

Exact quantum relaxation and metastability of lattice bosons with cavity-induced long-range interactions

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Extended Bose-Hubbard model

Bipartite lattice, lattice sites: \mathbf{r}, \mathbf{r}'

$\hat{b}_{\mathbf{r}}^\dagger, \hat{b}_{\mathbf{r}}$ boson operators, $\hat{n}_{\mathbf{r}} = \hat{b}_{\mathbf{r}}^\dagger \hat{b}_{\mathbf{r}}$ number operators

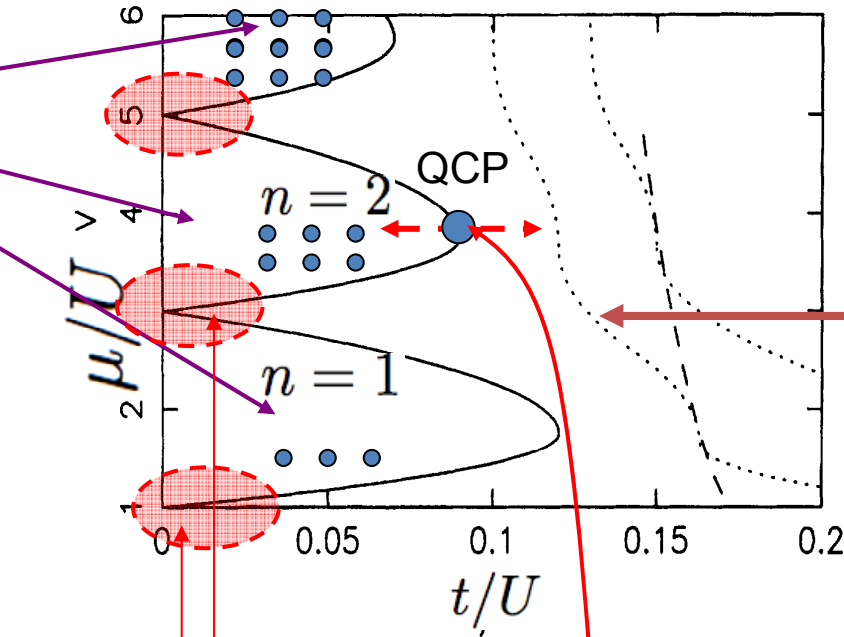
$$\hat{H} = -\mathcal{T} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left(\hat{b}_{\mathbf{r}}^\dagger \hat{b}_{\mathbf{r}'} + \text{H.c.} \right) + \frac{U}{2} \sum_{\mathbf{r}} \hat{n}_{\mathbf{r}} (\hat{n}_{\mathbf{r}} - 1) \\ - \mu \sum_{\mathbf{r}} \hat{n}_{\mathbf{r}} - \varepsilon \frac{1}{N} \left(\sum_{\mathbf{r} \in e} \hat{n}_{\mathbf{r}} - \sum_{\mathbf{r} \in o} \hat{n}_{\mathbf{r}} \right)^2$$

\mathcal{T} tunneling constant, U onsite repulsion
 μ chemical potential, ε infinite-range interaction
 N number of lattice sites

Bose-Hubbard model, $\varepsilon = 0$

Zimanyi et. al (1994)

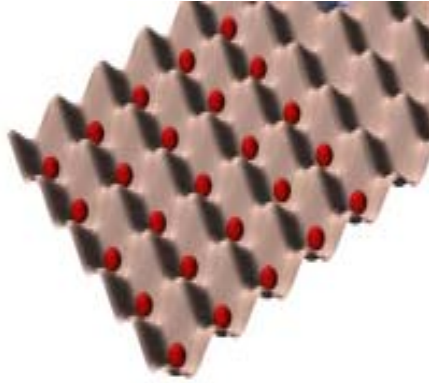
Mott insulators
incompressible



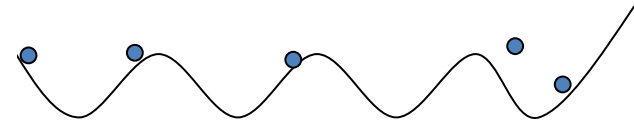
S=1/2, XXZ model

S=1, O(2) relativistic GP

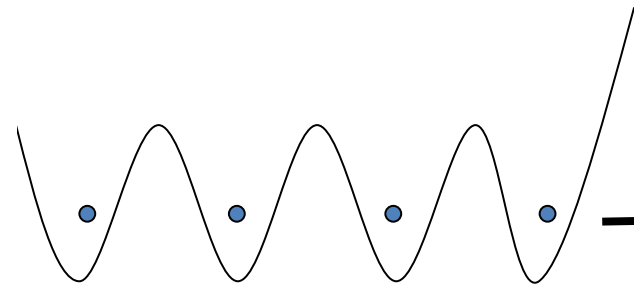
Boson on an Optical Lattice



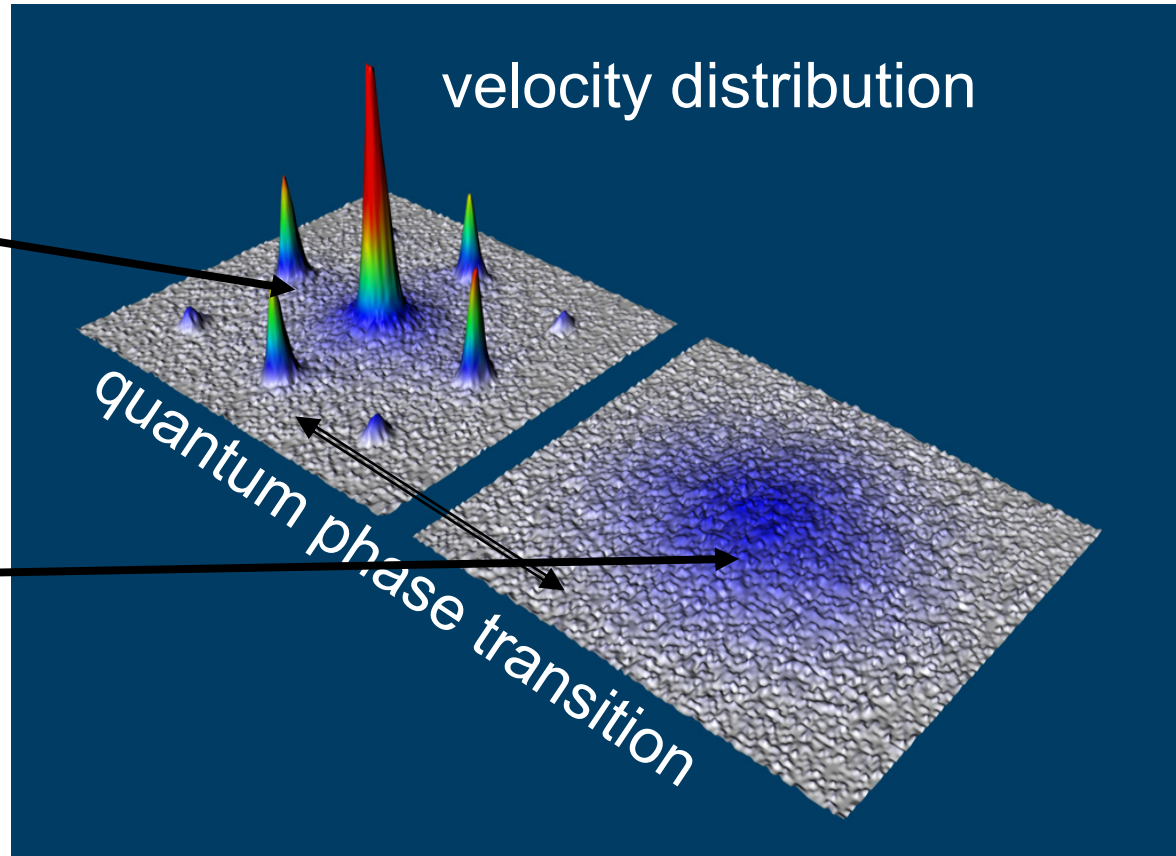
I. Bloch



superfluid

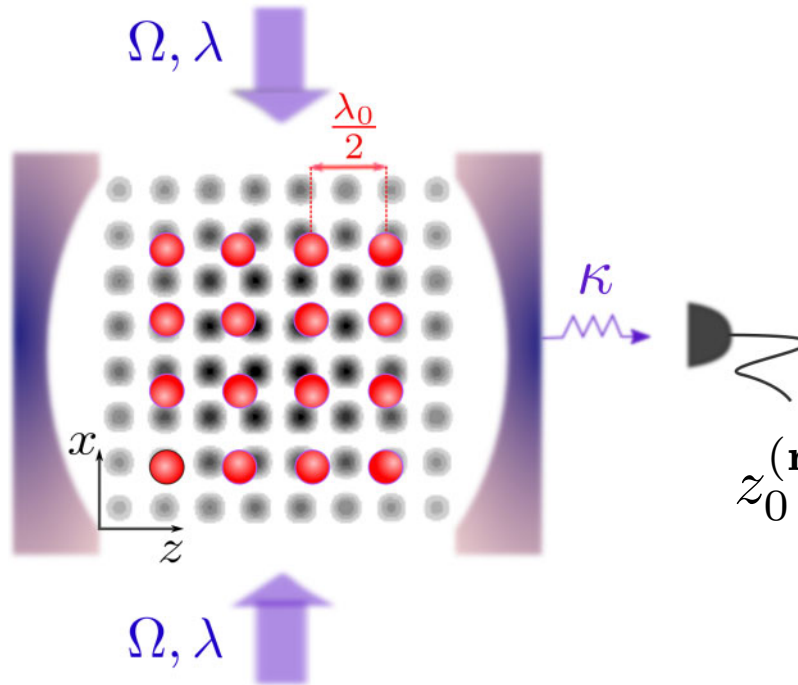


Mott insulator



Cavity induced long-range interactions

H. Habibian, A. Winter, S. Paganelli, H. Rieger and G. Morigi, PRL **110**, 075304 (2013)



$$V_{\text{cavity}} = -\varepsilon N \hat{x}^2$$

$$\hat{x} = \frac{1}{N} \sum_{\mathbf{r}} z_0^{(\mathbf{r})} \hat{n}_{\mathbf{r}}$$

$$z_0^{(\mathbf{r})} = \int d\mathbf{r}' \cos(kx') \cos(kz') W_{\mathbf{r}}^2(\mathbf{r}')$$

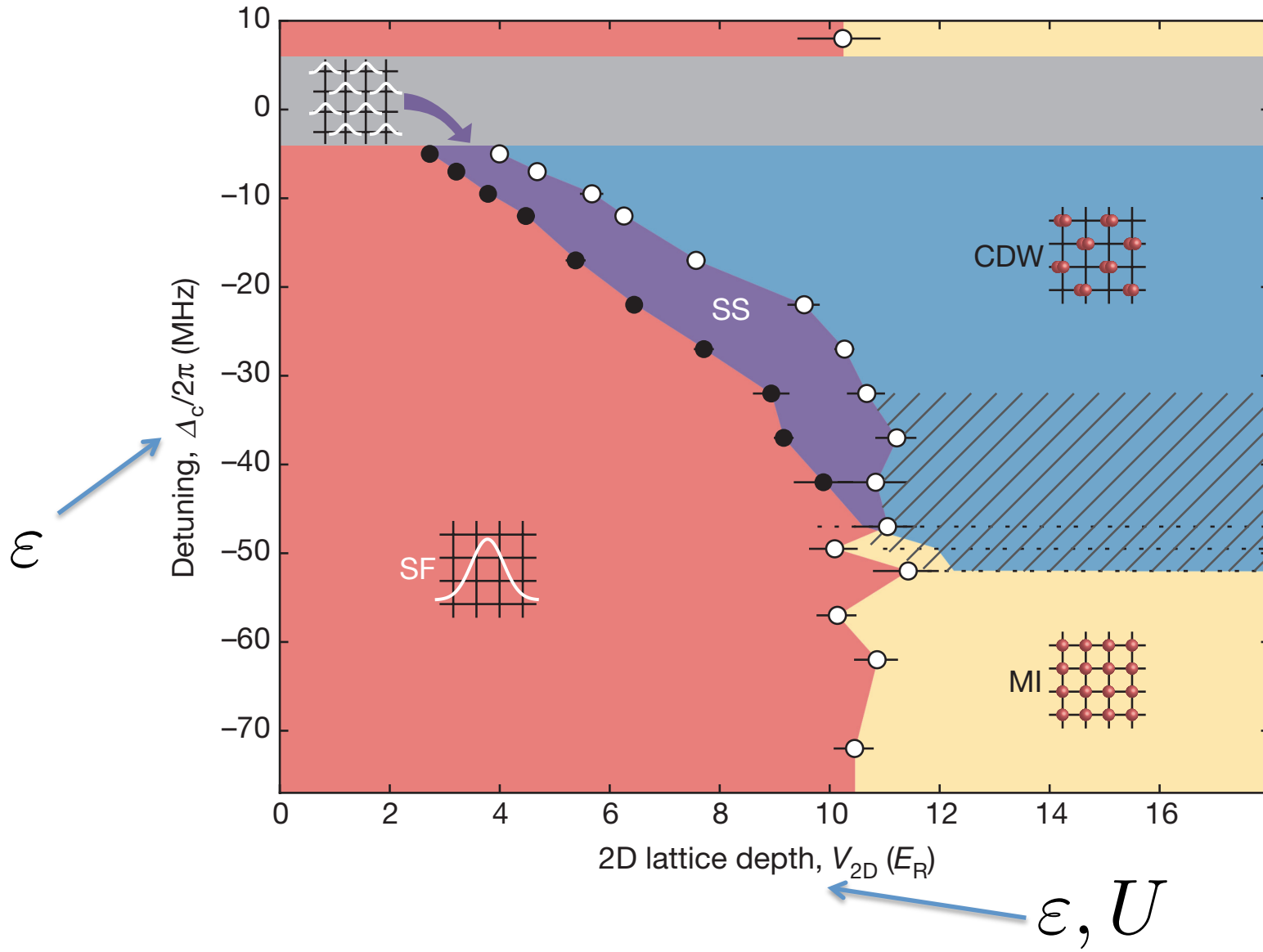
$$k = \pi, \quad W_{\mathbf{r}}^2(\mathbf{r}') = \delta(\mathbf{r}')$$

$$\hat{x} = \frac{1}{N} \left(\sum_{\mathbf{r} \in e} \hat{n}_{\mathbf{r}} - \sum_{\mathbf{r} \in o} \hat{n}_{\mathbf{r}} \right)$$

DW order parameter

Experiment in 2d

Esslinger group, Nature 2016



Linearization of the global-range interaction term

$$\hat{\mathcal{T}} = -\mathcal{T} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left(\hat{b}_{\mathbf{r}}^\dagger \hat{b}_{\mathbf{r}'} + \text{H.c.} \right) \quad \text{kinetic energy}$$

$$\hat{\mathcal{V}} = \frac{U}{2} \sum_{\mathbf{r}} \hat{n}_{\mathbf{r}} (\hat{n}_{\mathbf{r}} - 1) - \mu \sum_{\mathbf{r}} \hat{n}_{\mathbf{r}} - \varepsilon \frac{1}{N} \left(\sum_{\mathbf{r} \in e} \hat{n}_{\mathbf{r}} - \sum_{\mathbf{r} \in o} \hat{n}_{\mathbf{r}} \right)^2 \quad \text{potential energy}$$

Suzuki-Trotter decomposition of the canonical partition function:

$$Z = \text{Tr} e^{-\beta H} = \lim_{M \rightarrow \infty} \text{Tr} \left(e^{-\Delta\tau \hat{\mathcal{T}}} e^{-\Delta\tau \hat{\mathcal{V}}} \right)^M, \quad \Delta\tau = \beta/M$$

using $\hat{n}_{\mathbf{r}}$ eigenstates $|S_1, \dots, S_N\rangle \rightarrow \sum_{\underline{S}^k} |\underline{S}^k\rangle \langle \underline{S}^k| = \mathbf{I}$

Feynman path-integral expression:

$$Z = \lim_{M \rightarrow \infty} \sum_{\underline{S}^1, \dots, \underline{S}^M} \prod_{k=1}^M \langle \underline{S}^k | e^{-\Delta\tau \hat{\mathcal{T}}} | \underline{S}^{k+1} \rangle \exp \left(-\Delta\tau \sum_{k=1}^M V(\underline{S}^k) \right)$$

Hubbard-Stratonovic transformation for the quadratic term in $V(\underline{S}^k)$

$$e^{\lambda A^2} = \int \frac{dx}{\mathcal{N}} e^{-\lambda x^2 + 2\lambda x A}$$

$$\exp\left(-\epsilon N \left[\left(\frac{1}{N} \sum_{\mathbf{r} \in e} S_{\mathbf{r}}^k - \frac{1}{N} \sum_{\mathbf{r} \in o} S_{\mathbf{r}}^k\right)\right]^2\right) = \int \frac{dx_k}{\mathcal{N}} \exp\left(-\epsilon N \left[-x_k^2 + 2x_k \left(\frac{1}{N} \sum_{\mathbf{r} \in e} S_{\mathbf{r}}^k - \frac{1}{N} \sum_{\mathbf{r} \in o} S_{\mathbf{r}}^k\right)\right]\right)$$

For large N the integral is evaluated by the saddle-point method:

$$V_{\text{cavity}} = -\epsilon N \hat{x}^2 \rightarrow -2\epsilon N x \hat{x} + \epsilon N x^2$$

where x is calculated self-consistently: $x = \langle \hat{x} \rangle_{\text{GS}(\hat{H}(x))}$

in a (non-equilibrium) dynamical process $x(t)$ has to be calculated at each time-step

Large U -limit, one dimension

Excluding multiple site occupancies, $\hat{a}_j^\dagger, \hat{a}_j$ hard-core Bose operators

$$\hat{H}(x) = -\mathcal{T} \sum_{j=1}^L \left(\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_j \hat{a}_{j+1}^\dagger \right) + \varepsilon L x^2 - \sum_{j \text{ even}} (\mu + 2\varepsilon x) \hat{a}_j^\dagger \hat{a}_j - \sum_{j \text{ odd}} (\mu - 2\varepsilon x) \hat{a}_j^\dagger \hat{a}_j$$

Jordan-Wigner transformation, followed by Fourier transformation:

$$\hat{H}(x) = - \sum_{k>0} (c_k^\dagger, c_{k-\pi}^\dagger) \begin{pmatrix} \alpha_k & \gamma_k \\ \gamma_k & \beta_k \end{pmatrix} \begin{pmatrix} c_k \\ c_{k-\pi} \end{pmatrix} + 2\varepsilon x^2$$

With c_k^\dagger, c_k fermion operators,

$$k = (2n - 1) \cdot \pi / L, \quad n = 1, 2, \dots, L/2,$$

$$\alpha_k = \mu + 2\mathcal{T} \cos(k), \quad \beta_k = \mu - 2\mathcal{T} \cos(k), \quad \gamma = 2\varepsilon x$$

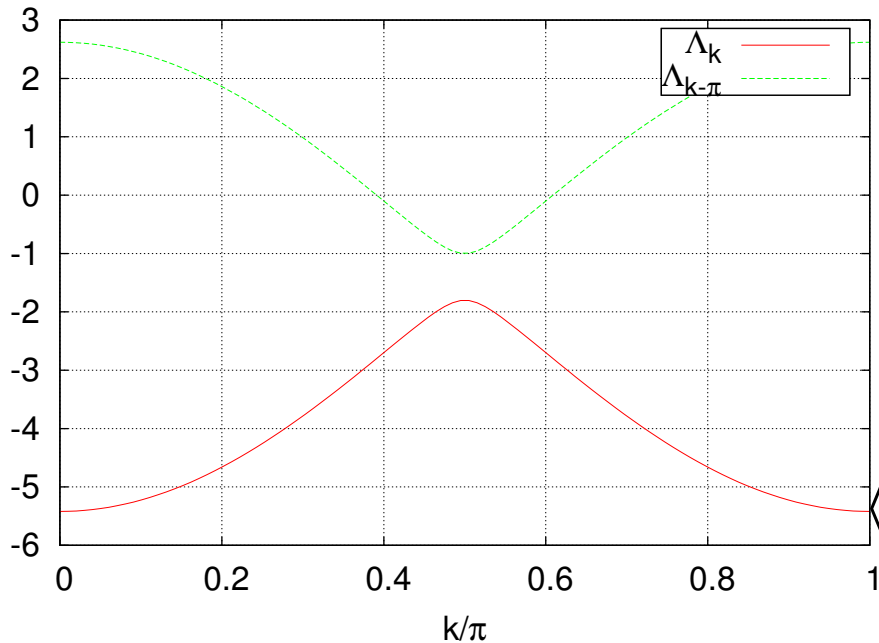
The k and the $k - \pi$ modes can be decoupled:

$$\hat{H}(x) = \sum_{0 < k < \pi/2} 2[\Lambda_k \hat{\eta}_k^\dagger \hat{\eta}_k + \Lambda_{k-\pi} \hat{\eta}_{k-\pi}^\dagger \hat{\eta}_{k-\pi} + 2\varepsilon x^2]$$

With energy of eigenmodes:

$$\Lambda_k = -\mu - \lambda_k, \quad \Lambda_{k-\pi} = -\mu + \lambda_k, \quad \lambda_k = (\mathcal{T}^2 \cos^2(k) + \varepsilon^2 x^2)^{1/2}$$

$\lambda_k = \lambda_{\pi-k}$, thus it is enough to consider $0 < k < \pi/2$



One state is characterized by k_m

$$\left\langle \hat{\eta}_k^\dagger \hat{\eta}_k \right\rangle_{k_m} = 1 \quad \text{for all } k$$

$$\left\langle \hat{\eta}_{k-\pi}^\dagger \hat{\eta}_{k-\pi} \right\rangle_{k_m} = 0, \quad k \in (0, k_m)$$

$$\left\langle \hat{\eta}_{k-\pi}^\dagger \hat{\eta}_{k-\pi} \right\rangle_{k_m} = 0, \quad k \in (k_m, \pi/2)$$

energy per site:

$$\begin{aligned} e(k_m) &= \frac{1}{L} \sum_{k \in (0, k_m)} \Lambda_k + \frac{1}{L} \sum_{k \in (k_m, \frac{\pi}{2})} (\Lambda_k + \Lambda_{k-\pi}) + \varepsilon x^2 \\ &= -\frac{\mu}{2} \left(1 - \frac{2k_m}{\pi} \right) - \frac{1}{\pi} \int_0^{k_m} dk \sqrt{\mathcal{T}^2 \cos^2 k + \varepsilon^2 x^2} + \varepsilon x^2 \end{aligned}$$

DW order parameter

$$\hat{x} = \frac{1}{L} \sum_{k > 0} \left(\hat{c}_k^\dagger \hat{c}_{k-\pi} + \hat{c}_{k-\pi}^\dagger \hat{c}_k \right)$$

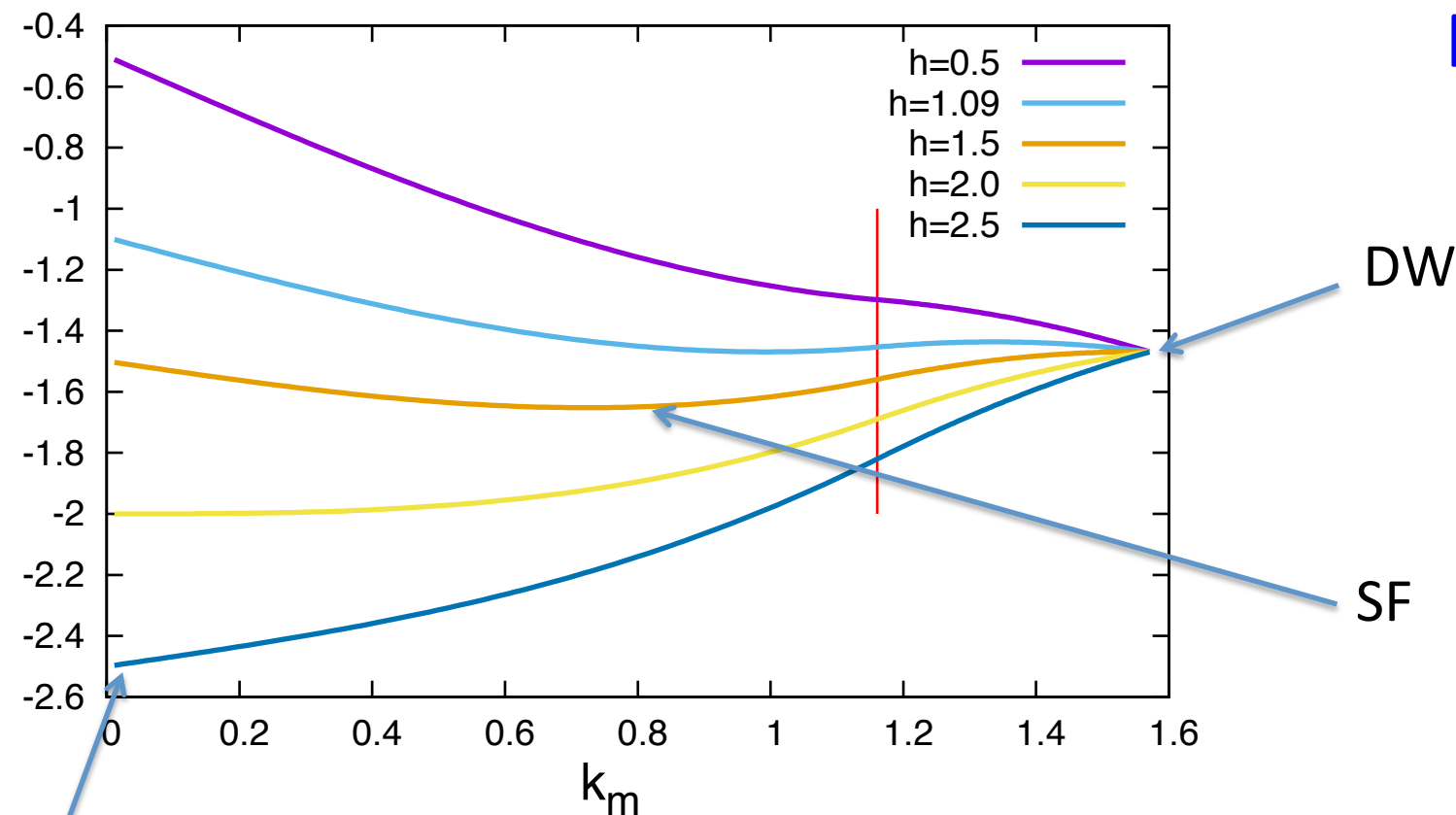
self-consistency equation:

$$x = \frac{\varepsilon x}{\pi} \int_0^{k_m} dk \frac{1}{\sqrt{\mathcal{T}^2 \cos^2(k) + \varepsilon^2 x^2}}$$

in the ground state: $e_0 = \min_{k_m} e(k_m)$

energy vs k_m - epsilon=1.0

Phases



MI

MI

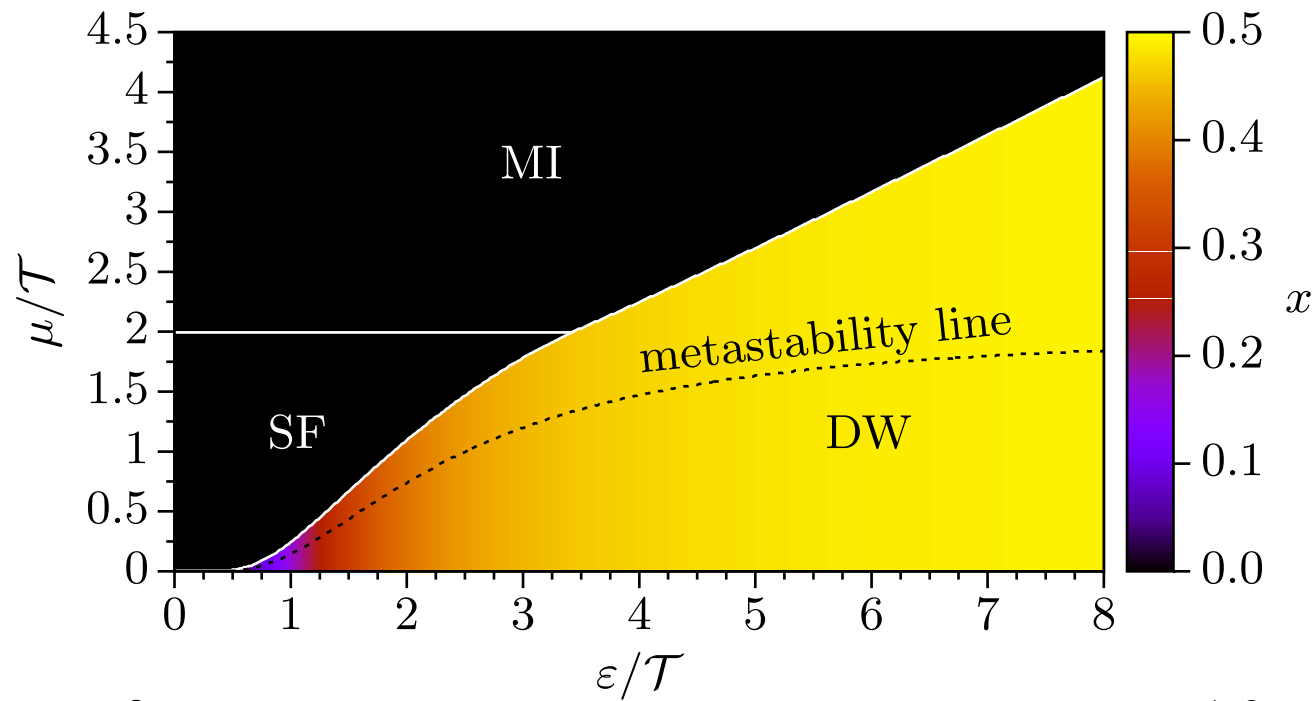
SF

DW

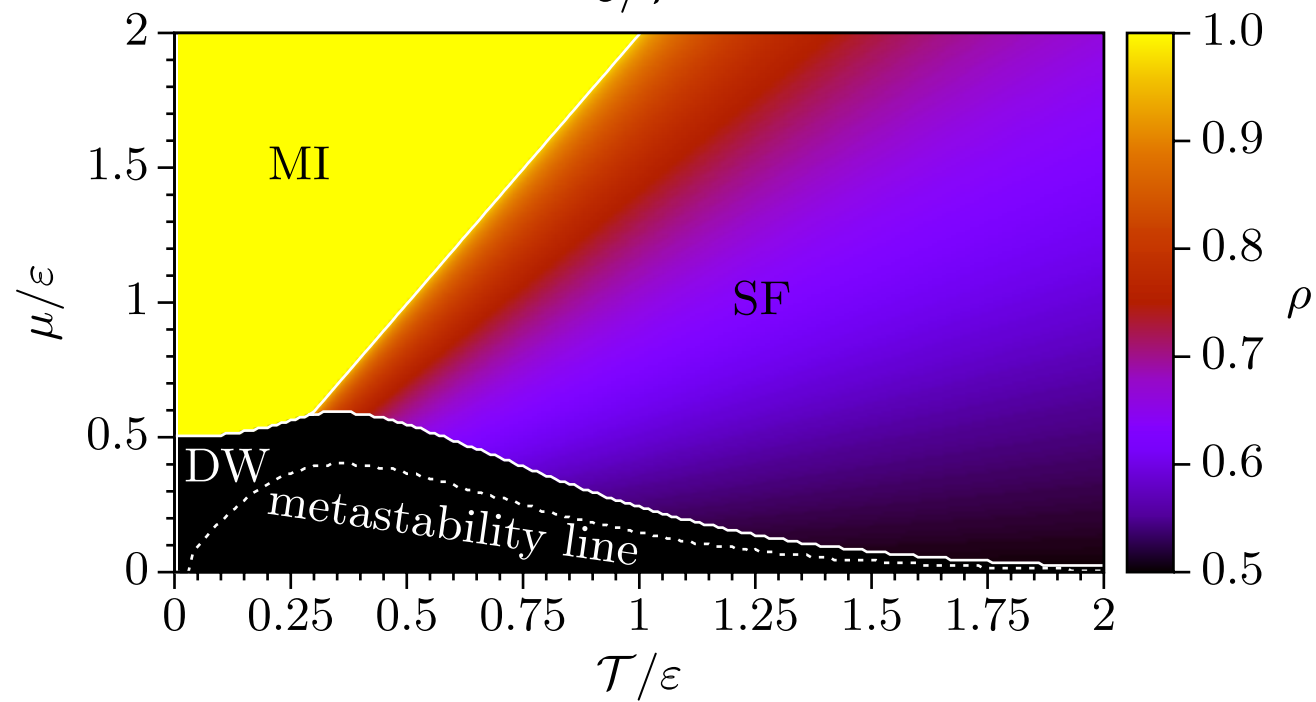
$$\rho = \frac{1}{L} \sum_j \langle \hat{a}_j^\dagger \hat{a}_j \rangle_{\text{GS}}$$

energy gap

	MI	SF	DW
k_m	0	$\in (0, \tilde{k}_m)$	$\pi/2$
x	0	0	$\in (0, 1/2]$
ρ	1	$\in (1/2, 1)$	1/2
Δe	> 0	0	> 0

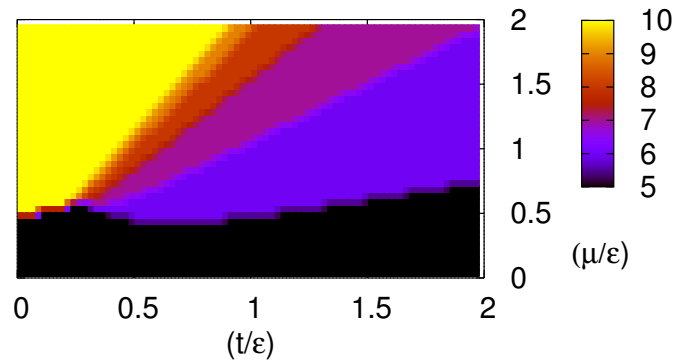


Phase diagram

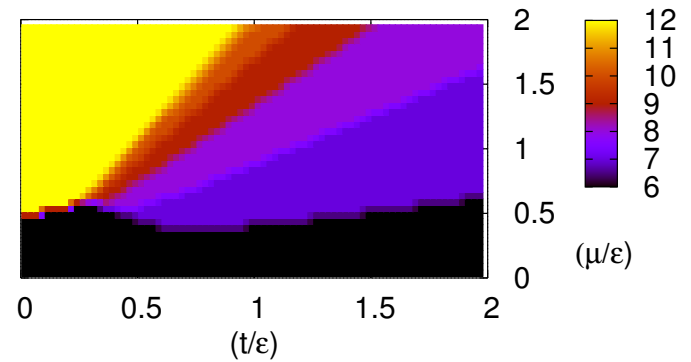


Phase diagram in finite systems without linearization of V_{cavity}

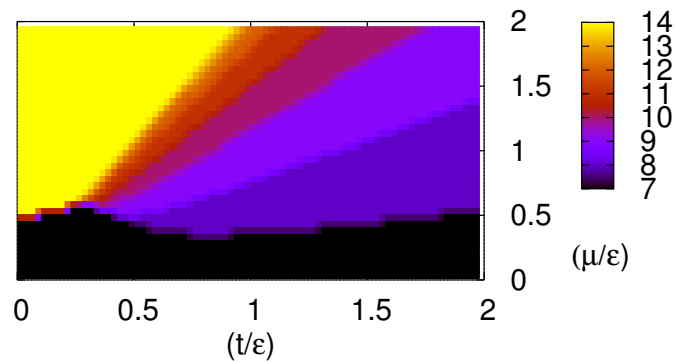
$N, L=10$



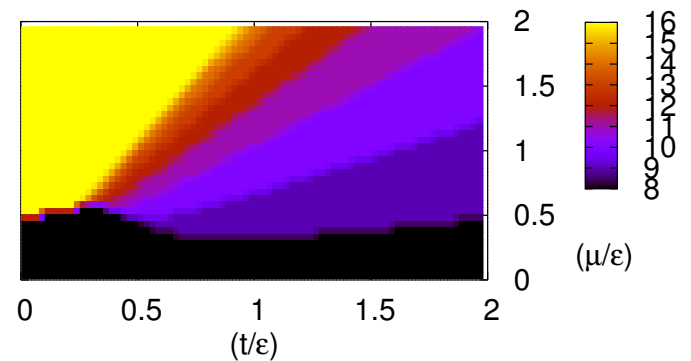
$N, L=12$



$N, L=14$



$N, L=16$



Non-equilibrium dynamics after a sudden quench

change of parameters $\begin{matrix} t < 0 & t \geq 0 \\ (\mathcal{T}_0, \mu_0, \varepsilon_0) & \rightarrow (\mathcal{T}, \mu, \varepsilon) \end{matrix}$

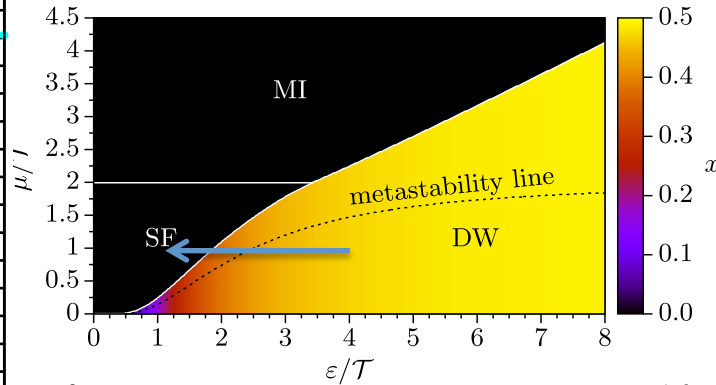
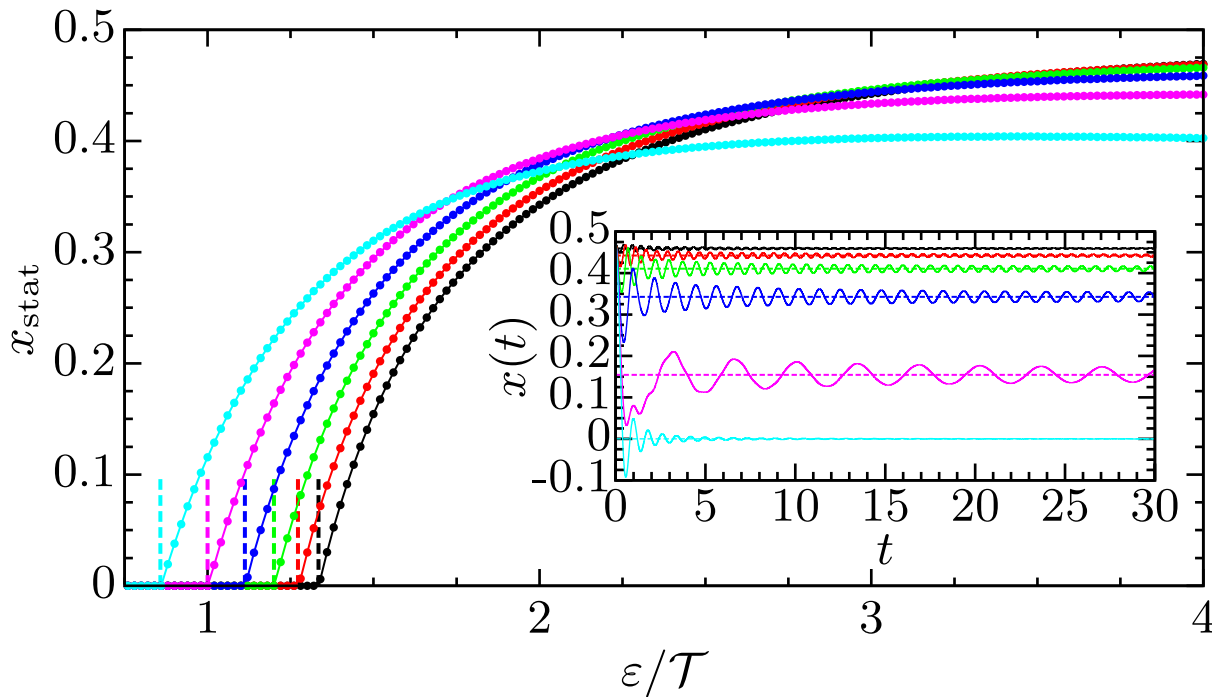
state of the system: $|\Psi(t)\rangle = \exp[-i\mathcal{H}(\mathcal{T}, \mu, \varepsilon)t] |\Psi_0\rangle$

using the linearized Hamiltonian:

$$\mathcal{H}'(\mathcal{T}, \mu, \varepsilon, x(t)), \quad x(t) = \langle \Psi(t) | \hat{x} | \Psi(t) \rangle$$

$$|\Psi(t)\rangle = \exp \left[-i \int_0^t dt' \mathcal{H}'(\mathcal{T}, \mu, \varepsilon, x(t')) \right] |\Psi_0\rangle$$

Quench from the DW to SF phase



**Dynamic
phase transition**

inset:

$$(\mu_0/\mathcal{T}_0 = 1, \epsilon_0/\mathcal{T}_0 = 4) \rightarrow (\mu/\mathcal{T} = 1, \epsilon/\mathcal{T})$$

$$\epsilon/\mathcal{T} = 3.5, 3, 2.5, 2, 1.5, 1 \quad \text{up to bottom}$$

main:

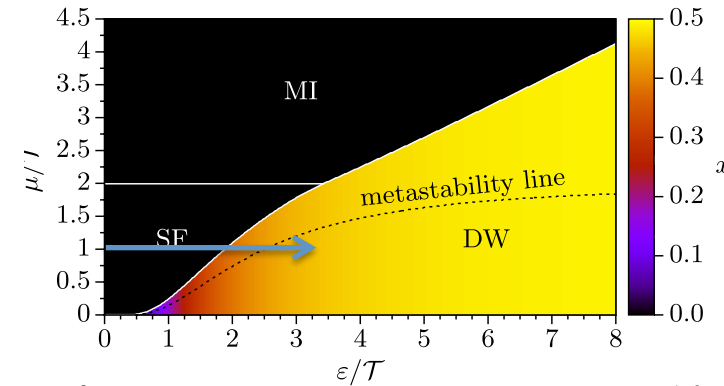
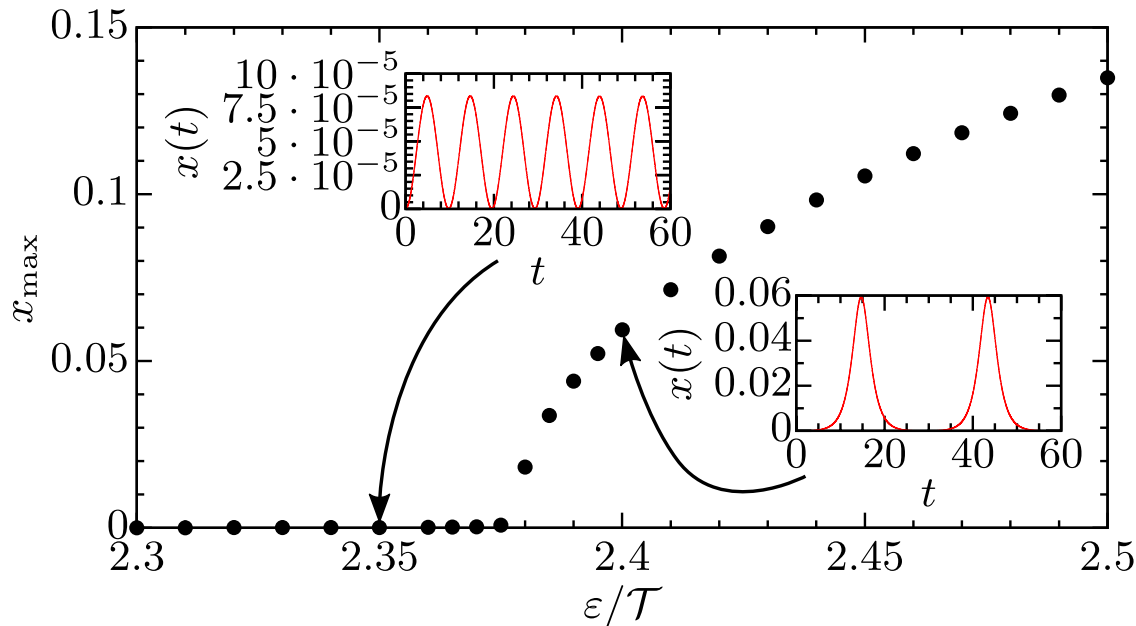
$$(\mu_0/\mathcal{T}_0 = 0.5, \epsilon_0/\mathcal{T}_0) \rightarrow (\mu/\mathcal{T} = 0.5, \epsilon/\mathcal{T})$$

$$\epsilon_0/\mathcal{T}_0 = 4, 3.5, 3, 2.5, 2, 1.5 \quad \text{right to left}$$

Quench from the SF to DW phase

If $x(t = 0) = 0 \rightarrow x(t > 0) = 0$

Stability test: $x(t = 0) = x_0 > 0$, $x_0 \ll 1 \rightarrow x(t > 0) = ?$



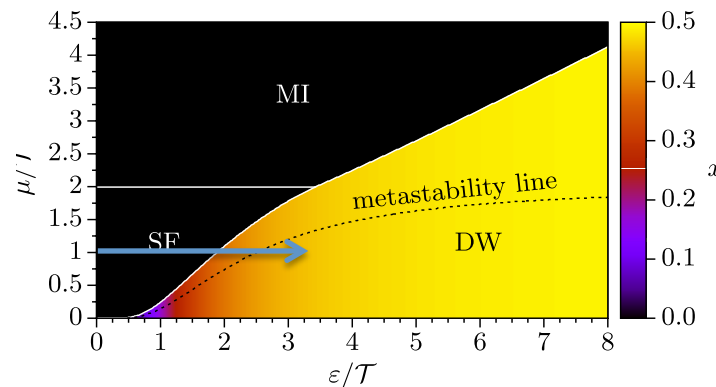
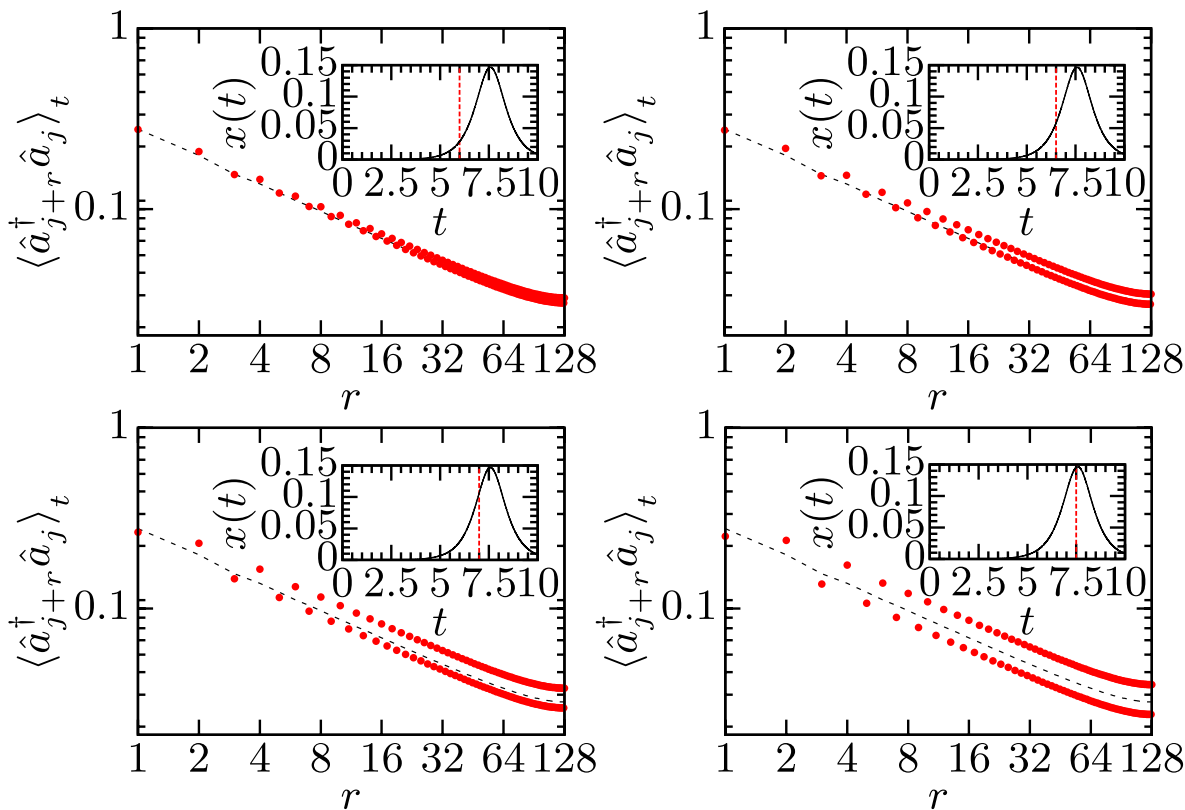
$(\mu_0/\mathcal{T}_0 = 1, \epsilon_0/\mathcal{T}_0 = 0) \rightarrow (\mu/\mathcal{T} = 1, \epsilon/\mathcal{T})$

$x_0 = 10^{-6}$

**Dynamic
phase transition**

Quench from the SF to DW phase

SF correlations: $\langle \hat{a}_{j+r}^\dagger \hat{a}_j \rangle_t \rightarrow$ algebraic decay

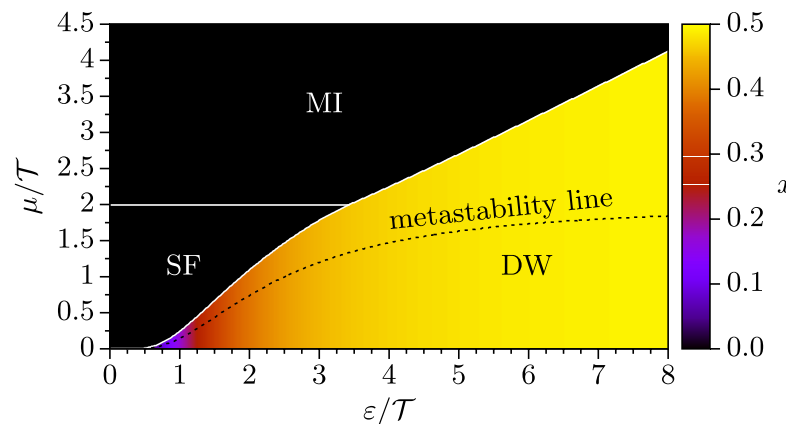


Supersolid
quasi-long-
range order

$$(\mu_0/\mathcal{T}_0 = 1, \varepsilon_0/\mathcal{T}_0 = 0) \rightarrow (\mu/\mathcal{T} = 1, \varepsilon/\mathcal{T} = 2.5)$$

Summary

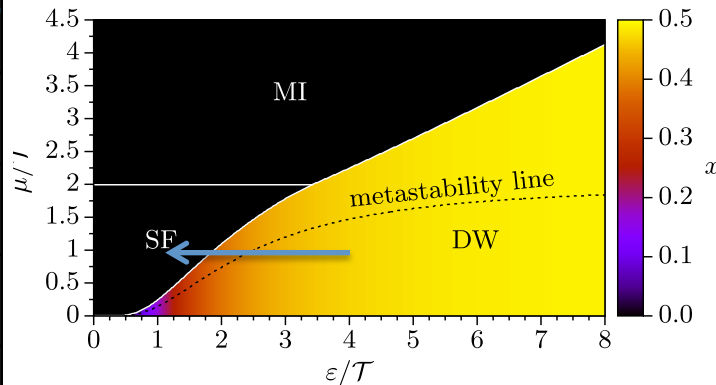
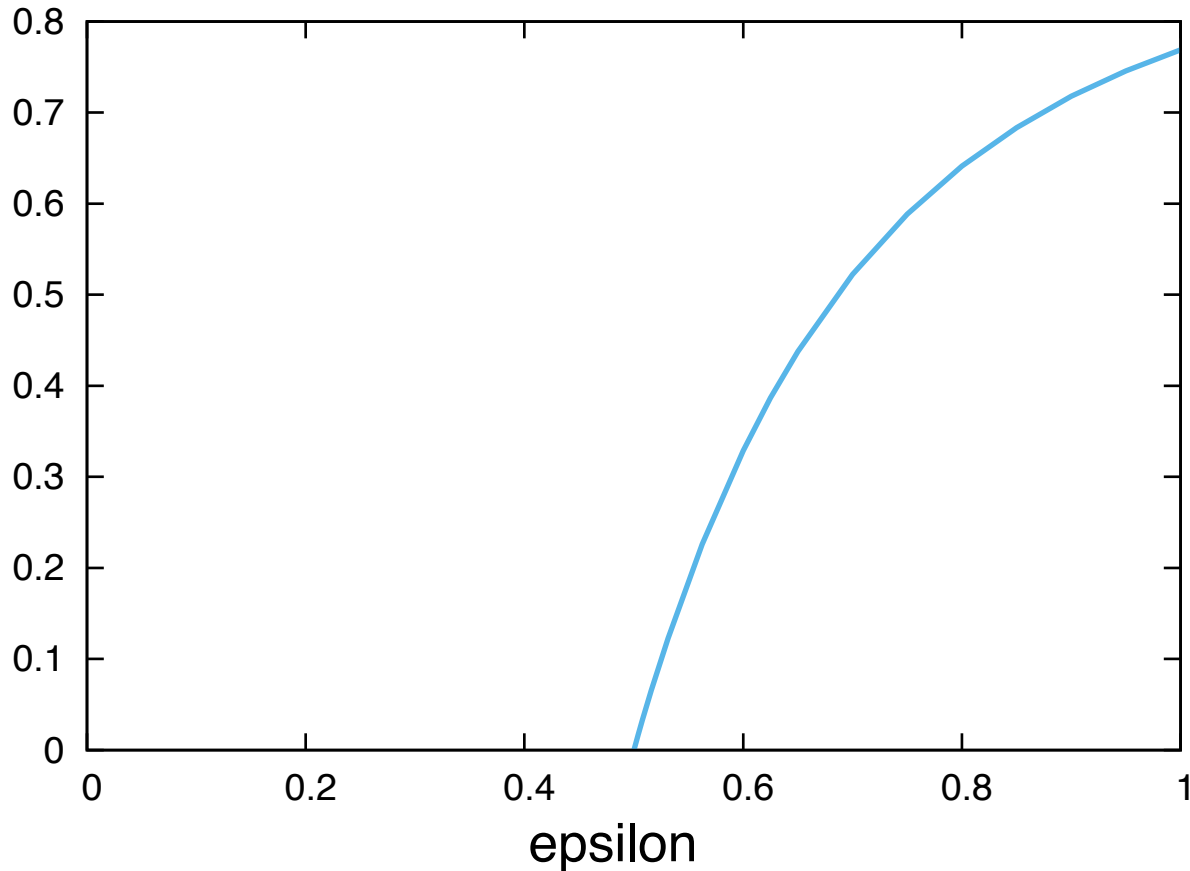
- We have solved exactly a system of hard-core bosons with cavity-induced long-range interactions in 1d
- Ground-state phase diagram consists of MI, SF and DW phases
- In non-equilibrium quench dynamics there are dynamic phase transitions
- Remanent DW order at DW to SF quenches
- Dynamically generated SS state at SF to DW quenches
- Possible experimental realization by cold atoms



Thank you for your attention!

Quench from the DW to SF phase

$h_0=h=1, \epsilon_0=1, J_0=J=1$



**Dynamic
phase transition**

$$(\epsilon/\mathcal{T})_{\text{crit}} = \frac{2\epsilon_0/\mathcal{T}_0}{2 + \epsilon_0/\mathcal{T}_0}$$