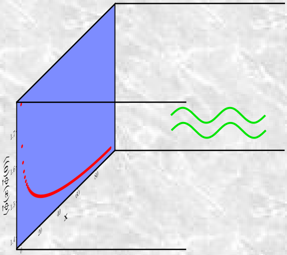
Universal finite-size scaling in the extraordinary-log phase



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Collaboration

Max Metlitski (MIT)

Boundary critical phenomena

In the vicinity of a second-order phase transitions:

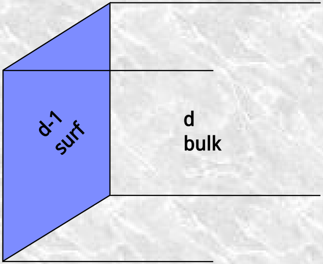
- Power-laws, universality
- Renormalization Group:

$$K \to \mathcal{R}(K) \to \mathcal{R}(\mathcal{R}(K)) \ldots$$

- Real systems: surfaces
- RG: bulk vs surface couplings
  Cardy book (1996)

$$\begin{array}{ll} & \nearrow K^{*(1)}_{\rm surf} \\ K^*_{\rm bulk} & \to K^{*(2)}_{\rm surf} \\ & \searrow \ldots \end{array}$$

- Also defects (planes, line, ...)



Reviews: Binder (1983); Diehl (1986)

A renewed interest

- Quantum spin models
  Bulk: classical O(3) universality class $d = 2 + 1$
  Boundary: unexpected exponents

  Suzuki, Sato (2012); Zhang, Wang (2017); Ding, Zhang, Guo (2018); Weber, FTP, Wessel (2018)

  Weber, Wessel (2019); Jian, Xu, Wu, Xu (2021); Zhu, Ding, Zhang, Guo (2021); Weber, Wessel (2021)

  Ding, Zhu, Guo, Zhang (2021); Wang, Zhang, Guo (2022); Sun, Lyu, Lv (2022)

  Yu, Huang, Song, Xu, Ding, Zhang (2022); Sun, Lv (2022); Xu, Peng, Xiong, Zhang (2022)

- Advancements in conformal field theory

  McAvity, Osborn (1995); Liend, Rastelli, van Rees (2013); Gliozzi, Liendo, Meineri, Rago (2015)

  Billò, Gonçalves, Lauria, Meineri (2016); Liendo, Meneghelli (2017); Lauria, Meineri, Trevisani (2018)

  Mazáč, Rastelli, Zhou (2019); Kaviraj, Paulos (2020); Dey, Hansen, Shpot (2020)

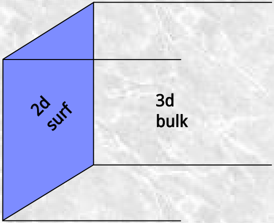
  Behan, Di Pietro, Lauria, van Rees (2020); Metlitski (2020); Gimenez-Grau, Liendo, van Vliet (2021)

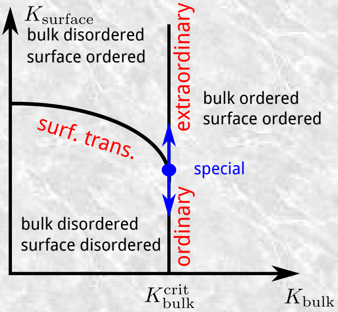
  Padayasi, Krishnan, Metlitski, Gruzberg, Meineri (2022); Krishnan, Metlitski (2023)

- Surface critical behavior of 3D $O(N)$ model
  not fully understood!

O(N) phase diagram $d = 3$: $N = 1$ (Ising)

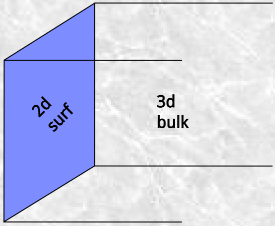
$$-\beta\mathcal{H} = K_{\text{bulk}} \sum_{\langle i\, j\rangle} S_i S_j + K_{\text{surface}} \sum_{\langle i\, j\rangle \in \text{surf.}} S_i S_j$$

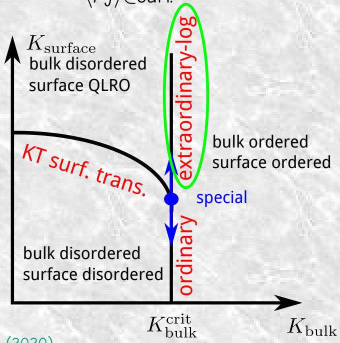




O(N) phase diagram $d = 3$: $N = 2$ (XY)

$$-\beta\mathcal{H} = K_{\text{bulk}} \sum_{\langle i\, j\rangle} \vec{S}_i \vec{S}_j + K_{\text{surface}} \sum_{\langle i\, j\rangle \in \text{surf.}} \vec{S}_i \vec{S}_j$$





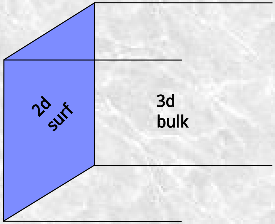
- New extraordinary-log phase: Metlitski, (2020)

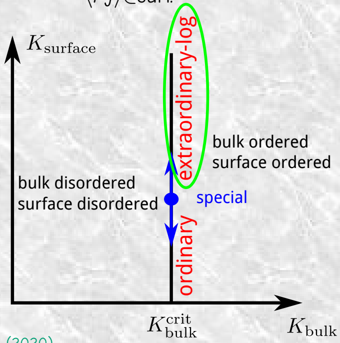
$$\langle \vec{\phi}(\mathsf{x}) \cdot \vec{\phi}(0) \rangle \sim \frac{1}{(\log \mathsf{x})^q}$$

- Ordinary UC: MC estimates FPT PRB 108, L020404 (2023)

O(N) phase diagram $d = 3$: $2 < N < N_c$

$$-\beta\mathcal{H} = K_{\text{bulk}} \sum_{\langle i\,j \rangle} \vec{S}_i \vec{S}_j + K_{\text{surface}} \sum_{\langle i\,j \rangle \in \text{surf.}} \vec{S}_i \vec{S}_j$$





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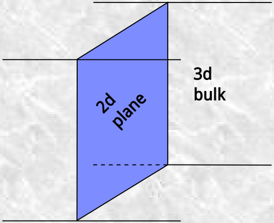
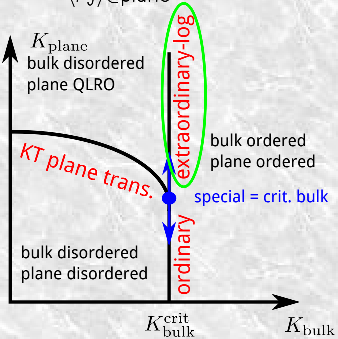
- Ordinary UC: MC estimates FPT PRB 108, L020404 (2023)
- $N = 3$ surface transition FPT, PRL 126, 135701 (2021)

O(N) phase diagram $d = 3$: $N = 2$ (XY)

$$-\beta\mathcal{H} = K_{\text{bulk}} \sum_{\langle i\, j\rangle} \vec{S}_i \vec{S}_j + K_{\text{plane}} \sum_{\langle i\, j\rangle \in \text{plane}} \vec{S}_i \vec{S}_j$$





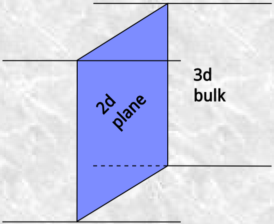
- Extraordinary-log phase: Metlitski, (2020); Krishnan, Metlitski (2023)

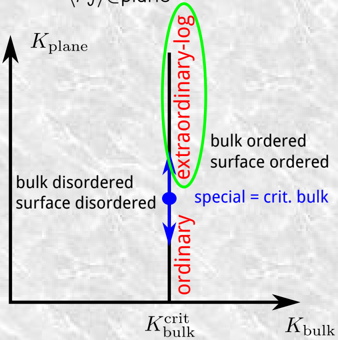
$$\langle \vec{\phi}(x) \cdot \vec{\phi}(0) \rangle \sim \frac{1}{(\log x)^q}$$

- Ordinary UC: MC estimates FPT PRB 108, L020404 (2023)

O(N) phase diagram $d = 3$: $N > 2$

$$-\beta\mathcal{H} = K_{\text{bulk}} \sum_{\langle i\, j\rangle} \vec{S}_i \vec{S}_j + K_{\text{plane}} \sum_{\langle i\, j\rangle \in \text{plane}} \vec{S}_i \vec{S}_j$$





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Extraordinary-log phase

- Boundary action:

$$S_{\rm bdy} = \int d^2x \frac{1}{2g} (\partial_\mu \vec{n})^2 \qquad \vec{n}^2 = 1$$

- RG-flow of g near $g^* = 0$

$$\frac{dg}{dl} = -\beta(g) = -\alpha g^2 + O(g^3)$$

$\alpha(N)$ determined by universal boundary OPE of *normal* UC Metlitski (2020)

- Log-power two-point function

$$\langle \vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0) \rangle \sim \frac{1}{(\log \mathbf{x})^q} \qquad q = \frac{N-1}{2\pi\alpha}$$

- Values of α from MC of *normal* UC FPT, Metlitski, PRL 128, 215701 (2022)

Finite-Size Scaling in the Extraordinary-log phase

- Logarithmic violation of FSS

$$U_4 - 1 \propto 1/(\ln L)^2$$

$$(\xi/L)^2 \simeq A + \alpha/(N-1)\ln L$$

$$\Upsilon L \simeq A + 4\alpha/N \ln L \quad \text{open b.c.}$$

$$\Upsilon L \simeq A + 2\alpha/N \ln L \quad \text{plane defect}$$

$$(U_4 - 1)\,(\xi/L)^4 \underset{L\to\infty}{\approx} \frac{0.00773}{N-1}$$

$U_4 = \langle M_s^4 \rangle / \langle M_s^2 \rangle^2$: Binder ratio
ξ: finite-size correlation length
Υ: helicity modulus (spin stiffness)

MC results *normal* UC

FPT, Metlitski, PRL 128, 215701 (2022)

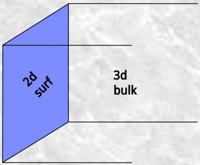
	α	
N	open BCs	plane defect
2	0.300(5)	0.600(10)
3	0.190(4)	0.540(8)

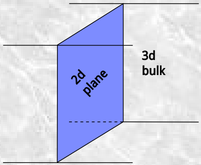
- On a standard fixed point: U_4, ξ/L, $\Upsilon L \sim$ const

Model

- ϕ^4 $N = 3$ lattice model, $L \times L \times L$ lattice, open b.c. or plane defect

$$\mathcal{H} = -\beta \sum_{\langle i\, j \rangle} \vec{\phi}_i \cdot \vec{\phi}_j - \beta_s \sum_{\langle i\, j \rangle \in \partial} \vec{\phi}_i \cdot \vec{\phi}_j + \sum_i [\vec{\phi}_i^{\,2} + \lambda(\vec{\phi}_i^{\,2} - 1)^2]$$



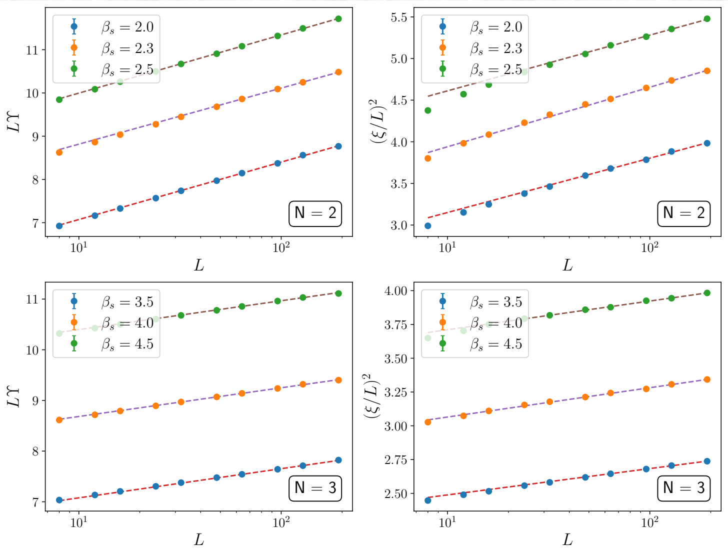


- $N = 2, 3$, $\lambda = \lambda^*$ *improved* Hasenbusch (2020), FPT (2022)
  $\rightarrow$ leading *bulk* scaling corrections suppressed

- Fix $\lambda = \lambda^*$, $\beta = \beta_c(\lambda^*)$, and β_s in the extraordinary-log phase

Goal: study FSS

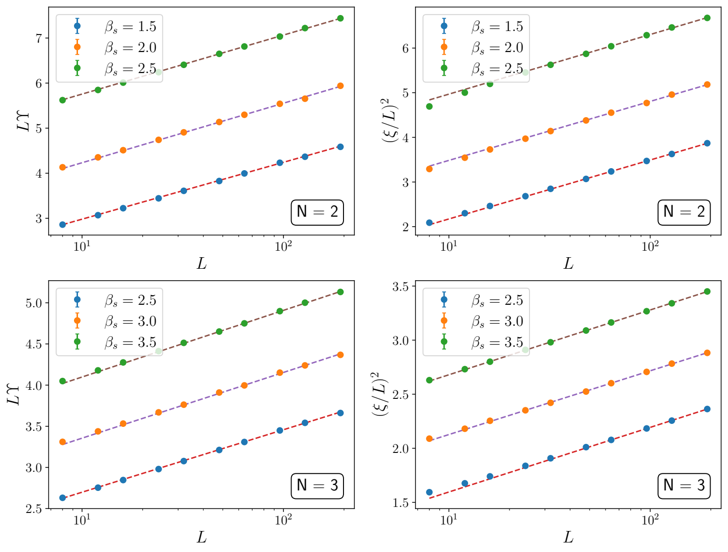
MC results: open b.c.



Fits: $\alpha(N = 2) \approx 0.28 - 0.31 \quad \alpha(N = 3) \approx 0.18 - 0.19$

Normal UC: $\alpha(N = 2) = 0.300(5) \quad \alpha(N = 3) = 0.190(4)$

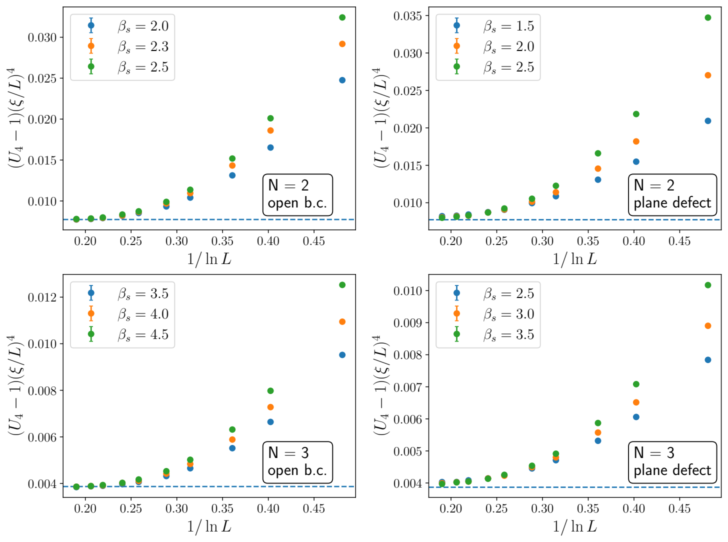
MC results: plane defect



Fits: $\alpha(N = 2) \approx 0.55 - 0.58 \quad \alpha(N = 3) \approx 0.49 - 0.53$

Normal UC: $\alpha(N = 2) = 0.600(10) \quad \alpha(N = 3) = 0.540(8)$

MC results: combination of RG-invariants



$$(U_4 - 1)\,(\xi/L)^4 \underset{L\to\infty}{\approx} \frac{0.00773}{N-1}$$

Summary

- 3D O(N) with a 2D surface ($N < N_c$) or plane defect (any N): new "Extraordinary-log" phase at strong boundary coupling
- Power-log correlations and logarithmic Finite-Size Scaling
- Field theory: universal parameter $\alpha(N, \text{b.c.})$ from the *normal* UC
- MC simulations: good *quantitative* agreement with predicted log-dependence of FSS quantities

Summary

- 3D $O(N)$ with a 2D surface ($N < N_c$) or plane defect (any N): new "Extraordinary-log" phase at strong boundary coupling
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Thank you!