-Wu Model



Δ	τ_{int}		τ_2	
	no corrections	corrections	no corrections	corrections
-10	1.172(10)	1.163(8)	1.171(3)	1.162(4)
-1	1.260(6)	1.234(27)	1.209(3)	1.202(7)
0	1.276(7)	1.262(9)	1.280(3)	1.273(5)
0.5	1.435(15)	1.422(10)	1.413(3)	1.421(5)
0.8902	1.786(35)	1.791(20)	1.802(12)	1.811(14)
-1 (HB)	2.145(18)	2.140(38)	-	-

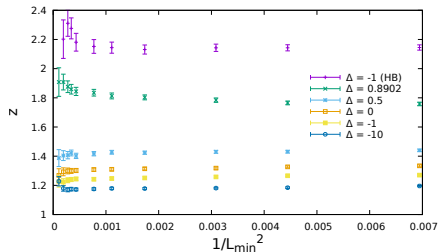
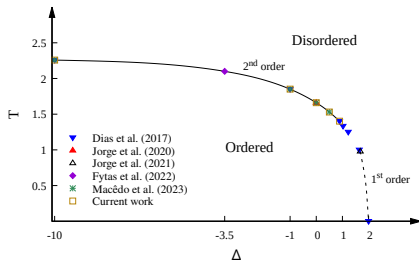


The image is a composite of two photographs. The top photograph shows the spire of Glasgow Cathedral on the left and the roof of Glasgow City Hall on the right, set against a clear blue sky with a few wispy clouds. A tall construction crane is visible in the background between the two buildings. A solid blue horizontal banner is superimposed over the middle of the image, containing the word 'Discussion' in white, sans-serif font. The bottom photograph shows the ground-level view of the same two buildings. On the left is the Gothic architecture of Glasgow Cathedral, and on the right is the modern, brick-clad Glasgow City Hall with its distinctive diamond-patterned facade. A large bronze sculpture of a winged figure stands in front of the City Hall. A wide set of stone steps leads up to the entrance of the City Hall. In the foreground, there is a paved plaza with several spherical stone bollards and small concrete planters with green shrubs.

Discussion

Discussion

- Baxter-Wu:
 - spin-1/2 (SW cluster).
 - spin-1 in a crystal field (hybrid).
- Novotny & Evertz cluster.
- Dynamical critical exponent z .
- z increases with Δ . Why?
- Is the hybrid useful near $m.c.$ point?





Thank You!
Questions?