

**t.b.a.**

T. S.

19.12.2023

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**tolle bekannte Approximationen**

**Example 1**

## Monte Carlo integration and importance sampling

$$\langle f \rangle_{p(x)} = \int f(x) \cdot p(x) dx \approx \frac{1}{N} \sum_{\substack{n=1 \\ x_n \sim p(x)}}^N f(x_n)$$

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**tolle bekannte Approximationen**

**Example 1**

## Monte Carlo integration and importance sampling

$$\langle f \rangle_{p(x)} = \int f(x) \cdot p(x) dx \approx \frac{1}{N} \sum_{\substack{n=1 \\ x_n \sim p(x)}}^N f(x_n)$$

$$\langle f \rangle_{p(x)} = \int f(x) \frac{p(x)}{q(x)} \cdot q(x) dx \approx \frac{1}{N} \sum_{\substack{n=1 \\ x_n \sim q(x)}}^N f(x_n) \frac{p(x_n)}{q(x_n)}$$

$$\langle f \rangle_{p(x)} = \left\langle f \cdot \frac{p(x)}{q(x)} \right\rangle_{q(x)}$$

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**tolle bekannte Approximationen**

**Example 2**

## **Flat-histogram Monte Carlo**

All points in the configurational space are equally probable, new states are accepted with probability

$$p_{\text{acc}} \propto \frac{1}{g(U)}$$

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**tolle bekannte Approximationen**

**Example 2**

## **Flat-histogram Monte Carlo**

All points in the configurational space are equally probable, new states are accepted with probability

$$p_{\text{acc}} \propto \frac{1}{g(U)}$$

**Wang-Landau-type  
methods**

$$\ln g(U) \rightarrow \ln g(U) + \gamma_t$$

Update on every step

$$\gamma_t = \min \left\{ \gamma_0, \frac{t_0}{t} \right\}$$

**Multicanonical-type  
methods**

$$\ln g(U) \rightarrow \ln g(U) + \ln H(U)$$

Update after very final step

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**to bind approximations**

$$\hat{g}(U) = \int \delta(E(x) - U) dx$$

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**to bind approximations**

$$\hat{g}(U) = \int \delta(E(x) - U) dx$$

$$\hat{g}(U) = \int \frac{\delta(E(x) - U)}{q(E(x))} q(E(x)) dx \approx \sum_{x \sim q(E(x))} \frac{\delta(E(x) - U)}{q(E(x))}$$

## **t.b.a.** **to bind approximations**

$$\hat{g}(U) = \int \delta(E(x) - U) dx \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

$$\hat{g}(U) = \int \frac{\delta(E(x) - U)}{q(E(x))} q(E(x)) dx \approx \sum_{x \sim q(E(x))} \frac{\delta(E(x) - U)}{q(E(x))}$$

$$q(E) \propto \frac{1}{g(E)} \quad \text{for flat-histogram algorithms}$$

## t.b.a. to bind approximations

$$\hat{g}(U) = \int \delta(E(x) - U) dx \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

**Multicanonical-type  
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$$\ln g(U) \rightarrow \ln g(U) + \ln H(U)$$

## t.b.a. to bind approximations

$$\hat{g}(U) = \int \delta(E(x) - U) dx \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

### Multicanonical-type methods

$$\ln g(U) \rightarrow \ln g(U) + \ln H(U)$$

$$\hat{g}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(U)$$

$$\hat{g}(U) \approx C \cdot g(U) \underbrace{\sum_{x \sim q(E(x))} \delta(E(x) - U)}_{H(U)}$$

$$\ln \hat{g}(U) \approx \ln C + \ln g(U) + \ln H(U)$$

## t.b.a. to bind approximations

$$\hat{g}(U) = \int \delta(E(x) - U) dx \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

### Multicanonical-type methods

$$\ln g(U) \rightarrow \ln g(U) + \ln H(U)$$

### Wang-Landau-type methods

$$\ln g(U) \rightarrow \ln g(U) + \gamma_t$$

$$g_{\text{is}}(U) \approx \sum_t g(E(x_t), t) \delta(E(x_t) - U)$$

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**to bind approximations. trio: better approximation?**

$$p_{\text{acc}} \propto \frac{1}{g(U)}$$

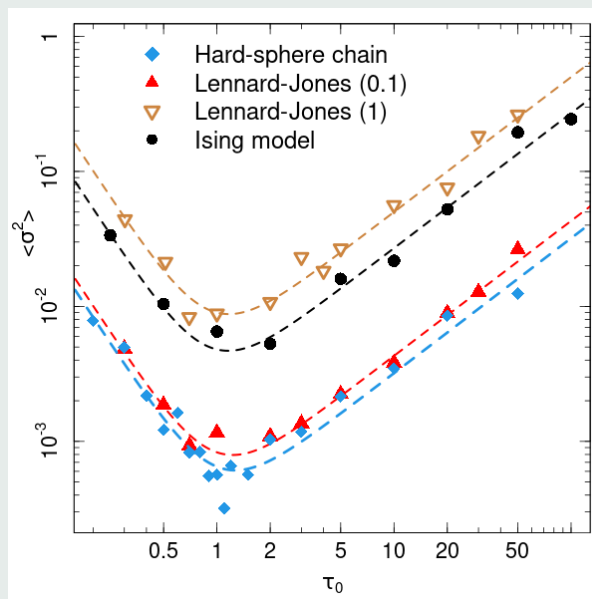
**Wang-Landau-type  
methods**

$$\ln g(U) \rightarrow \ln g(U) + \gamma_t$$

$$\gamma_t = \min \left[ \gamma_0; \frac{t_0}{t} \right]$$

$$\tau_0 = \frac{t_0}{N_{\text{bin}}}$$

**How accurate is it?**



**t.b.a. t.b.a.?**

**to bind approximations. trio: better approximation?**

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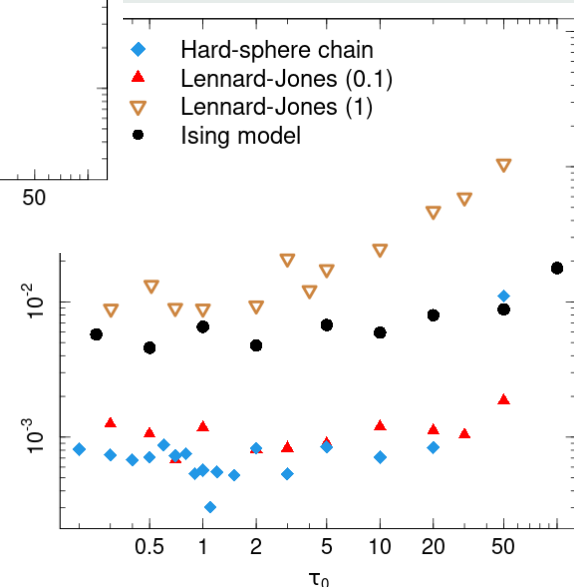
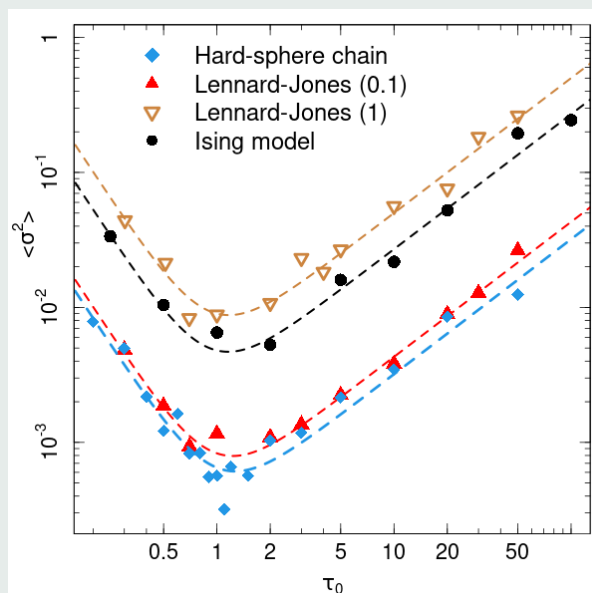
**Wang-Landau-type  
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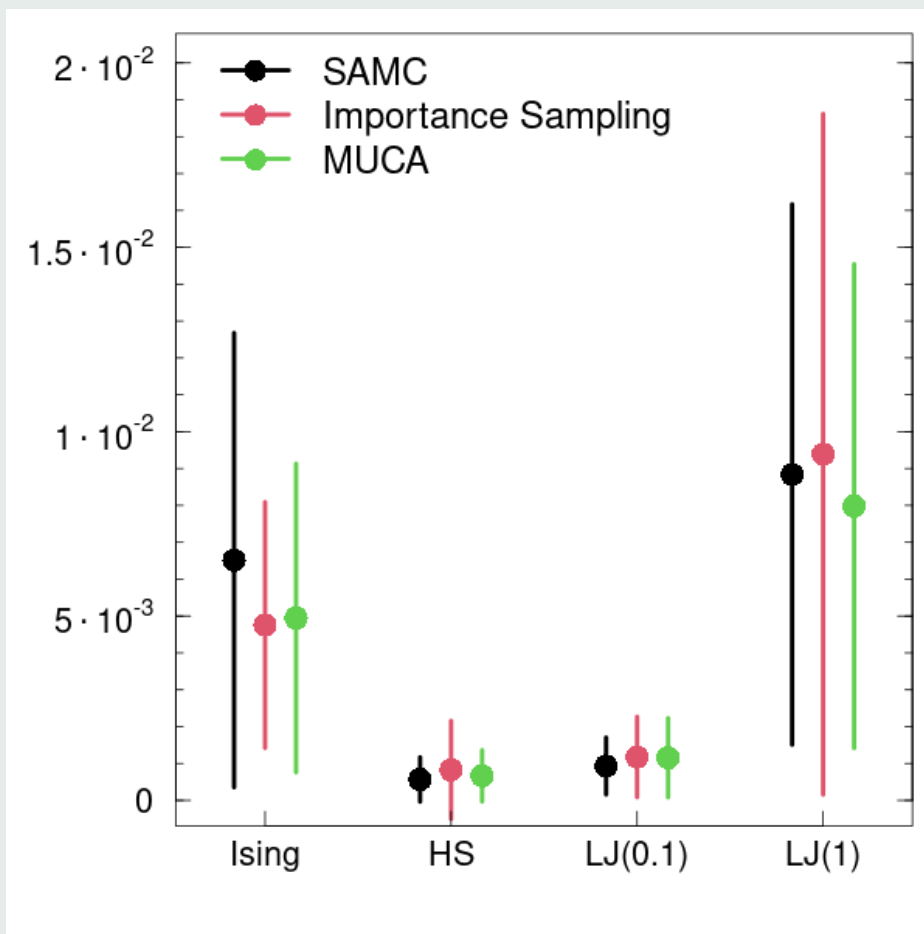
**How accurate is it?**



**t.b.a. t.b.a.?**

**to bind approximations. trio: better approximation?**

### How accurate is it?



**t.b.a.**  
**to broaden applications**

$$\hat{g}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

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**to broaden applications**

$$\hat{g}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

$$\hat{k}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(K(x) - U) g(E(x))$$

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**to broaden applications**

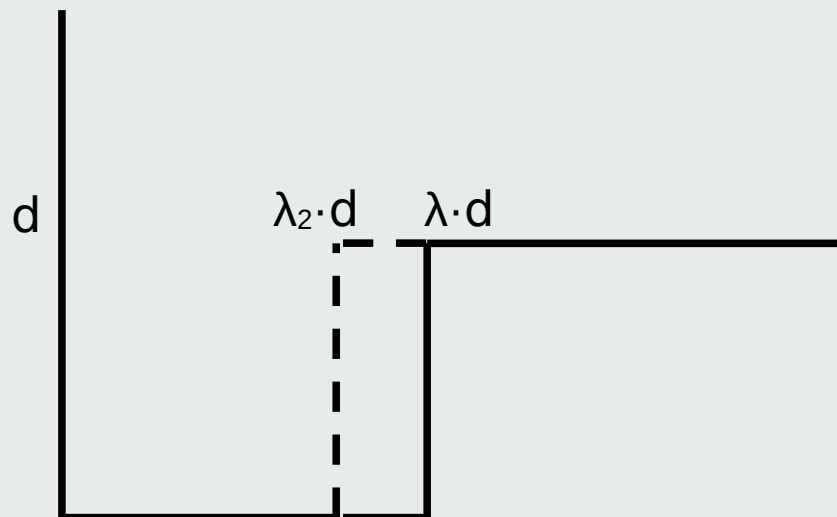
**Example 1**

$$\hat{g}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

$$\hat{k}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(K(x) - U) g(E(x))$$

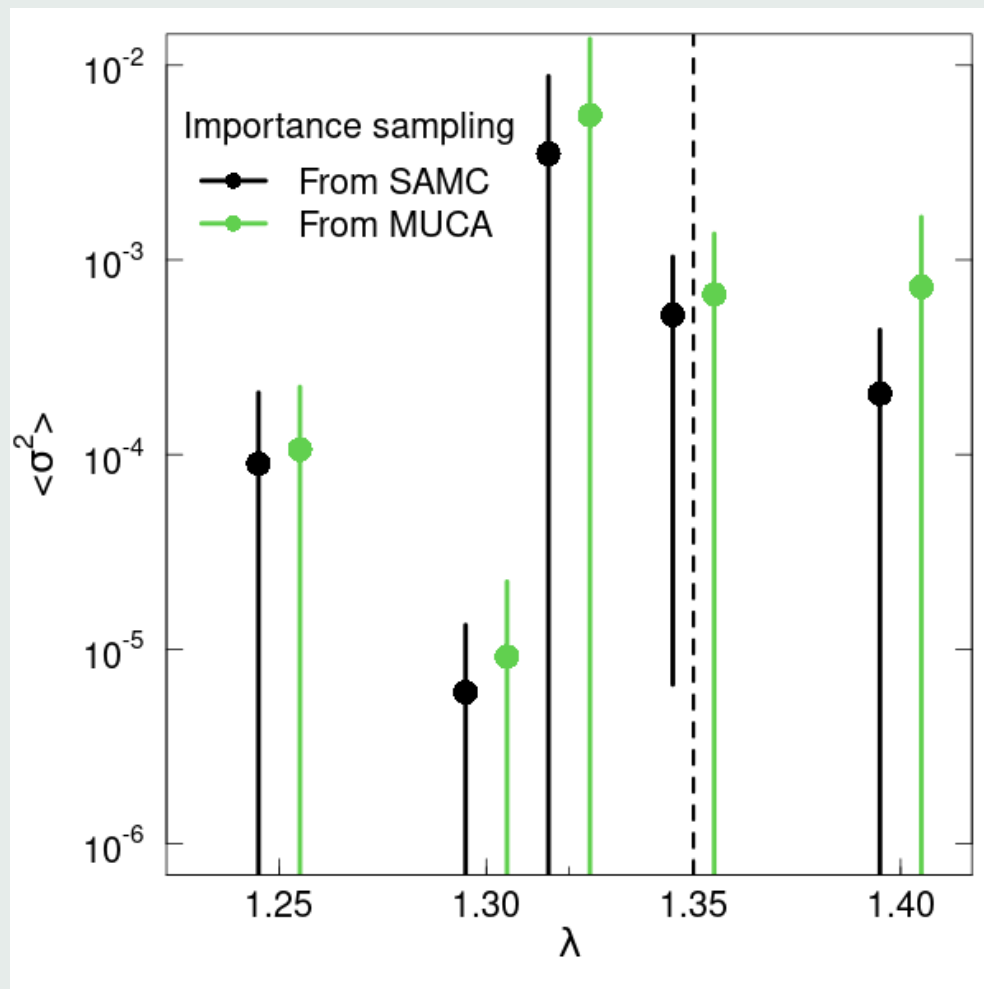
$$E(x) = - \sum_{\langle ij \rangle} \Theta(\lambda d - r_{ij})$$

$$K(x) = - \sum_{\langle ij \rangle} \Theta(\lambda_2 d - r_{ij})$$



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**to broaden applications**

**Example 1**



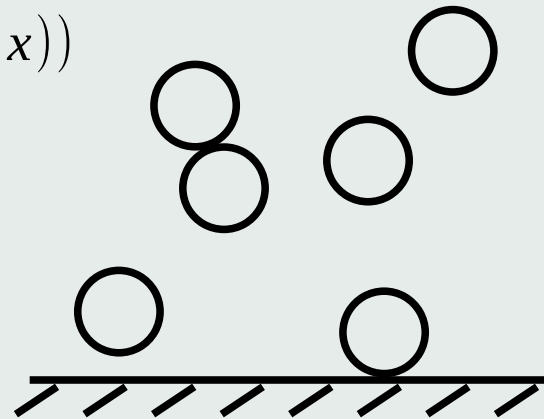
**t.b.a.**  
**to broaden applications**

**Example 2**

$$\hat{g}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(E(x) - U) g(E(x))$$

$$\hat{k}(U) \approx C \cdot \sum_{x \sim q(E(x))} \delta(K(x) - U) g(E(x))$$

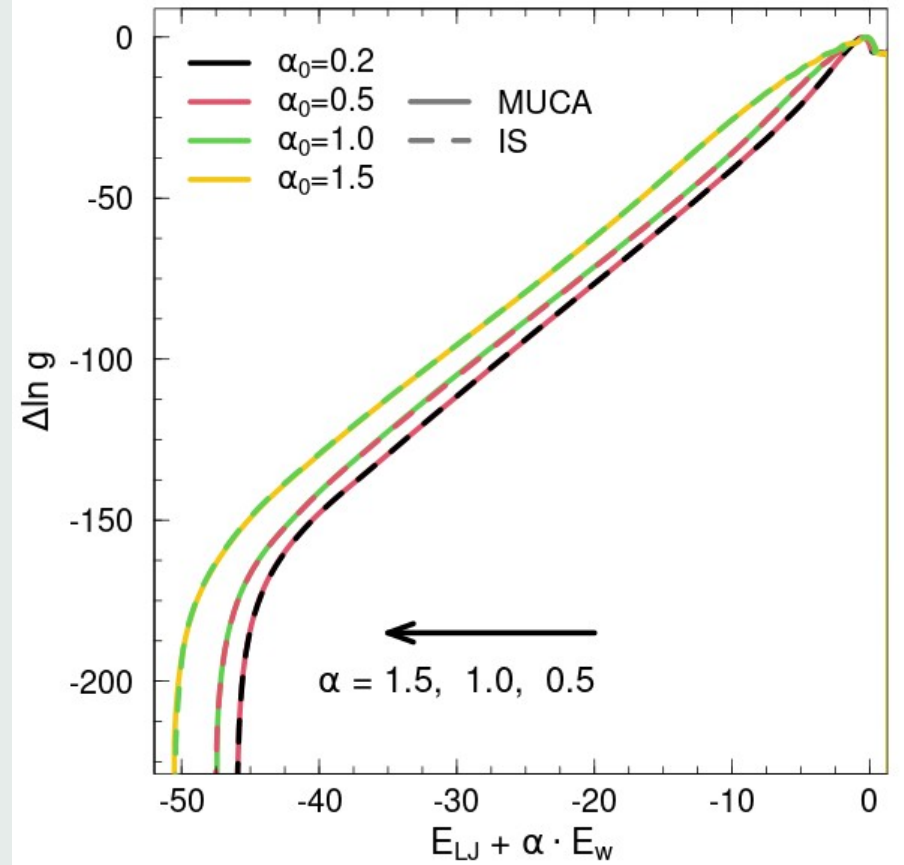
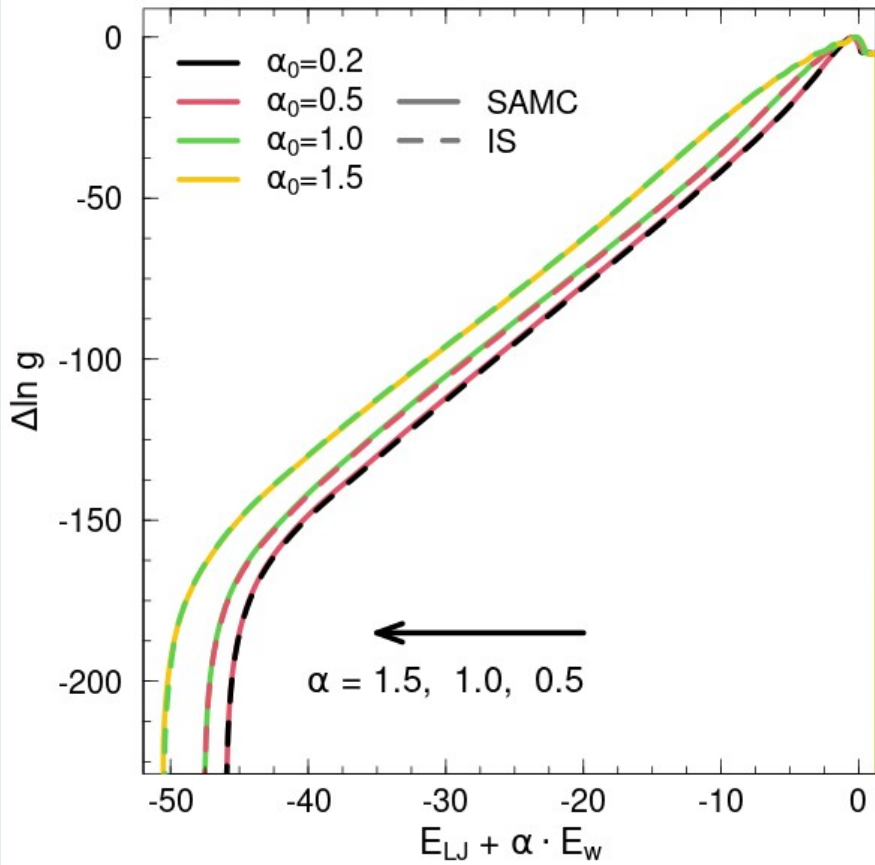
$$K(x) = \underbrace{E_{\text{LJ}}(x)}_{\text{bead-bead}} + \alpha \underbrace{E_{\text{w}}(x)}_{\text{bead-surface}}$$



13 LJ beads + LJ wall

# t.b.a. to broaden applications

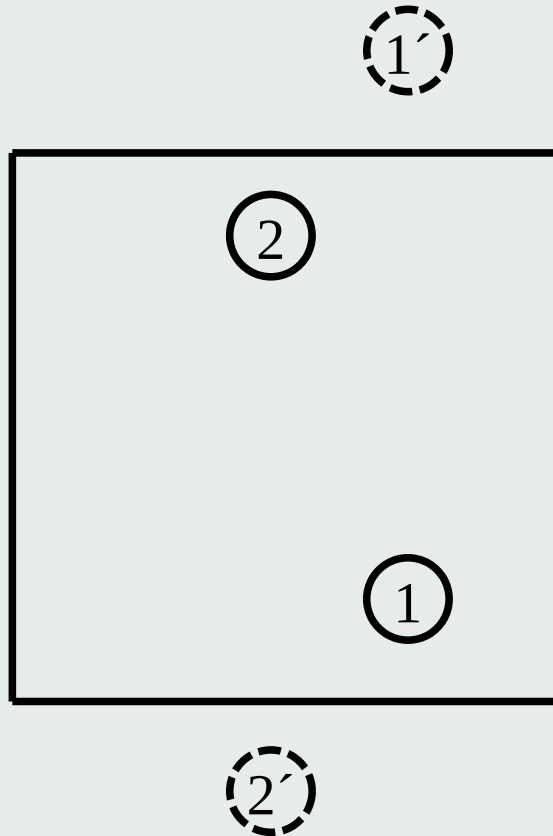
## Example 2



$$K(x) = E_{LJ}(x) + \alpha E_w(x)$$

**t.b.a.**  
**trial beyond accustomed**

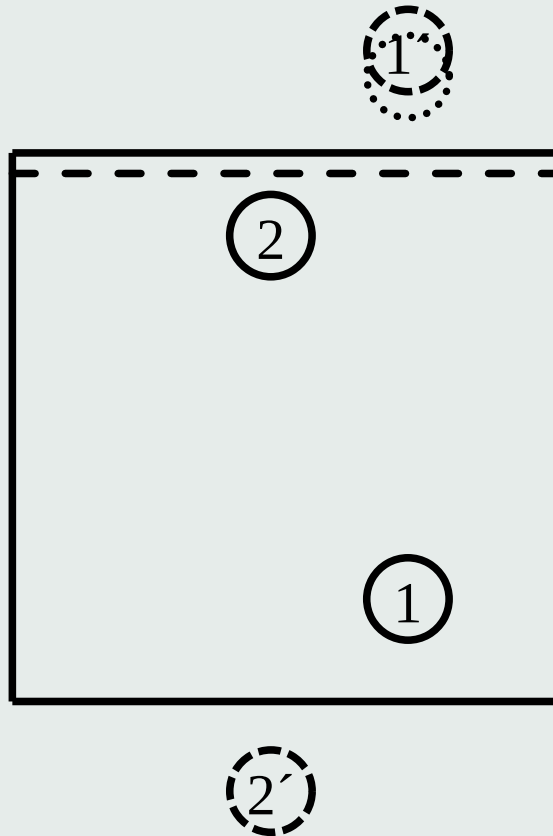
$$E = \sum_{\langle ij \rangle} U(\min(r_{ij})|L)$$



**t.b.a.**  
**trial beyond accustomed**

$$E = \sum_{\langle ij \rangle} U(\min(r_{ij})|L)$$

$$\beta p = \frac{\partial \ln g}{\partial V}$$

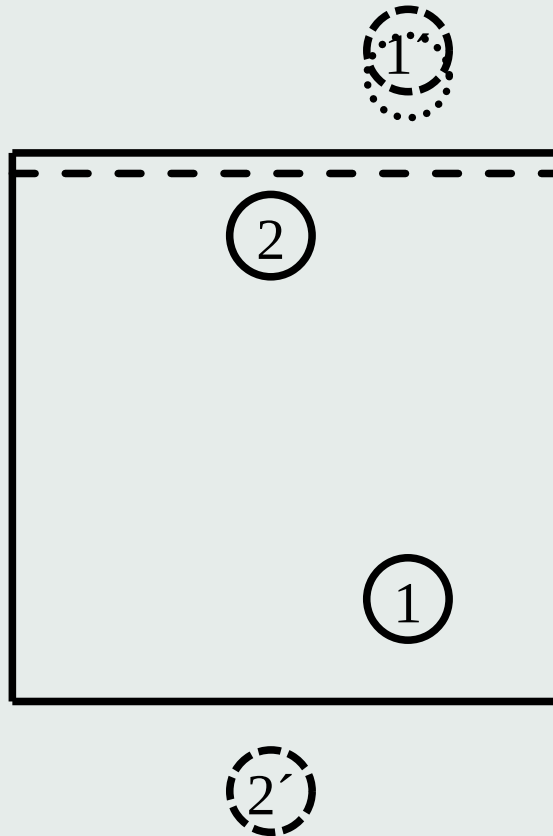


**t.b.a.**  
**trial beyond accustomed**

$$E = \sum_{\langle ij \rangle} U(\min(r_{ij}) | L)$$

$$\beta p = \frac{\partial \ln g}{\partial V}$$

$$pV = -\frac{1}{3} \langle r_{ij} \cdot F(r_{ij}) \rangle$$



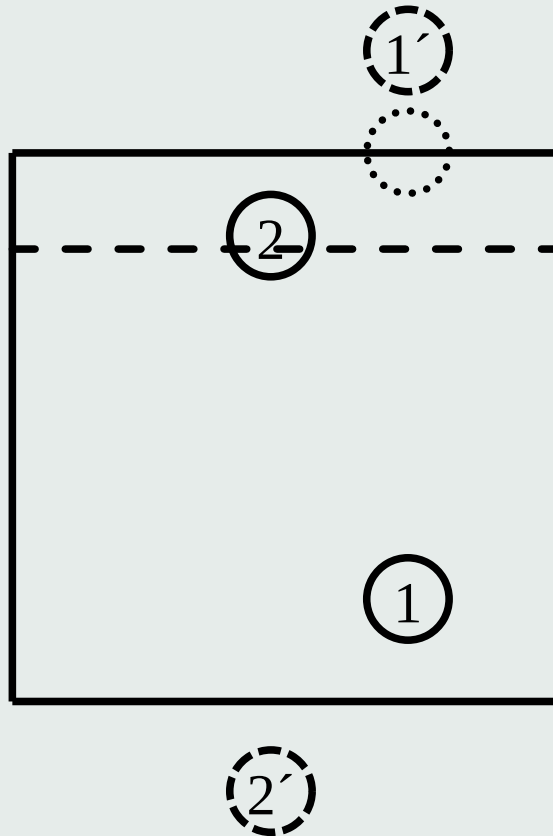
$$K = \sum_{\langle ij \rangle} U(\min(r_{ij}) | l < L)$$

**t.b.a.**  
**trial beyond accustomed**

$$E = \sum_{\langle ij \rangle} U(\min(r_{ij}) | L)$$

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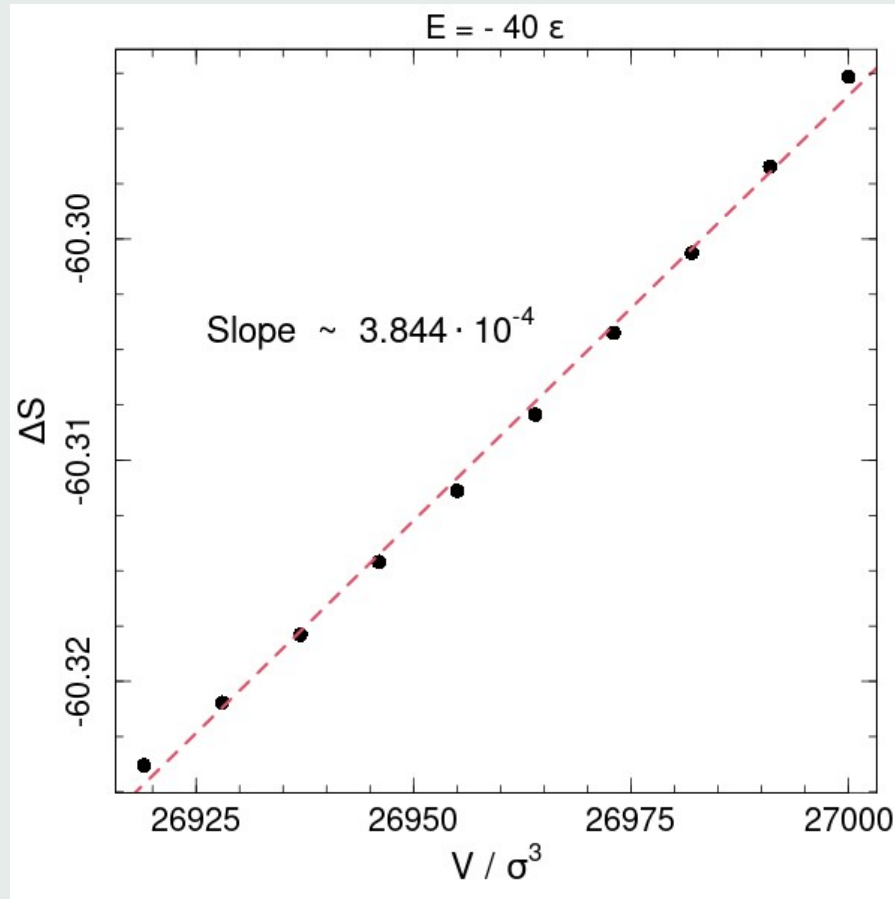
$$K = \sum_{\langle ij \rangle} U(\min(r_{ij}) | l < L)$$

**t.b.a.**

**trial beyond accustomed**

**Pressure of 13 LJ particles**

$$\beta p = \frac{\partial \ln g}{\partial V}$$

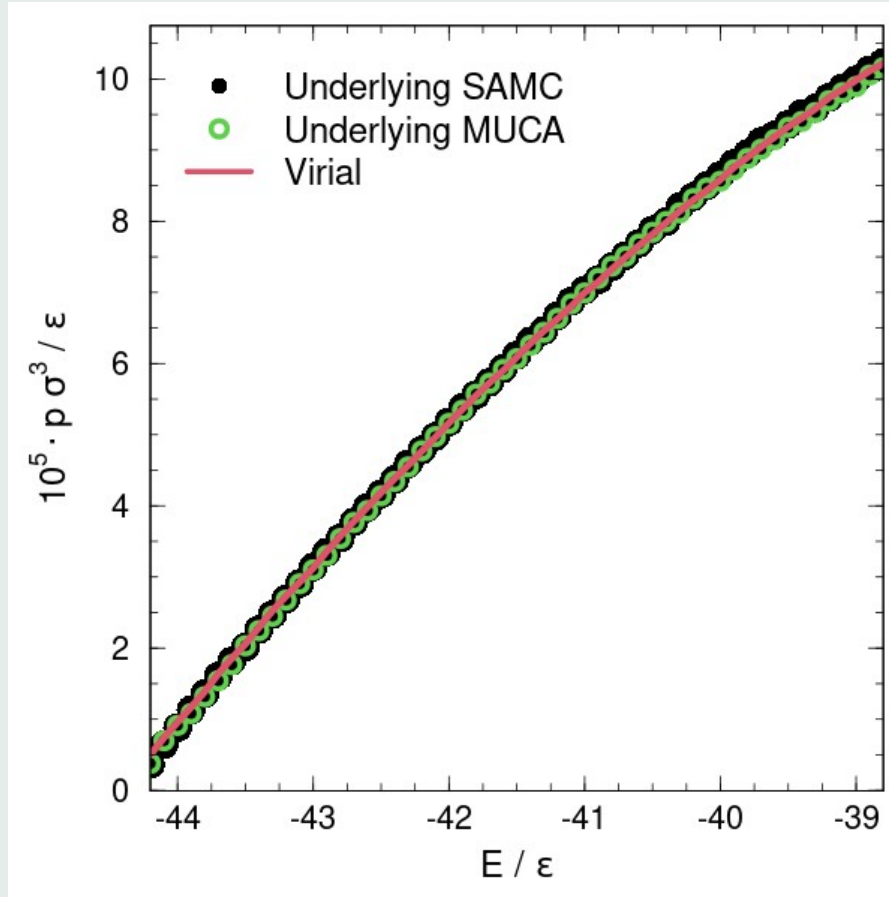


$$K = \sum_{\langle ij \rangle} U(\min(r_{ij}) | l_z < L)$$

**t.b.a.**  
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**Pressure of 13 LJ particles**

$$\beta p = \frac{\partial \ln g}{\partial V}$$



$$K = \sum_{\langle ij \rangle} U(\min(r_{ij}) | l_z < L)$$

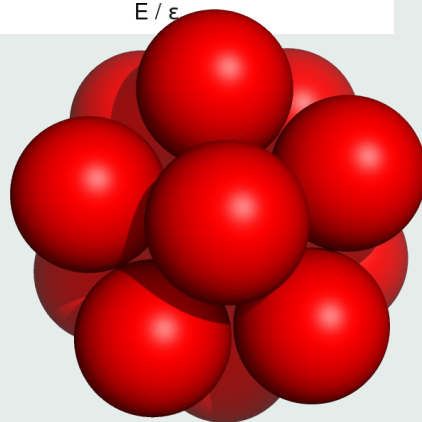
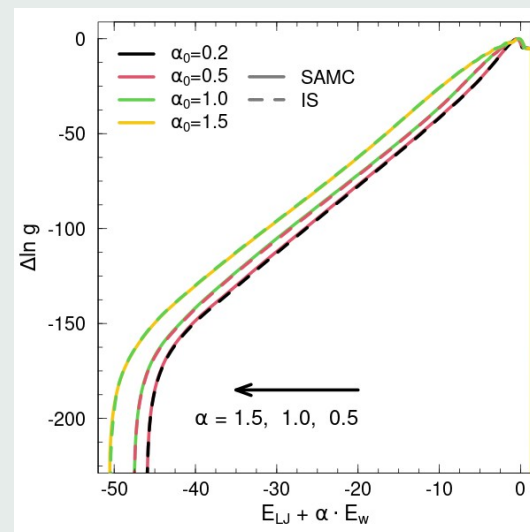
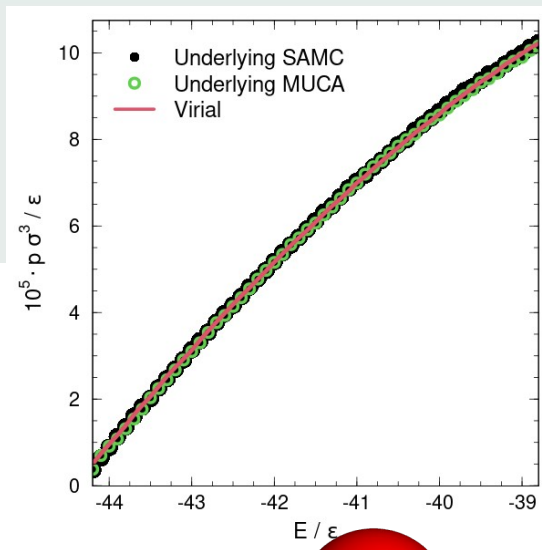
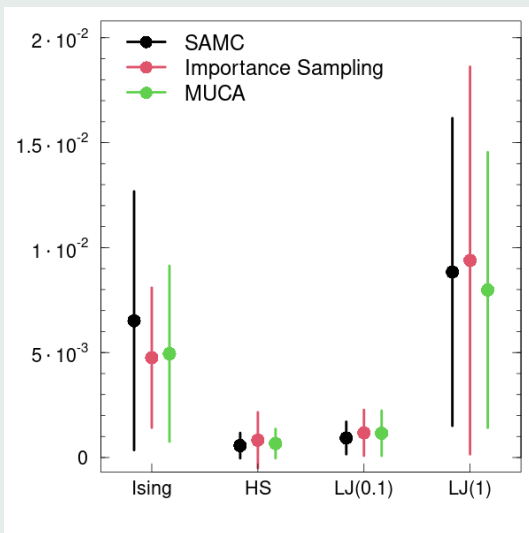
$$pV = -\frac{1}{3} \langle r_{ij} \cdot F(r_{ij}) \rangle$$

**t.b.a.**

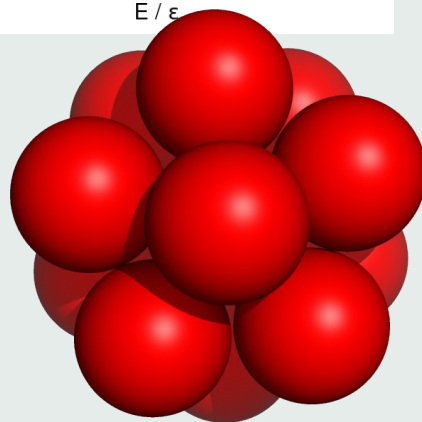
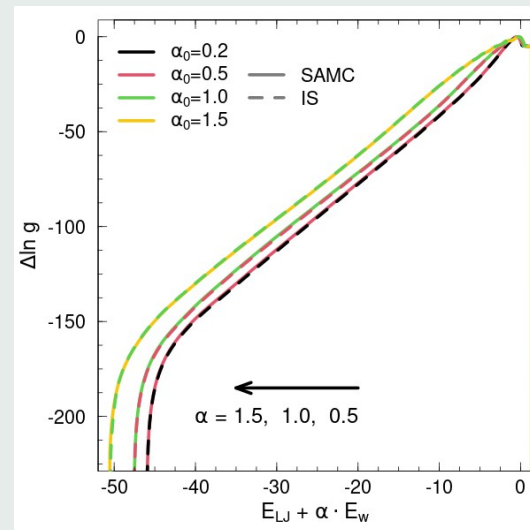
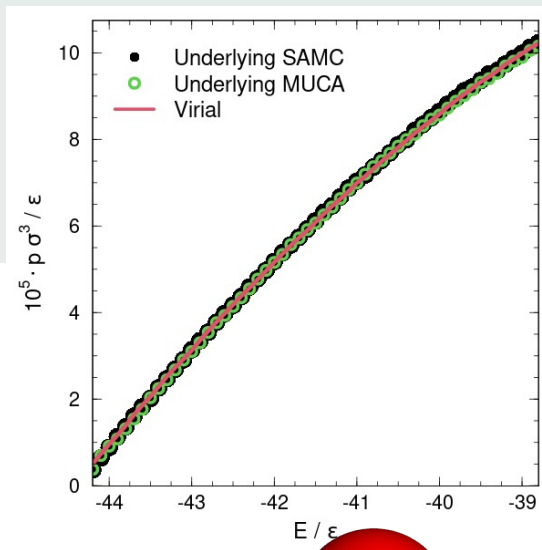
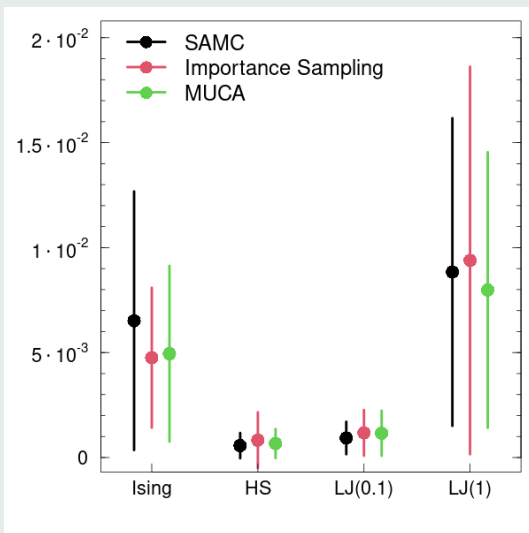
## **Take before abroad**

- Optimal convergence rate for SAMC is  $t_0 / N_b \approx 1$
- Importance sampling error depends not on the SAMC parameters and have the same accuracy as MUCA or optimal SAMC
- Sampling during a single run may be efficiently provided for a series of system or potential parameters within a single SAMC or MUCA simulation (running with other values of these parameters).

# Thank you for your attention!



# Thank you for your attention!



# t.b.a. to broaden applications

## Example 1

888 J. Chem. Phys., Vol. 118, No. 2, 8 January 2003

Mark P. Taylor

TABLE III. Density of states  $g_6^{(k)}$  for square-well 6-mer chains with well diameter  $\lambda\sigma$  and bond length  $L = \sigma$ . The partition function for the related hard-sphere 6-mer chain is  $Z_6^{\text{hs}} = 0.257\ 93$ .

$\lambda$	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$
1.10	0.196 15	0.052 77	$8.02 \times 10^{-03}$	$9.19 \times 10^{-04}$	$8.11 \times 10^{-05}$	$5.31 \times 10^{-06}$	$2.56 \times 10^{-07}$	$7.37 \times 10^{-09}$	0.0	0.0	0.0
1.15	0.168 42	0.070 09	0.016 18	$2.81 \times 10^{-03}$	$3.79 \times 10^{-04}$	$3.82 \times 10^{-05}$	$2.87 \times 10^{-06}$	$1.32 \times 10^{-07}$	0.0	0.0	0.0
1.20	0.142 80	0.082 20	0.025 65	$6.01 \times 10^{-03}$	$1.10 \times 10^{-03}$	$1.51 \times 10^{-04}$	$1.57 \times 10^{-05}$	$1.05 \times 10^{-06}$	$7.0 \times 10^{-14}$	0.0	0.0
1.25	0.119 36	0.089 59	0.035 51	0.010 52	$2.45 \times 10^{-03}$	$4.30 \times 10^{-04}$	$5.82 \times 10^{-05}$	$5.38 \times 10^{-06}$	$1.24 \times 10^{-09}$	0.0	0.0
1.30	0.098 13	0.092 77	0.044 99	0.016 20	$4.63 \times 10^{-03}$	$9.97 \times 10^{-04}$	$1.69 \times 10^{-04}$	$2.06 \times 10^{-05}$	$6.26 \times 10^{-08}$	0.0	0.0
1.32	0.090 26	0.093 00	0.048 55	0.018 75	$5.76 \times 10^{-03}$	$1.34 \times 10^{-03}$	$2.46 \times 10^{-04}$	$3.32 \times 10^{-05}$	$1.85 \times 10^{-07}$	$7.2 \times 10^{-15}$	0.0
1.35	0.079 10	0.092 33	0.053 47	0.022 78	$7.75 \times 10^{-03}$	$2.00 \times 10^{-03}$	$4.13 \times 10^{-04}$	$6.44 \times 10^{-05}$	$7.17 \times 10^{-07}$	$9.2 \times 10^{-11}$	0.0
1.40	0.062 29	0.088 80	0.060 44	0.029 87	0.011 86	$3.59 \times 10^{-03}$	$8.89 \times 10^{-04}$	$1.74 \times 10^{-04}$	$4.73 \times 10^{-06}$	$3.49 \times 10^{-08}$	0.0
1.43	0.053 26	0.085 46	0.063 72	0.034 16	0.014 79	$4.89 \times 10^{-03}$	$1.34 \times 10^{-03}$	$2.98 \times 10^{-04}$	$1.33 \times 10^{-05}$	$2.97 \times 10^{-07}$	$4.7 \times 10^{-14}$
1.45	0.047 68	0.082 78	0.065 52	0.036 95	0.016 90	$5.93 \times 10^{-03}$	$1.73 \times 10^{-03}$	$4.16 \times 10^{-04}$	$2.54 \times 10^{-05}$	$9.07 \times 10^{-07}$	$1.7 \times 10^{-11}$
1.50	0.035 24	0.074 76	0.068 60	0.043 51	0.022 62	$9.15 \times 10^{-03}$	$3.06 \times 10^{-03}$	$8.86 \times 10^{-04}$	$1.01 \times 10^{-04}$	$7.34 \times 10^{-06}$	$1.08 \times 10^{-08}$
1.60	0.016 65	0.054 38	0.068 71	0.053 84	0.034 75	0.018 23	$7.65 \times 10^{-03}$	$2.91 \times 10^{-03}$	$6.98 \times 10^{-04}$	$1.08 \times 10^{-04}$	$4.58 \times 10^{-06}$
1.80	$1.30 \times 10^{-03}$	0.012 37	0.039 52	0.058 41	0.055 81	0.042 20	0.025 24	0.014 09	$6.34 \times 10^{-03}$	$2.13 \times 10^{-03}$	$5.07 \times 10^{-04}$