Inferring dynamical properties of subsampled neural networks

F. Paul Spitzner, Jonas Dehning, Jens Wilting, and Viola Priesemann

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November 30, 2018

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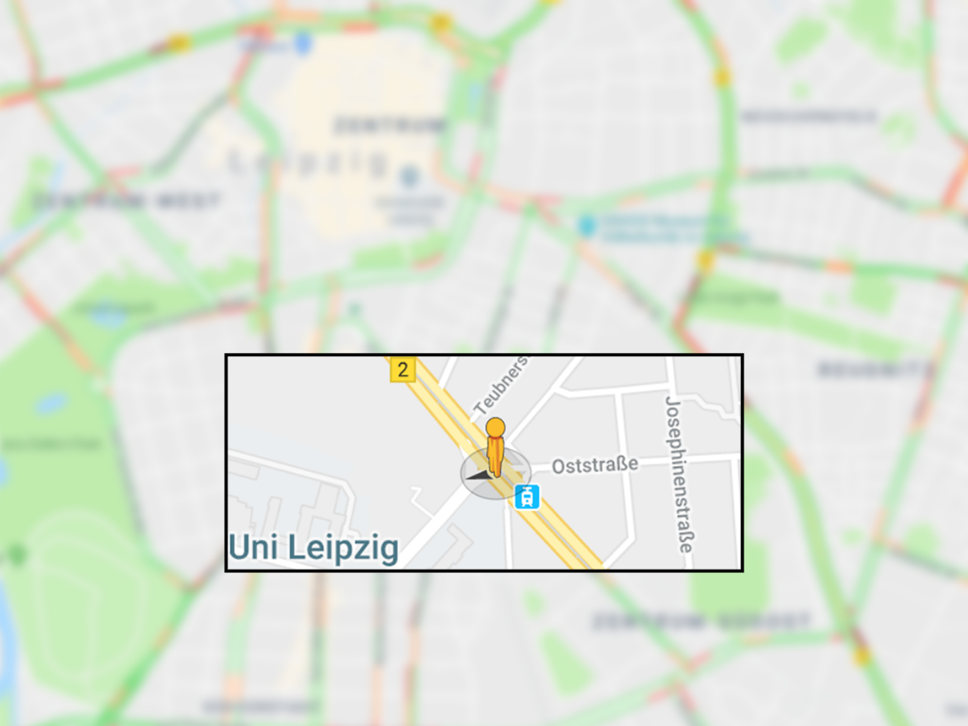
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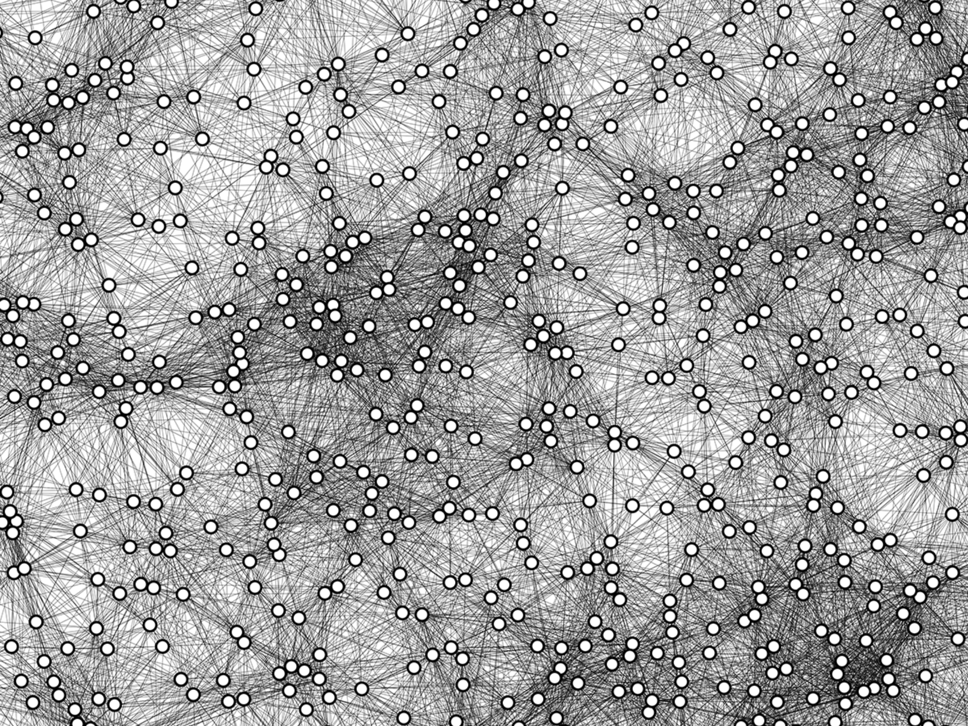
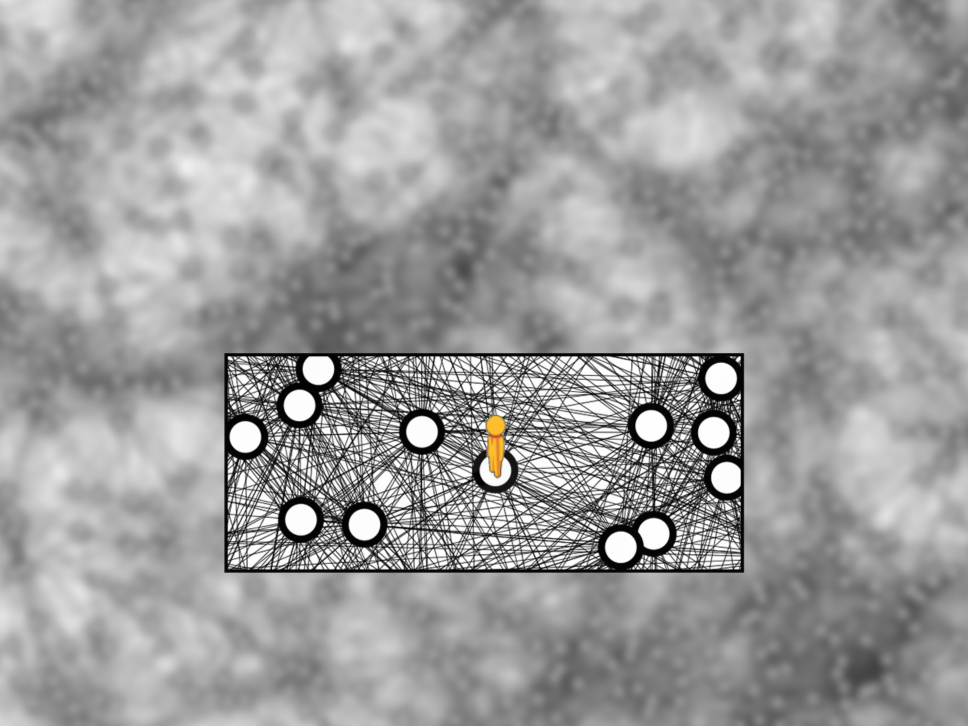
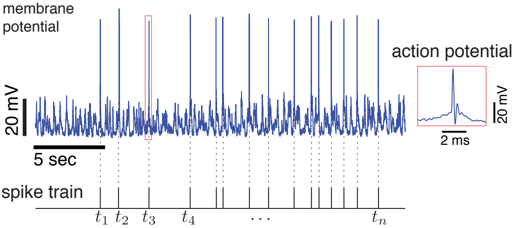
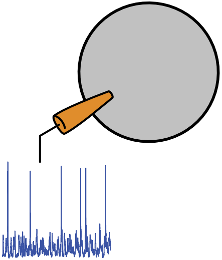
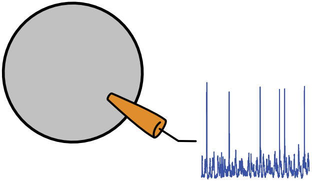




Activity propagates with Δt







I. M. Park et al., IEEE Signal Process. Mag. 30, 149 (2013)

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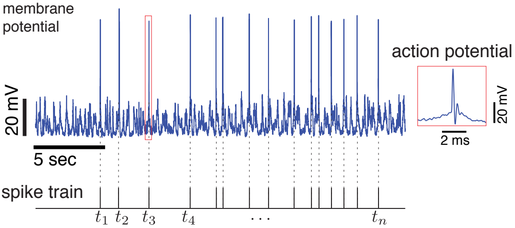

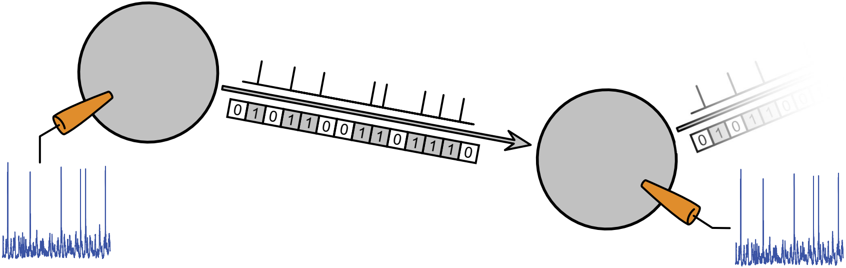

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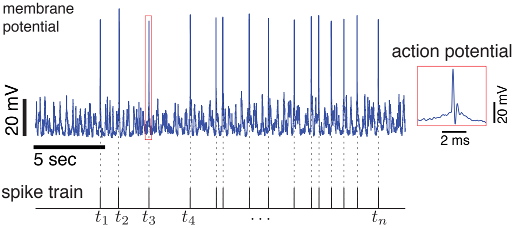
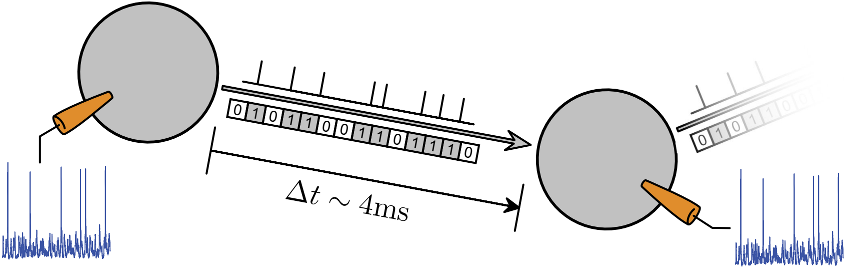
Activity propagates with Δt





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How to measure activity propagation?

- Average activity $A_t = \sum_{\text{active neurons}}$

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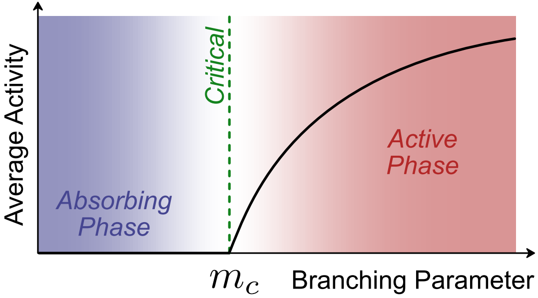
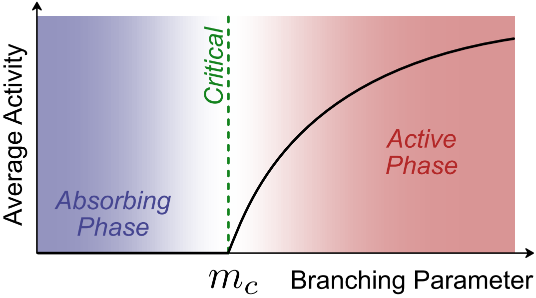


M. A. Muñoz, Rev. Mod. Phys. 90, 031001 (2018)

How to measure activity propagation?

- Average activity $A_t = \sum_{\text{active neurons}}$
- Average number of neurons each active neuron excites recurrently: *Branching Parameter* m
- Many connections $\leftrightarrow$ Mean field: $m_c = 1$



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Example model: $1\mathrm{d}$ Branching Network

- Neuron $\rightarrow$ node with state 0 or 1
- Each node connected to two other nodes (neighbours)

E. Domany and W. Kinzel, Phys. Rev. Lett. 53, 311 (1984)
H. Hinrichsen, Adv. Phys. 49, 815 (2000)

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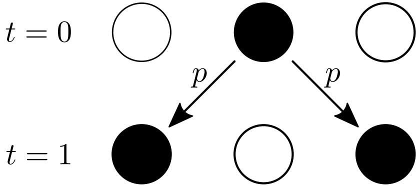
- Neuron $\rightarrow$ node with state 0 or 1
- Each node connected to two other nodes (neighbours)
- Time step:
  - Activate each neighbour with probability p
  - Become inactive (unless activated by a neighbour)



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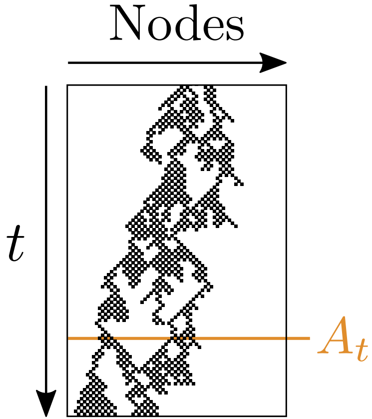
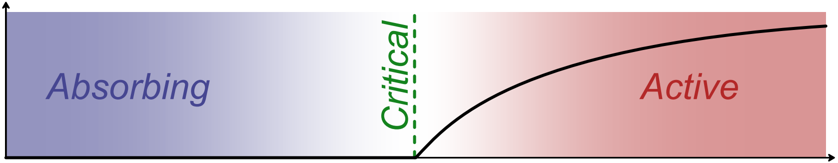
From 1d Network to Branching Process



From 1d Network to Branching Process

$$m = m_c$$



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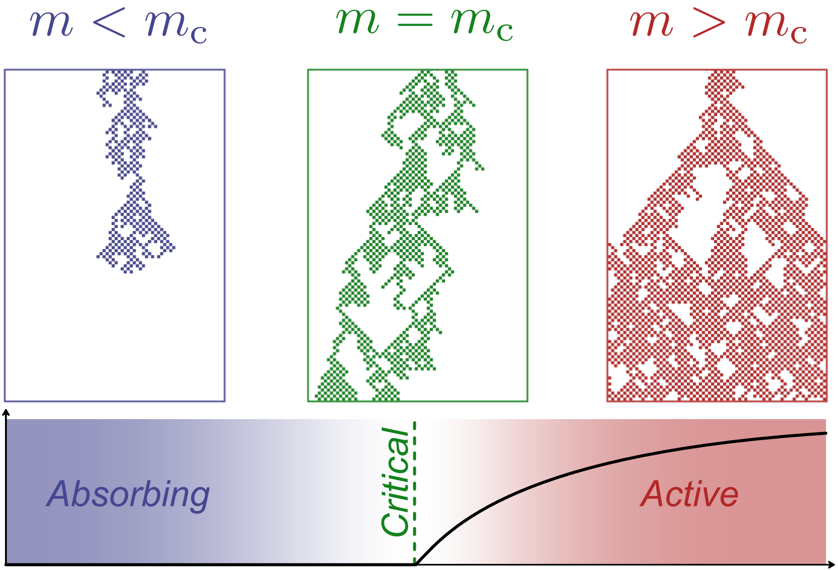
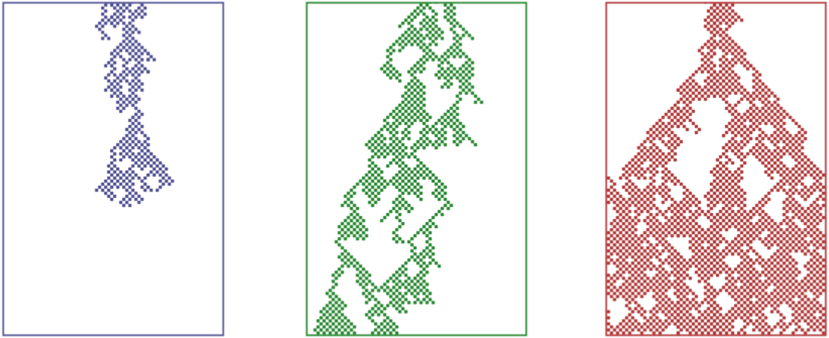


From 1d Network to Branching Process

$m < m_c$ $\qquad$ $m = m_c$ $\qquad$ $m > m_c$

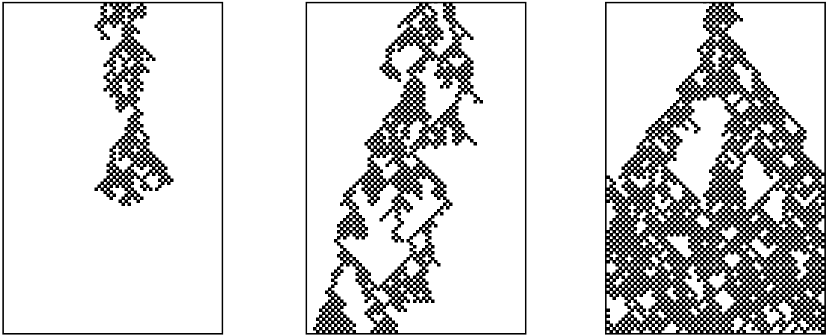


Mean field ($m_c = 1$):
$\rightarrow$ Branching process,
observed in experiments

$$\langle A_{t+1} | A_t \rangle = m A_t + H$$

Subsampling

$m < m_c$ $\quad$ $m = m_c$ $\quad$ $m > m_c$



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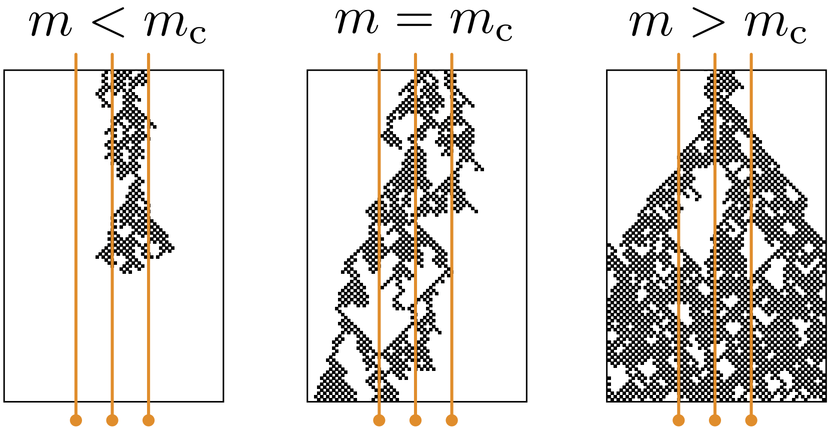

Sampling only a fraction of the system: $\alpha = \frac{n}{N}$

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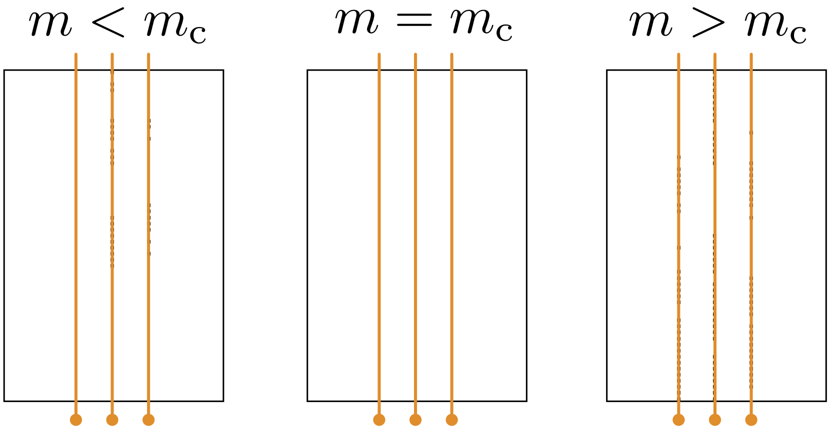

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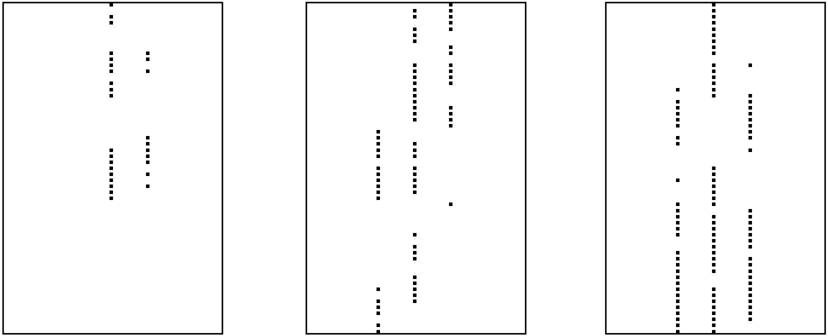
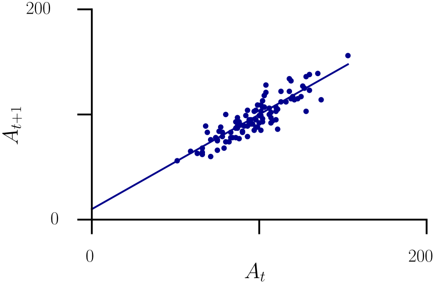


Sampling only a fraction of the system: $\alpha = \frac{n}{N}$

$$A_t \quad \rightarrow \quad a_t \approx \alpha A_t$$

Estimating m under Subsampling



$$\langle A_{t+1} | A_t \rangle = m A_t + H$$

$$\hat{m} = \frac{\sum_t (x_t - \overline{x})(y_t - \overline{y})}{\sum_t (x_t - \overline{x})^2}$$

$$x_t = A_t \qquad y_t = A_{t+1}$$

J. M. Beggs and D. Plenz, J. Neurosci. 23, 11167 (2003)
J. Wilting and V. Priesemann, Nat. Commun. 9, 2325 (2018)

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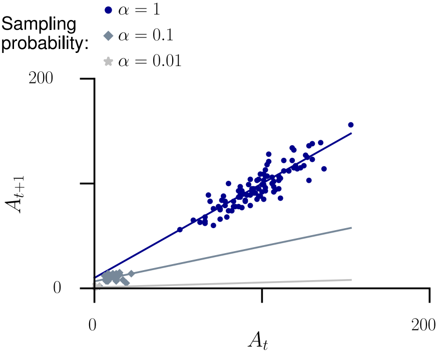

$$\langle A_{t+1} | A_t \rangle = m A_t + H$$

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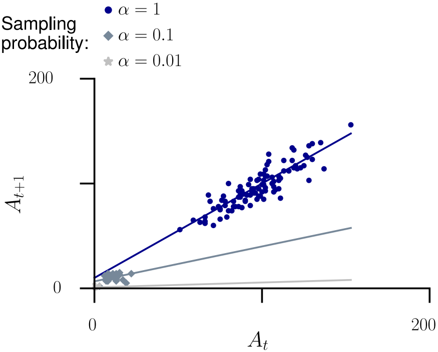

$$\langle A_{t+k} | A_t \rangle = m^k A_t + H \frac{1 - m^k}{1 - m}$$

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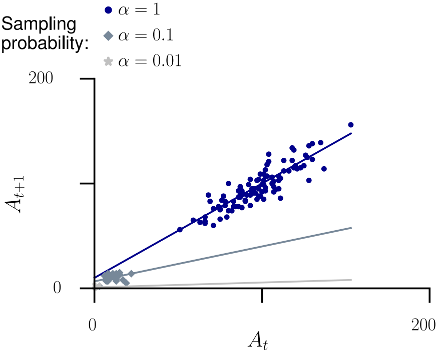

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The multi-step regression (MR) estimator:

$$\hat{r}_k = \alpha^2 \frac{\mathrm{Var}[A_t]}{\mathrm{Var}[a_t]} m^k$$

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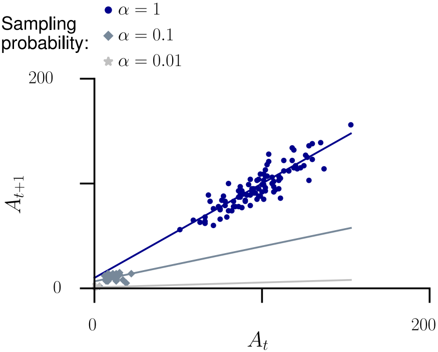

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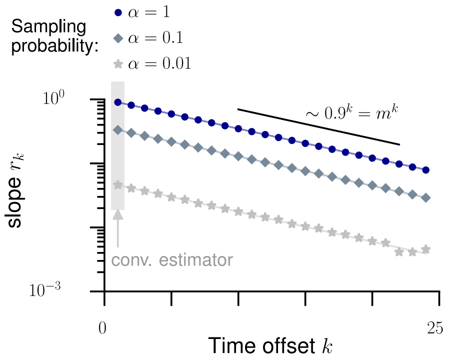
Estimating m under Subsampling



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What about Autocorrelations?

$$r_k \approx m^k = e^{-k\Delta t/\tau} \qquad \text{with} \qquad \tau = -\Delta t / \log m$$

$$\hat{r}_k = \frac{\left\langle (A_t - \overline{A_t})(A_{t+k} - \overline{A_{t+k}}) \right\rangle}{\sigma^2_{A_t}} \qquad C(s,t) = \frac{\langle (X_t - \mu_t)(X_s - \mu_s) \rangle}{\sigma_t \sigma_s}$$

Stationary Process ($m < 1$):

$$C(k) = \frac{\langle (X_t - \mu)(X_{t+k} - \mu) \rangle}{\sigma^2} \approx \hat{r}_k$$

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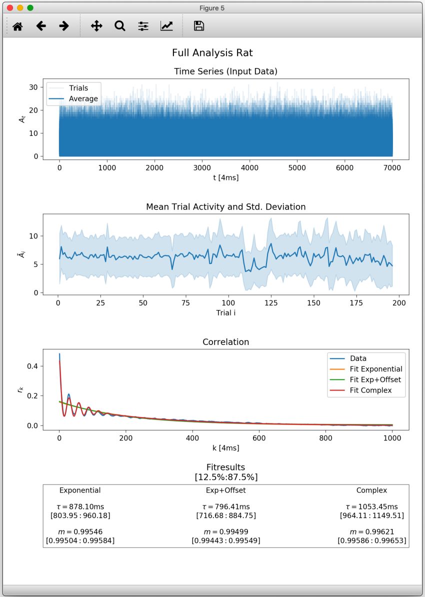
not in general

Mr. Estimator

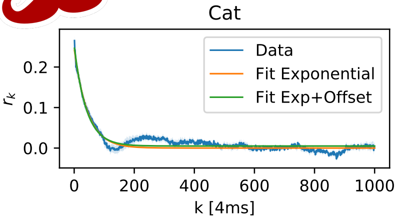
- open-source toolbox
- `https://github.com/Priesemann-Group/mrestimator`
- python (with numpy, scipy, matplotlib)
- one-step installation:

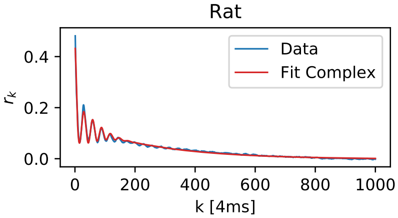
  `pip install mrestimator`
- estimates τ and m using multi-step regression

Mr. Estimator



Cat vs. Rat





$$Ae^{-k/\tau} + O$$

$$\tau = 180 \pm 40\text{ms}$$
$$m = 0.977(4)$$

$$Ae^{-k/\tau} + Be^{-(k/\tau_{osc})^{\gamma}} \cos(2\pi\nu k)$$
$$+ Ce^{-(k/\tau_{gs})^2} + O$$

$$\tau = 1.1 \pm 0.1\text{s}$$
$$m = 0.996(1)$$
$$\nu = 8.4\text{Hz}$$

T. Blanche, CRCNS.org, (2009). doi:10.6080/K0MW2F2J
K. Mizuseki, A. Sirota, E. Pastalkova, and G. Buzsáki, CRCNS.org, (2009). doi:10.6080/K0Z60KZ9

Where to go from here?

- Disease spreading
- Collaboration on epilepsy:

  Christina Stier (University Hospital Göttingen)
- Non-stationary processes:
  - Recurrent $m(t)$: Annika Hagemann
  - External $H(t)$: Jorge de Heuvel
- Hierarchy of time scales:

  Jonas Dehning
- Coalescence effects:

  Johannes Zierenberg