CTMRG study of an interacting-dimer model

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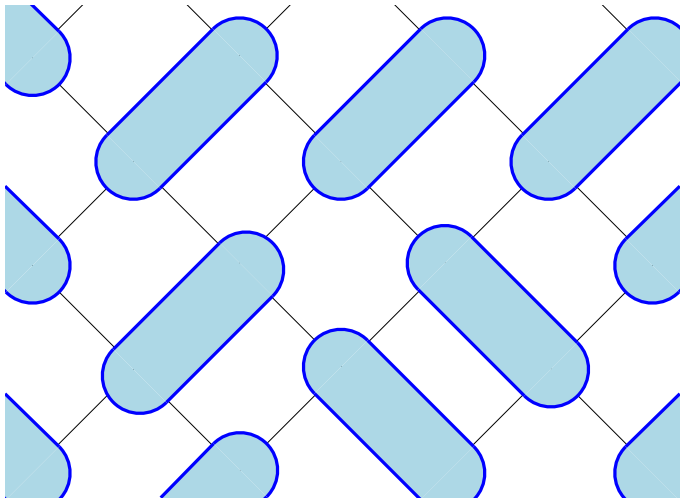
The model

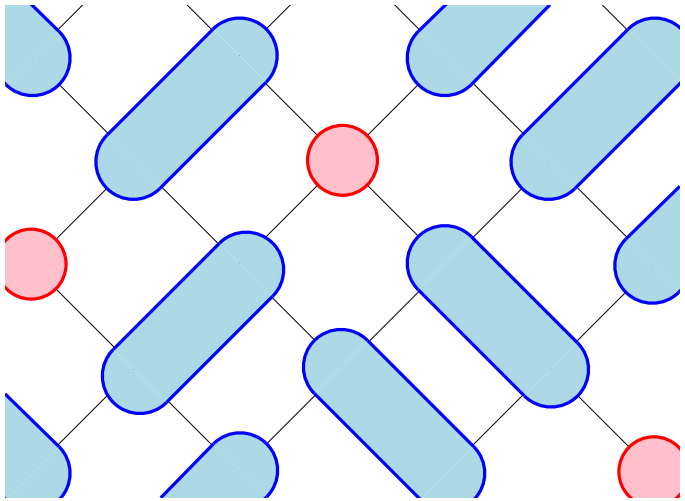
Non-overlapping dimers on the square lattice (diatomic molecules adsorbed on a surface, ground state of the Fully-Frustrated Ising model, ...)



The close-packed dimer model is exactly solvable (pfaffian).

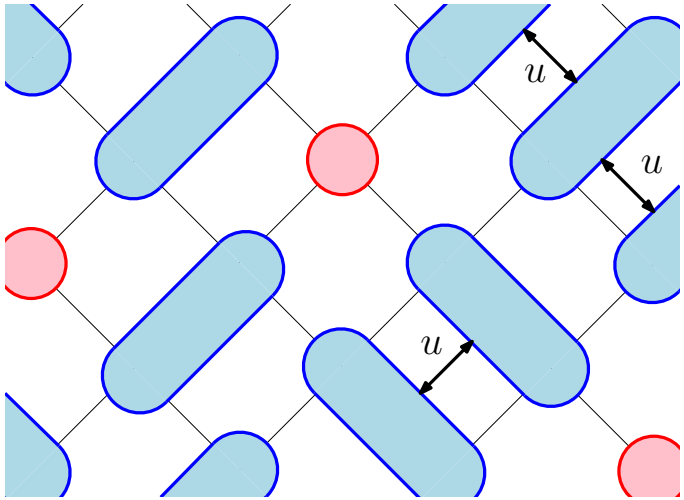
- macroscopic number of possible configurations,
- algebraic dimer-dimer correlation $C(r) \sim 1/r^2$.

Allow empty sites (or occupied by monomers) with a chemical potential μ.



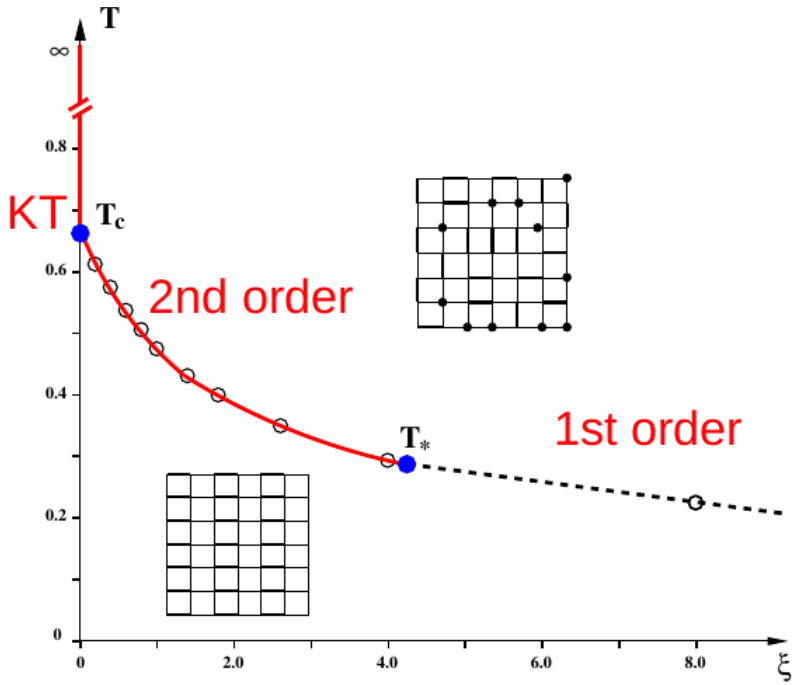
Algebraic monomer-monomer correlation $C(r) \sim 1/\sqrt{r}$
but no phase transition.

Interaction u between parallel dimers inside each plaquette.



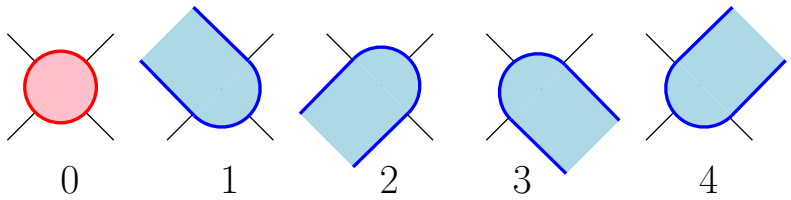
Monte Carlo simulations and transfer matrix calculations

(Alet, Ikhlef, Jacobsen, Misguich, Pasquier, 2006)



- Columnar low-temperature phase, critical high-temperature phase
- Kosterlitz-Thouless transition in the close-packed limit ($\mu \to -\infty$),
- Second order phase transition. Varying critical exponents (μ is marginal) as the Ashkin-Teller model, explained by a mapping onto the Coulomb gas.
- Tricritical point

Tensor decomposition of the partition function



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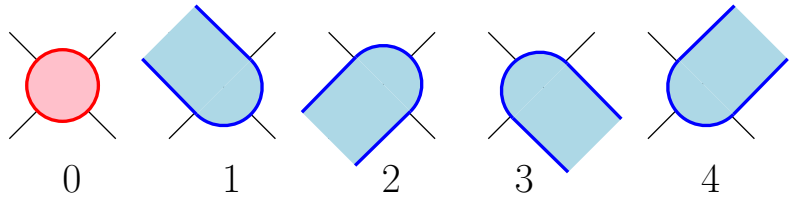

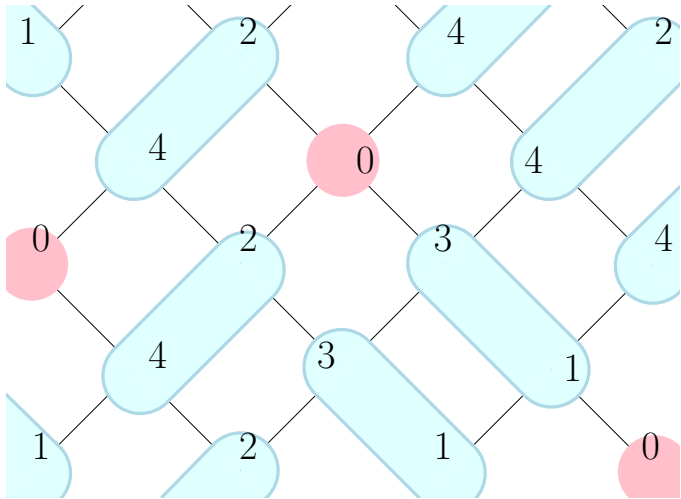

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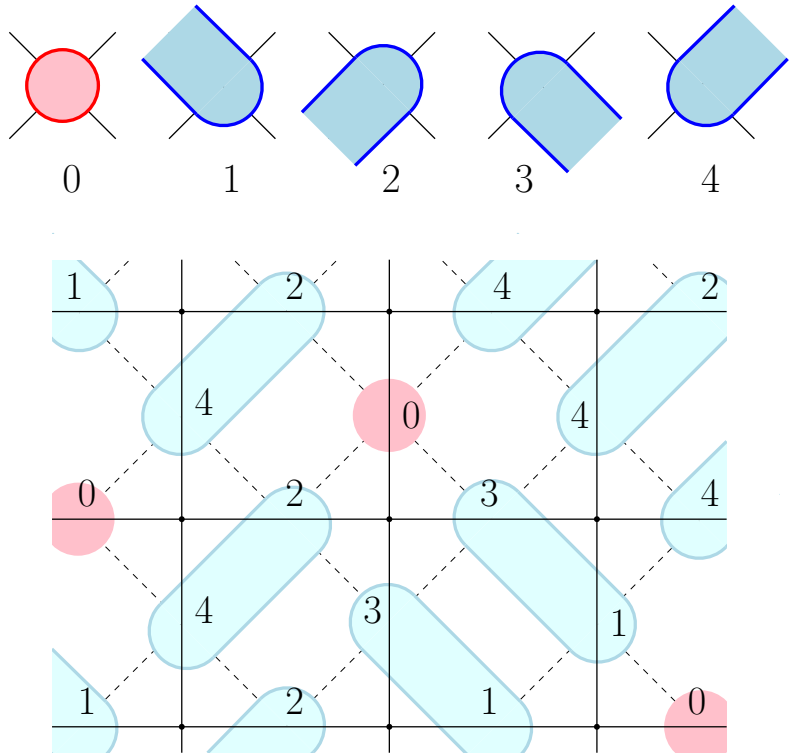

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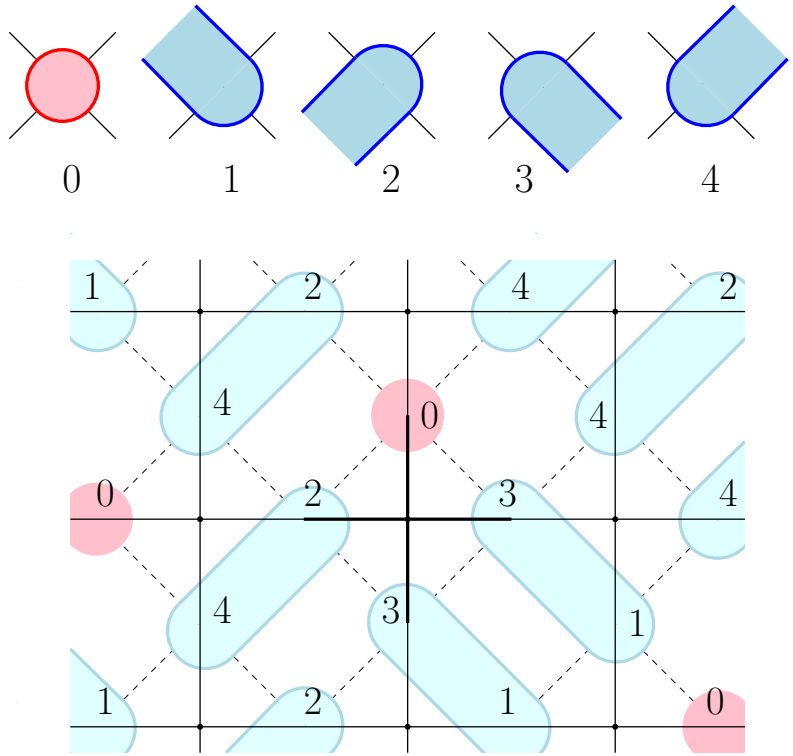

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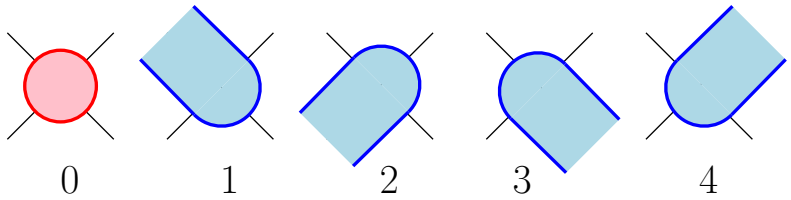
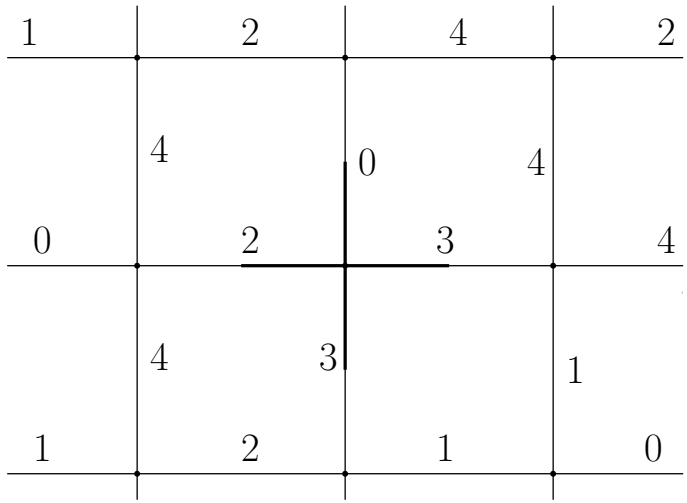
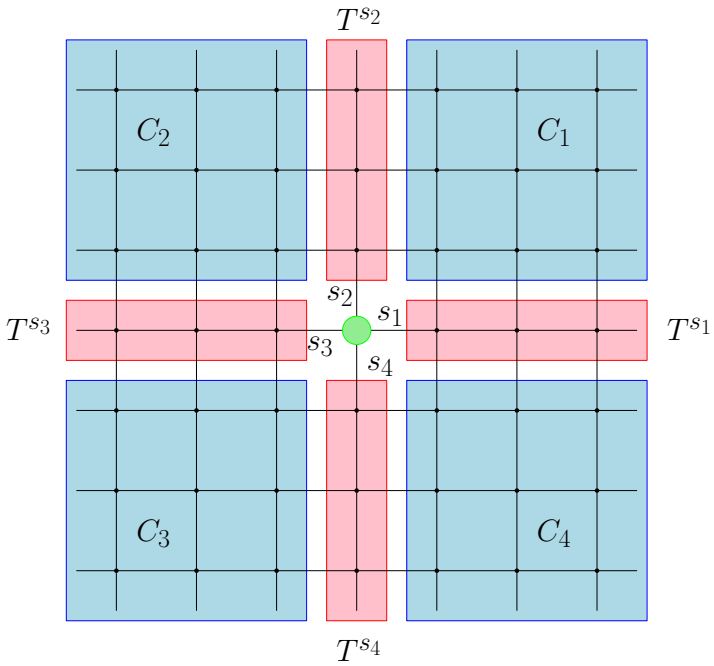




$$\mathcal{Z} = \sum_{\{s\}} \prod_{\alpha \in V} \prod_{\substack{i,j,k,l \\ \in E_\alpha}} w_{s_i s_j s_k s_l}$$

CTMRG algorithm



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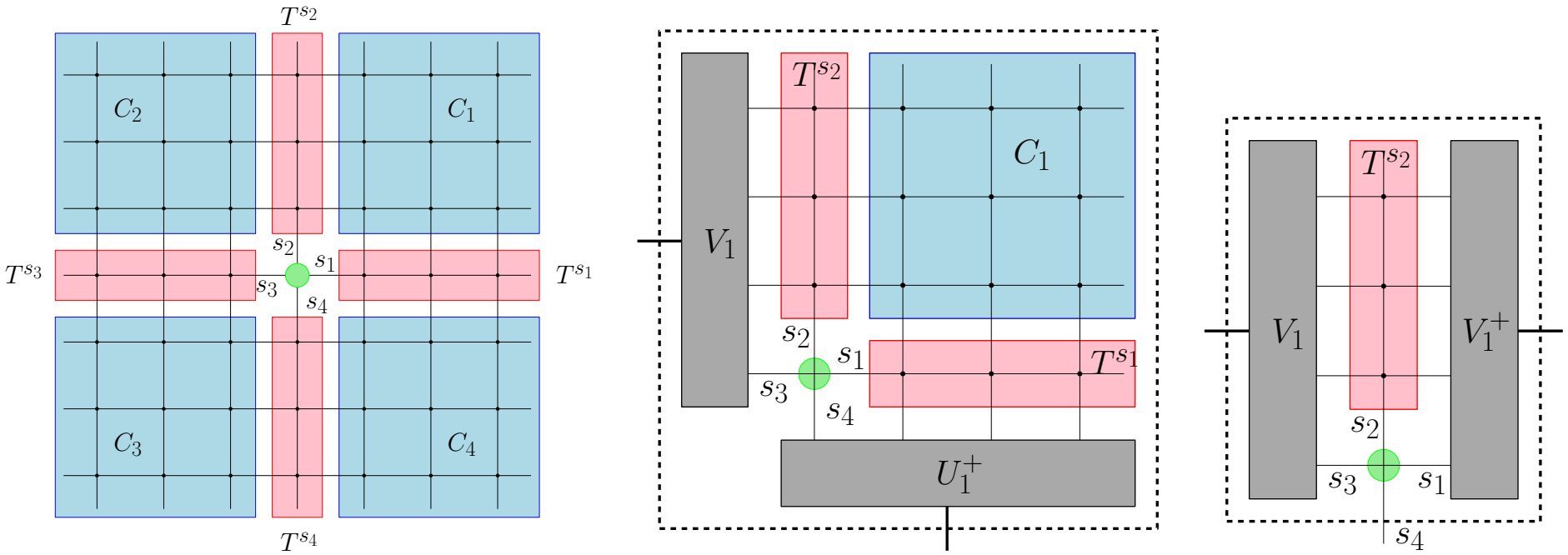
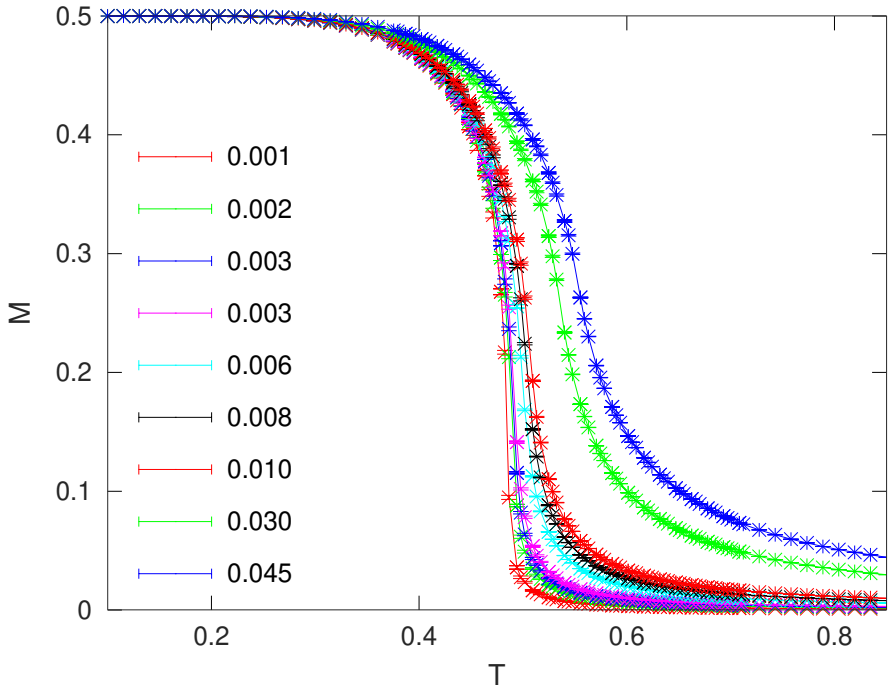


Singular Value Decomposition of the Corner Transfer Matrices and truncation.

The results

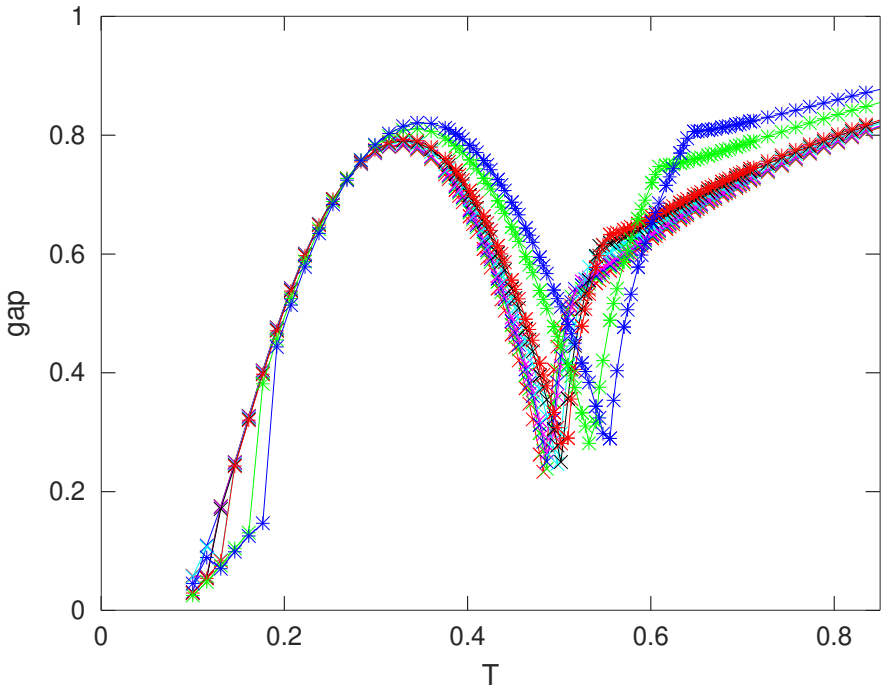
Due to truncation, Tensor Network algorithms are efficient away from criticality

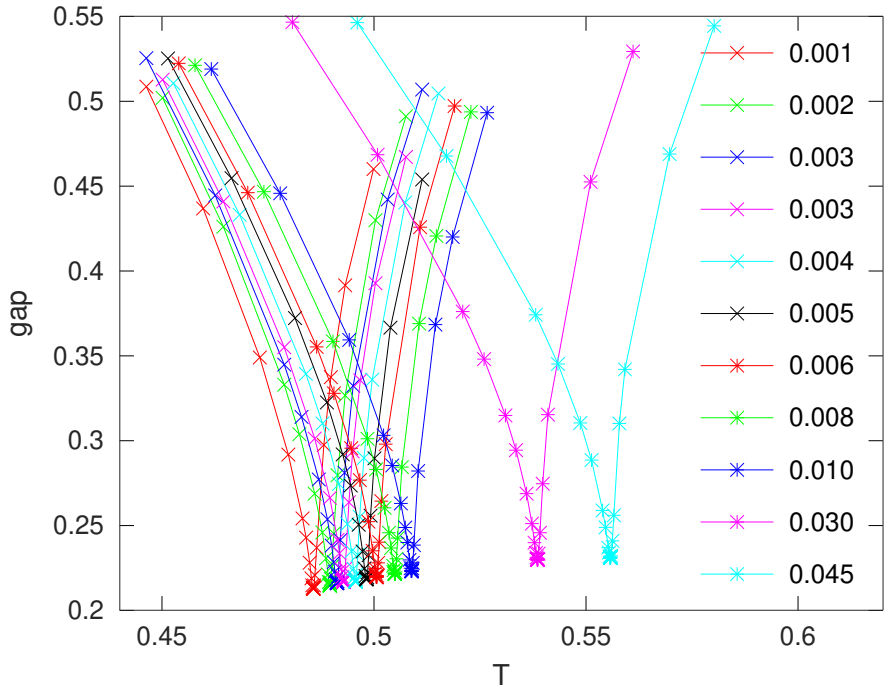


Order parameter (diff. of the number of horizontal and vertical dimers).
Rotation symmetry broken by a small difference $\Delta\mu$ of the chemical potentials of horizontal and vertical dimers.

Transition temperatures can be estimated from the two largest singular values of the Corner Transfer Matrix (similar to transfer matrix)

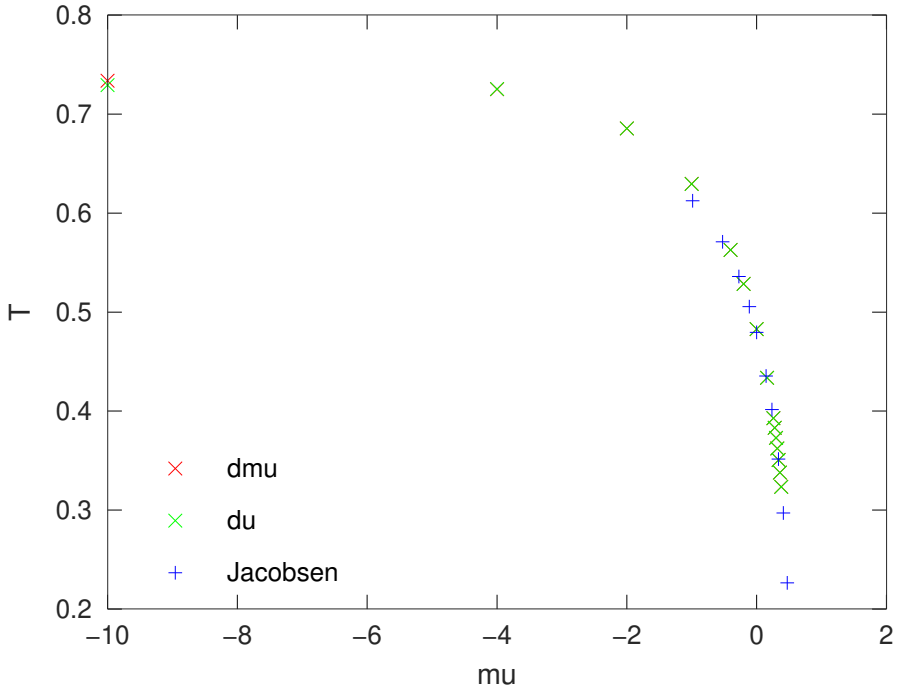
$$g = \ln \frac{\Lambda_2}{\Lambda_1}$$





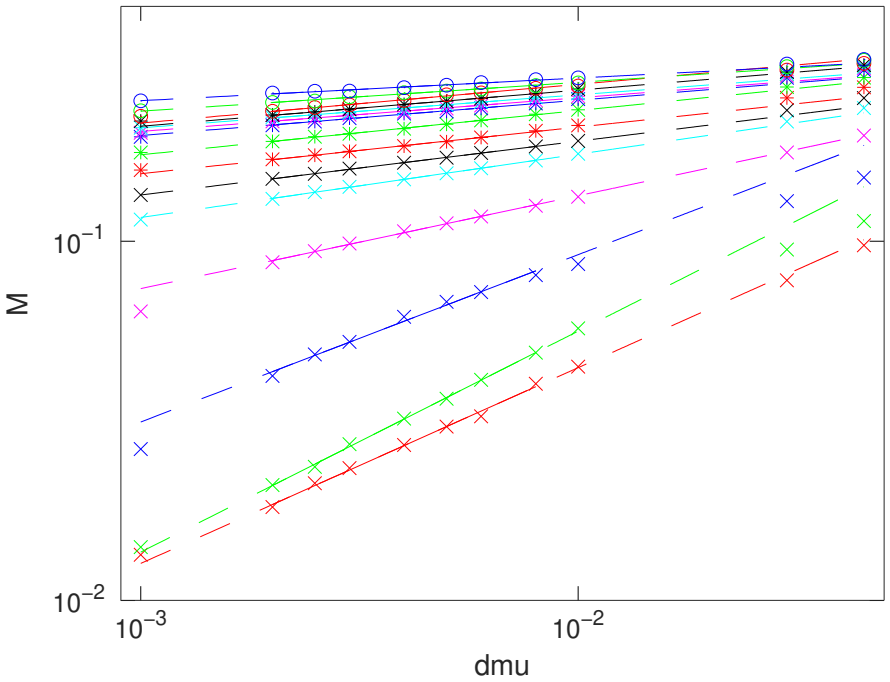
Extrapolate to the limit $\Delta\mu \to 0$.

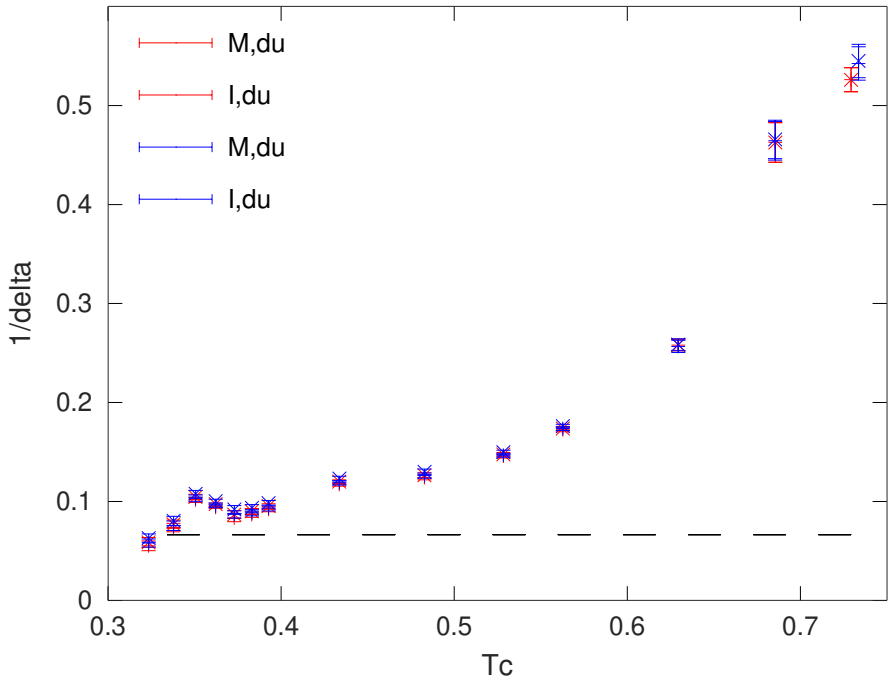
Good agreement with Alet et al. for large chemical potential μ



Scaling of the order parameter M with the symmetry-breaking field $\Delta\mu$

$$M \sim \Delta\mu^{1/\delta}$$





Conclusions

- No critical slowing down but systematic deviations due to truncation,
- Results are agreement with Alet et al. (Monte Carlo and transfer matrix) but so far not as accurate.