Extending the dynamic range by an ensemble of neural networks

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MAX-PLANCK-GESELLSCHAFT

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Living organisms have to discriminate sensory stimuli that cover many orders of magnitude

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light intensity

$> 10^5$lx	too bright
10^4lx	daylight
10^2lx	indoor lighting
1lx	moonlight
< 0.01lx	starlight

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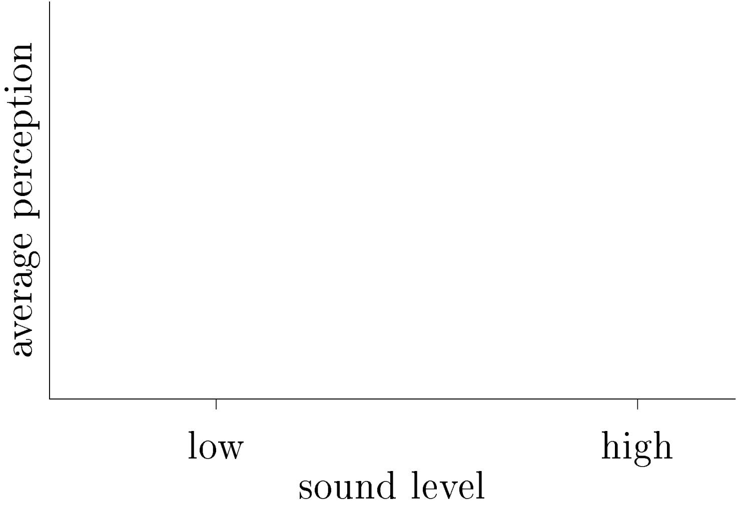
sound level

plane takeoff	> 100dB
busy city traffic	70dB
restaurant	60dB
quiet speech	50dB
rustle of leaves	20dB

What is the dynamic range?

Psychometric response curve to sound level:



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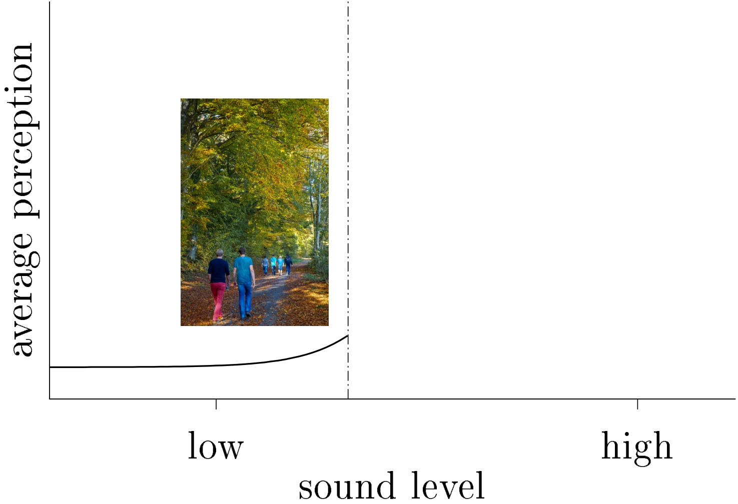

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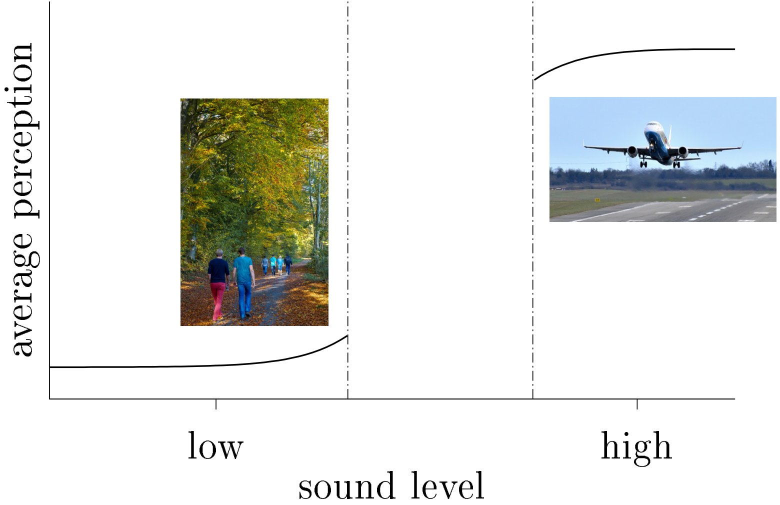

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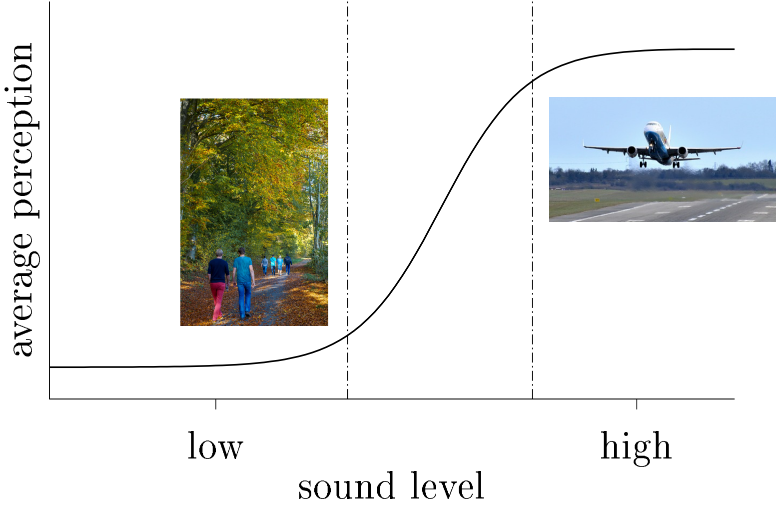

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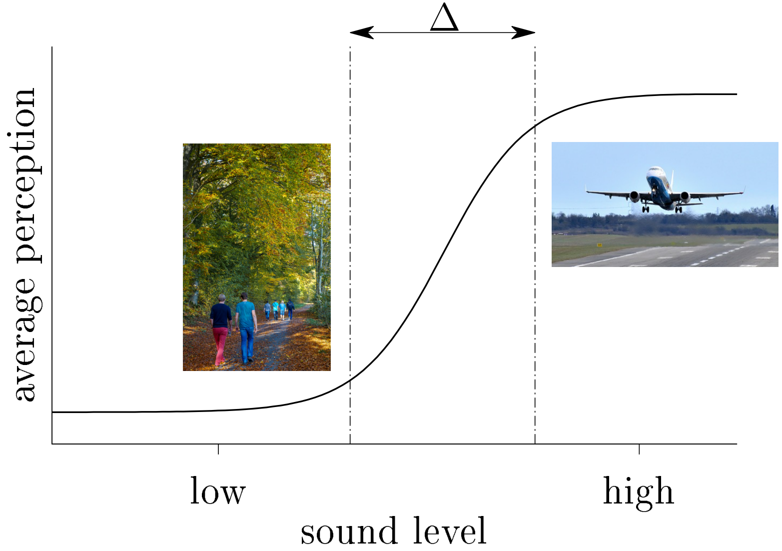

From perception to neural population response

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- correlation:
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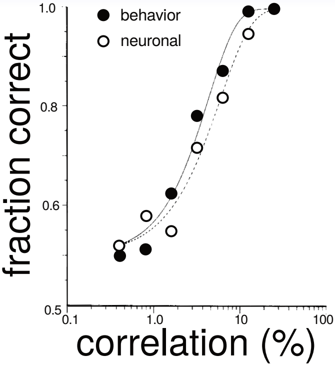

motion discrimination
in monkey visual cortex (MT)

Britten et al., J. Neurosci. (1992)

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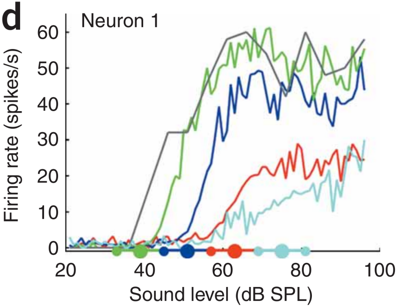

sound level discrimination
in auditory midbrain of guinea pig

Britten et al., J. Neurosci. (1992)
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From perception to neural population response

- correlation:
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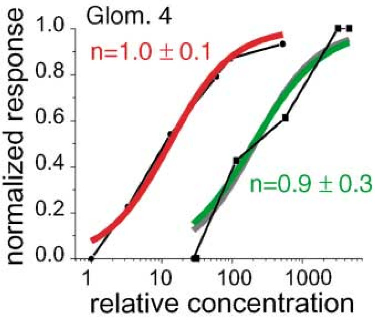
- single neuron response
  adjusted to input statistics
- population of neurons
  increases dynamic range



odor discrimination
in mouse olfactory bulb glomeruli

Britten et al., J. Neurosci. (1992)
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Wachowiak & Cohen, Neuron (2001)

Neural population dynamics viewed as a branching process

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Time discrete (Δt) activity propagation with external drive (H):

$$\langle A_{t+1} | A_t \rangle = m A_t + H$$

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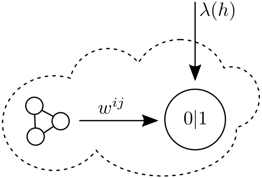

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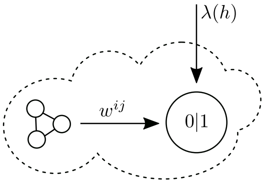

- recurrent excitation:
  - connectivity matrix w^{ij}
  - $P_{\mathrm{rec}}[s^j_{t+1} = 1 | s^i_t = 1] = w^{ij}$

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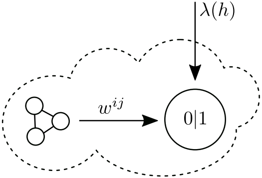

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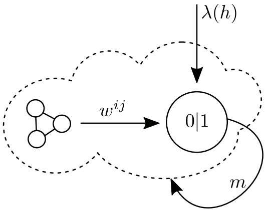

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$$m = \mathbb{E}_i \left(\sum_j w^{ij} \right)$$

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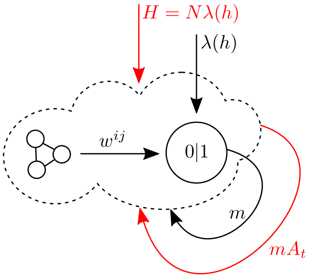

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- network size N

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Branching network exhibits non-equilibrium phase transition

Consider all-to-all connected network (mean-field)

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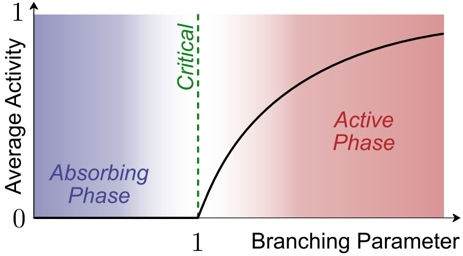
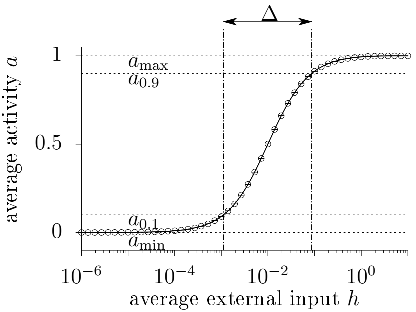


Muñoz, Rev. Mod. Phys. (2018)

Dynamic range is maximal at non-equilibrium phase transition

$$\Delta = 10 \log_{10} [h(a_{0.9})/h(a_{0.1})]$$



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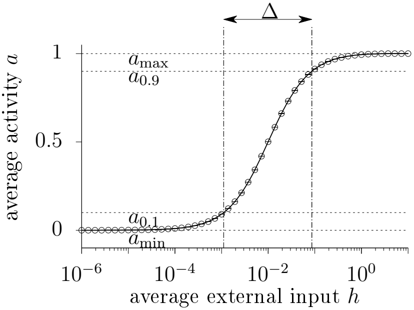

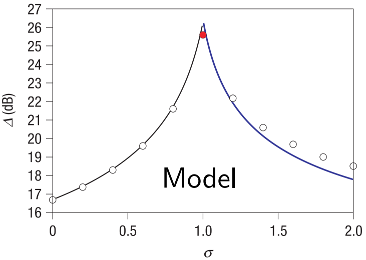

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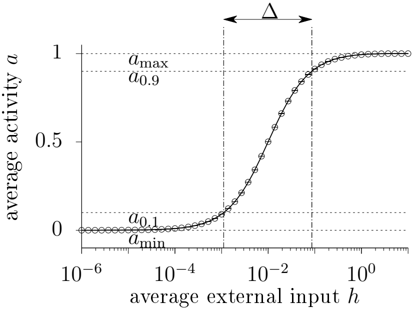

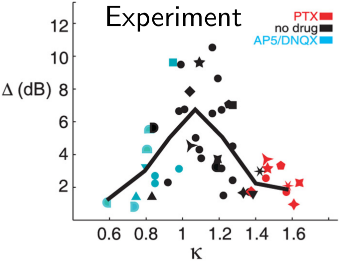

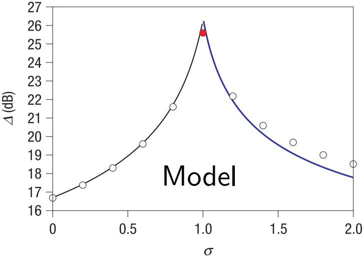

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Shew et al., J. Neurosci. (2009)

Dynamic range vs discriminable interval

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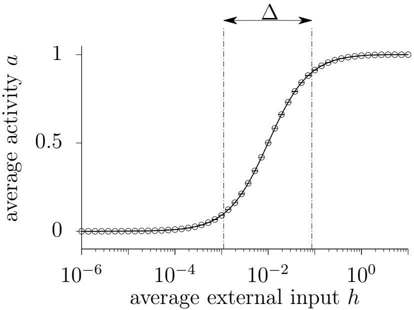

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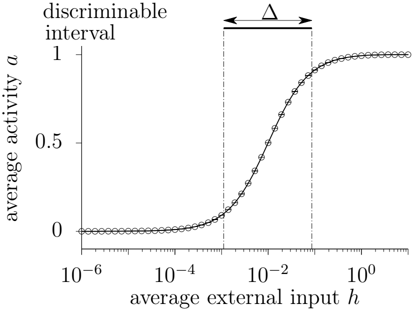

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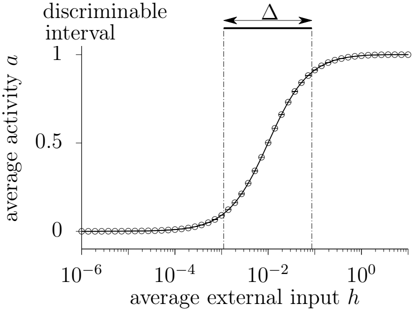

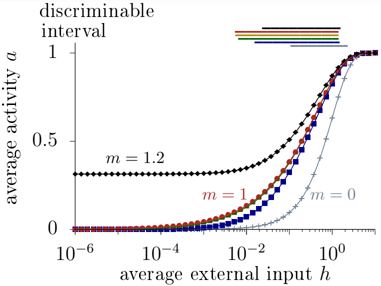

Branching network:
analytic and numeric

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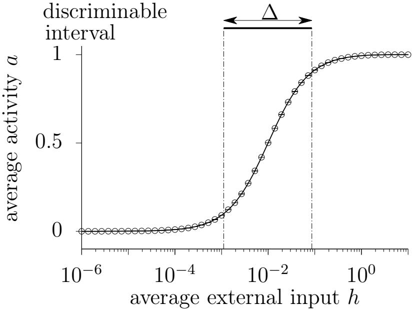

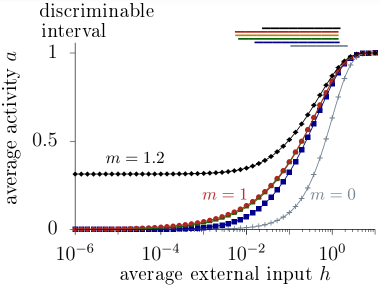

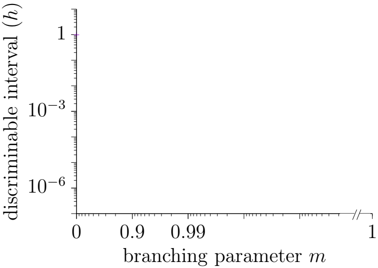

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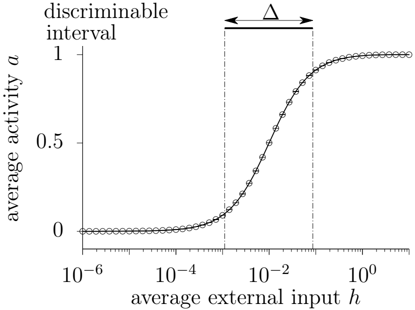
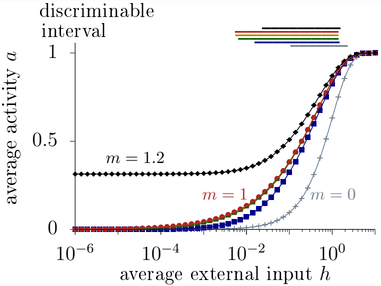
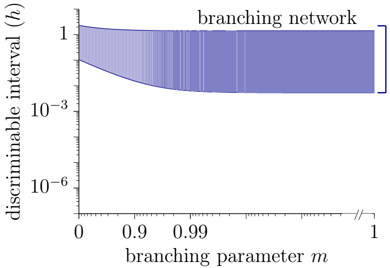
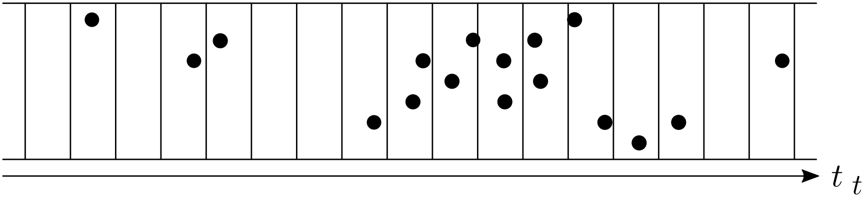






Constraint discriminable interval as a result of coalescence



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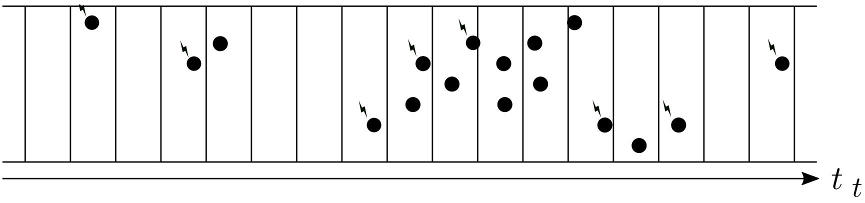

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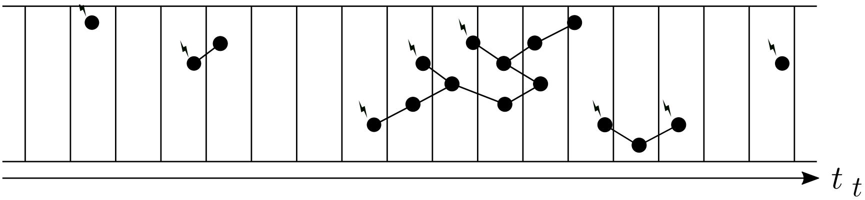

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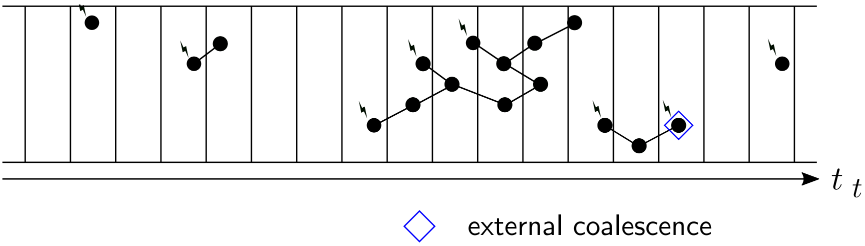

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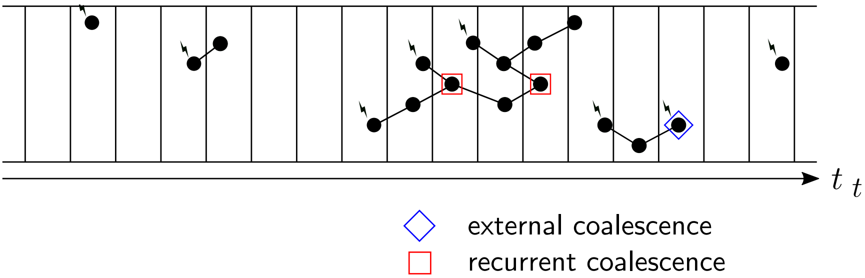
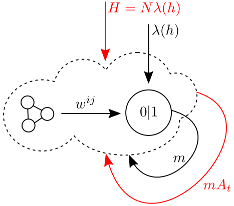


Coalescence can be compensated by adaptive synaptic weights

Motivation to approximate a branching process



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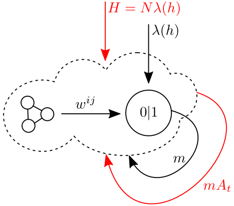

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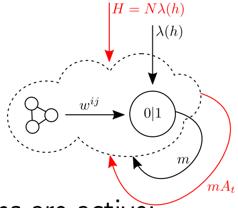

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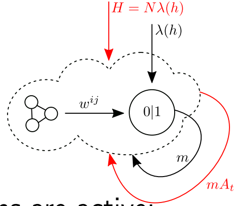

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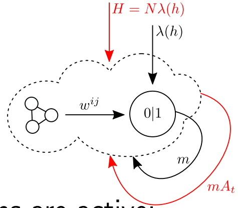

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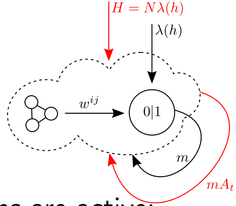

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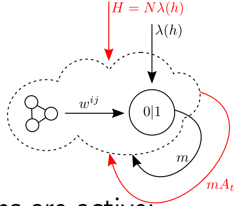

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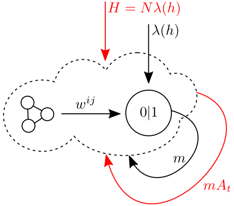

To compensate coalescence events we need adaptive synaptic weights

$$\widetilde{w}_{\text{cc}}(A_t) = 1 - \left(1 - \frac{mA_t}{N(1-\lambda(h))}\right)^{1/A_t}$$

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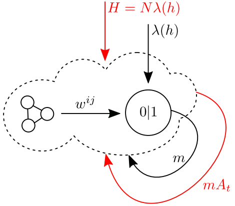
$$P_{\mathrm{rec}}[s_{t+1}^{j} = 1 | s_t^i = 1] = \cancel{w = \frac{m}{N}}$$
$$P_{\mathrm{ext}}[s_{t+1}^{i} = 1] = \lambda(h)$$



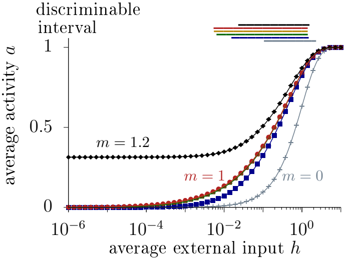
To compensate coalescence events we need adaptive synaptic weights

$$\widetilde{w}_{\mathrm{cc}}(A_t) = 1 - \left(1 - \frac{mA_t}{N(1-\lambda(h))}\right)^{1/A_t}$$

But the network cannot have access to h

$$w_{\mathrm{cc}}(A_t) = 1 - \left(1 - \frac{mA_t}{N}\right)^{1/A_t}$$

Coalescence-compensated network has specific discriminable intervals



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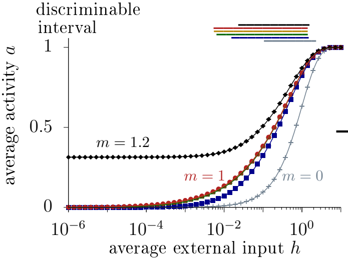

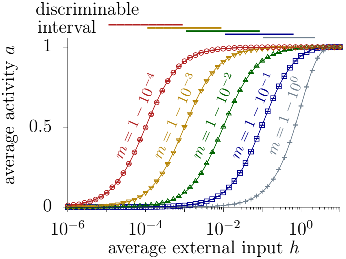

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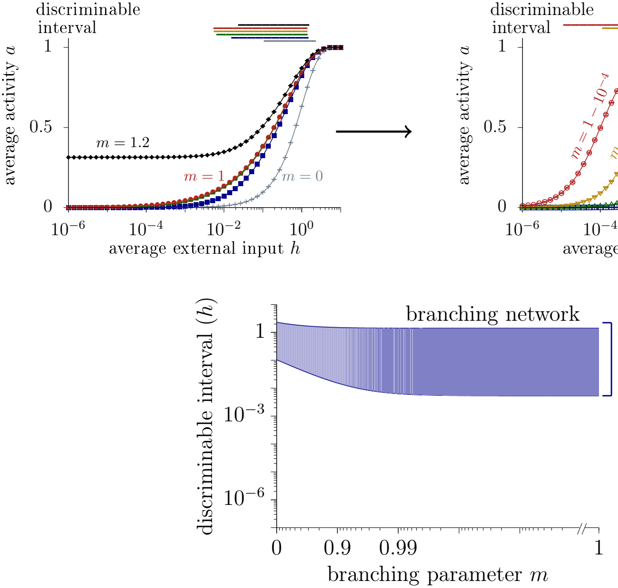

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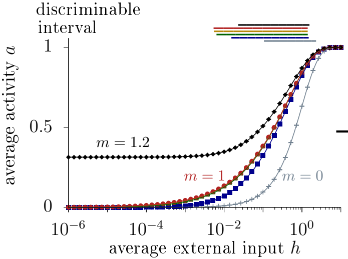
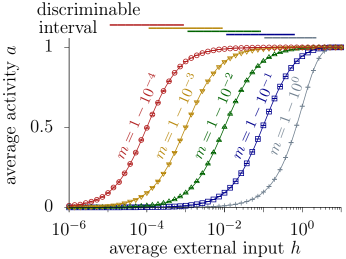
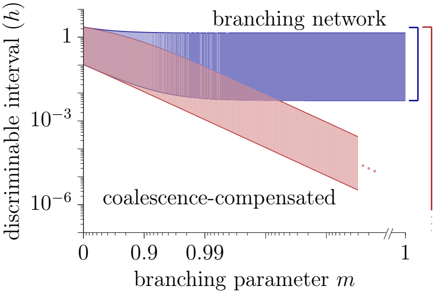
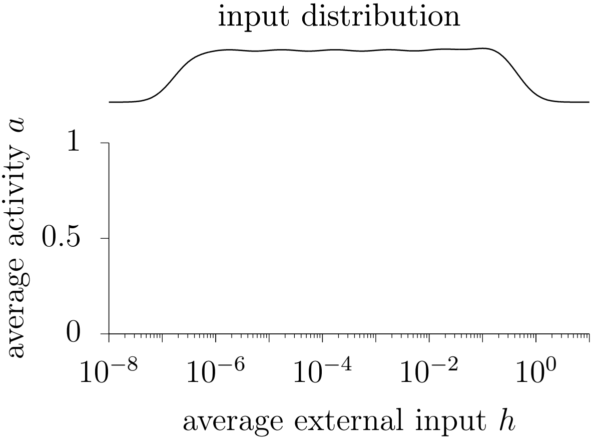






Ensemble of networks: Tailor neural response to task



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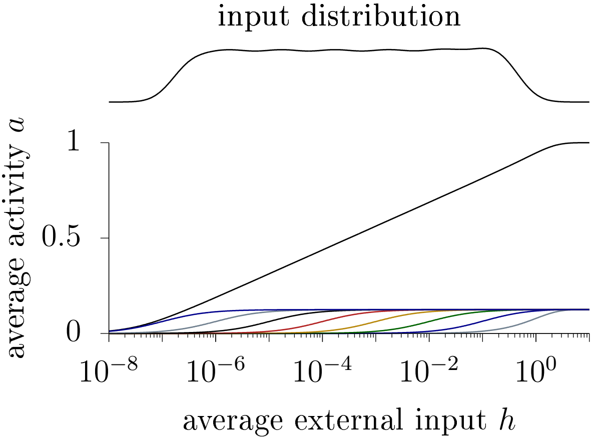

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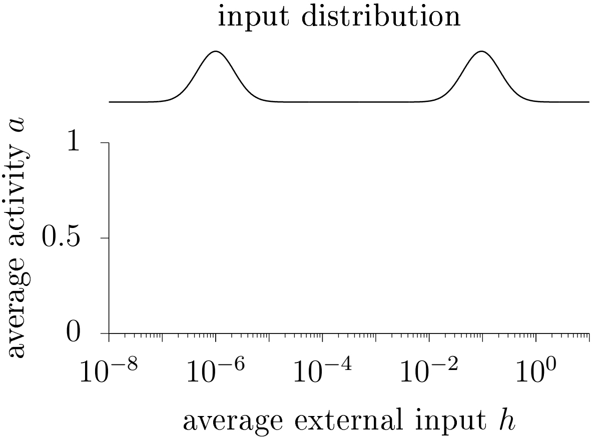

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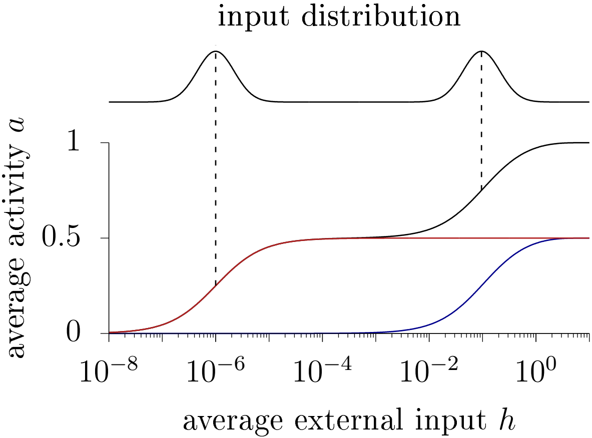


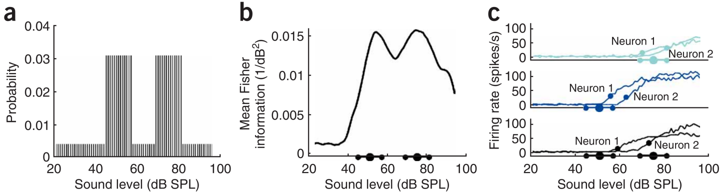
Applications

Tailored neural response to input statistics

Dean et al., Nat. Neurosci., 2005

Tailored neural response to input statistics

- is implemented, e.g., in auditory midbrain of guinea pigs

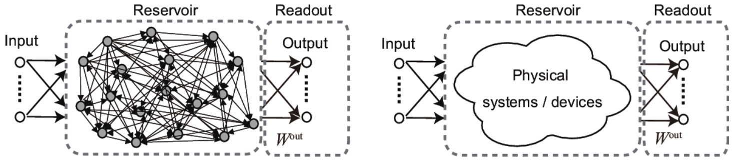


Dean et al., Nat. Neurosci., 2005

Applications

Tailored neural response to input statistics

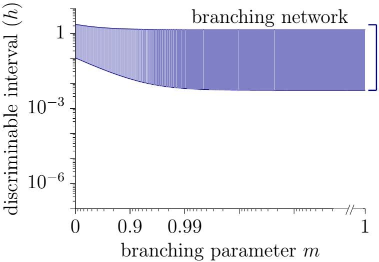
- is implemented, e.g., in auditory midbrain of guinea pigs
- may be beneficial for reservoir computing applications



(a) Conventional RC (b) Physical RC

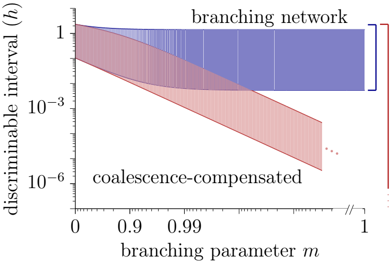
Dean et al., Nat. Neurosci., 2005
Tanaka et al., arxiv, 2018

Summary



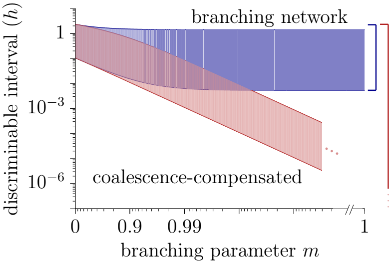
- Discriminable interval is constraint in branching network

Summary



- Discriminable interval is constraint in branching network
- Compensating coalescences leads to specific discriminable intervals

Summary



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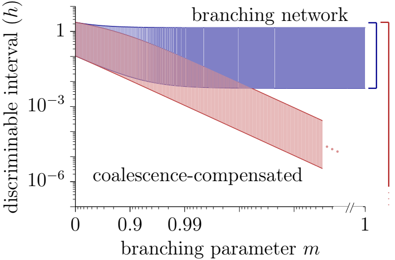
Summary



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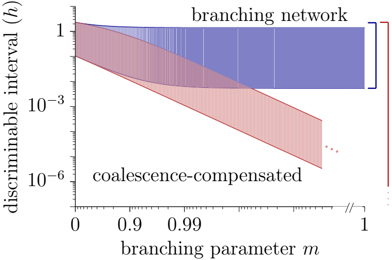
Announcement:
Focus session on "**Collective dynamics in neural networks**" at upcoming DPG Spring meeting (31.03.-05.04.2019)

Summary



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MAX-PLANCK-GESELLSCHAFT

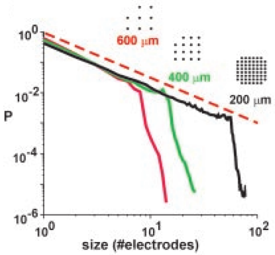
Bernstein Centers Göttingen

Appendix

Neural avalanches in experiments and branching model

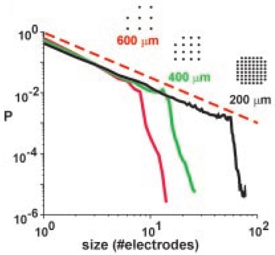
Neural avalanches in experiments and branching model

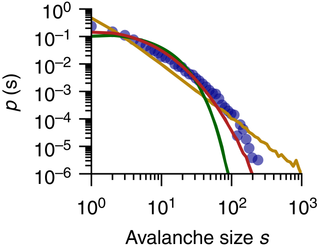
Experiment:



Neural avalanches in experiments and branching model

Experiment:





Neural avalanches in experiments and branching model

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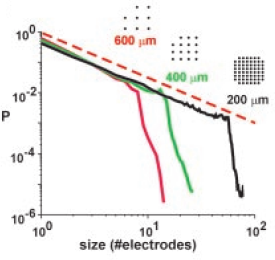
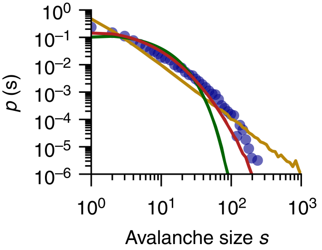
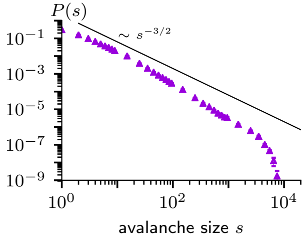
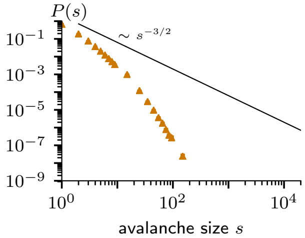




Model:





Neural avalanches in coalescence compensated network

At criticality: