Phase Transitions in Disordered Systems: The Example of the 4D Random–Field Ising Model

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Work in collaboration with V. Martín-Mayor (Madrid), M. Picco (Paris), and N. Sourlas (Paris).

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- At low T and for $\sigma \ll J$ we encounter the ferromagnetic phase, provided that $D \geq 3$.
- For $D = 2$, the tiniest $\sigma > 0$ suffices to destroy the ferromagnetic phase.
- Perturbative RG (PRG) computations suggest $D_{\mathrm{u}} = 6$ (for $D \geq D_{\mathrm{u}}$: mean-field exponents).

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- Supersymmetry predicts dimensional reduction: $\mathrm{RFIM}^{(\mathrm{D})} \to \mathrm{Ising}^{(\mathrm{D}-2)}$. Yet, the RFIM orders in $D = 3$ while the Ising ferromagnet in $D = 1$ does not.
- The failure of the PRG begs the question: Is there an intermediate dimension $D_{\mathrm{int}} < D_{\mathrm{u}}$ such that the PRG is accurate for $D > D_{\mathrm{int}}$?

RG fixed–point

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- The relevant RG fixed-point lies at $T = 0$ and the flow is described by *three* independent critical exponents, ν, η, and $\overline{\eta}$, and *two* correlation functions, $C_{xy}^{(\mathrm{con})}$ (connected) and $C_{xy}^{(\mathrm{dis})}$ (disconnected):

$$C_{xy}^{(\mathrm{con})} \equiv \frac{\partial \overline{\langle S_x \rangle}}{\partial h_y} \sim \frac{1}{r^{D-2+\eta}} \; ; \;\; C_{xy}^{(\mathrm{dis})} \equiv \overline{\langle S_x \rangle \langle S_y \rangle} \sim \frac{1}{r^{D-4+\overline{\eta}}} \, .$$

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- The relationship between the anomalous dimensions η and $\overline{\eta}$ is hotly debated for many years now and is one of the main themes of the present work.

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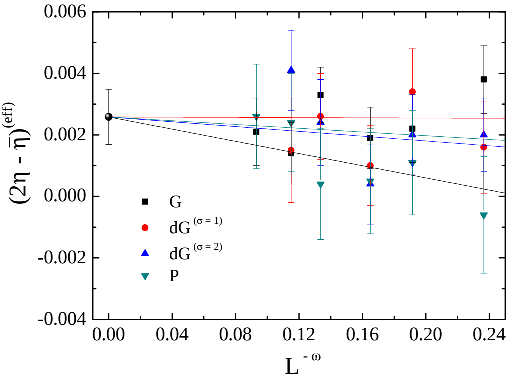
  - For $D < D_{\rm int}$: $\overline{\eta} \neq \eta$.
  - $D_{\rm int} \approx 5.1$.

Latest numerical results at $D = 3$

$$2\eta - \overline{\eta} = 0.0026(9)\ ;\ \chi^2/\mathrm{DOF} = 10.5/17$$
$$2\eta - \overline{\eta} = 0\ \text{(fixed)}\ ;\ \chi^2/\mathrm{DOF} = 18.3/18$$ [1]



[1] N.G. Fytas and V. Martín-Mayor, PRL **110**, 227201 (2013)

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3. Examine previous claims of universality violations for the RFIM when comparing different distributions of random fields.
4. Check the validity of dimensional reduction.

Simulation details

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- We consider the RFIM on a $D = 4$ hyper-cubic lattice with periodic boundary conditions and energy units $J = 1$. Our random fields $\{h_x\}$ follow either a Gaussian or a Poissonian distribution:

$$\mathcal{P}_G(h,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{h^2}{2\sigma^2}} \ , \ \mathcal{P}_P(h,\sigma) = \frac{1}{2|\sigma|} e^{-\frac{|h|}{\sigma}} \ ,$$

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- We simulated lattice sizes from $L = 4$ to $L = 60$. For each pair (L, σ) we computed ground states for 10^7 samples.

Observables

[2] N.G. Fytas and V. Martín-Mayor, PRE **93**, 063308 (2016)

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- $U_{22} = \chi^{(\mathrm{dis})} / [\chi^{(\mathrm{con})}]^2 \Longrightarrow 2\eta - \overline{\eta}$.

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Quotients method: We compare observables computed in pairs $(L, 2L)$. Scale-invariance is imposed by seeking the L-dependent critical point: the value of σ such that $\xi_{2L}/\xi_L = 2$. Here, we consider both $\xi^{(\mathrm{con})}/L$ and $\xi^{(\mathrm{dis})}/L$.

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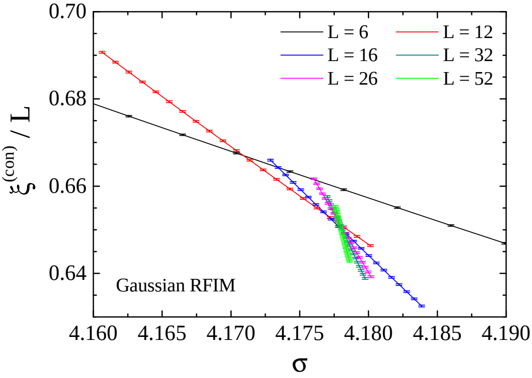

- For dimensionful quantities O, scaling in the thermodynamic limit as $\xi^{x_O/\nu}$, we consider the quotient $Q_O = O_{2L}/O_L$ at the crossing. For dimensionless magnitudes g, we focus on g_L or g_{2L}, whichever show less finite-size corrections. In either case, one has:

$$Q_O^{\rm cross} = 2^{x_O/\nu} + \mathcal{O}(L^{-\omega})\,,\; g_{(L);(2L)}^{\rm cross} = g^* + \mathcal{O}(L^{-\omega})\,,$$

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- Dimensionless quantities: $\xi^{(\mathrm{con})}/L$, $\xi^{(\mathrm{dis})}/L$ and U_4.
- Dimensionful quantities:
  - Derivatives of $\xi^{(\mathrm{con})}$, $\xi^{(\mathrm{dis})}$ [$x_\xi = 1+\nu$],
  - Derivatives of $\chi^{(\mathrm{con})}$ and $\chi^{(\mathrm{dis})}$ [$x_{\chi^{(\mathrm{con})}} = \nu(2-\eta)$, $x_{\chi^{(\mathrm{dis})}} = \nu(4-\overline{\eta})$],
  - U_{22} [$x_{U_{22}} = \nu(2\eta - \overline{\eta})$].

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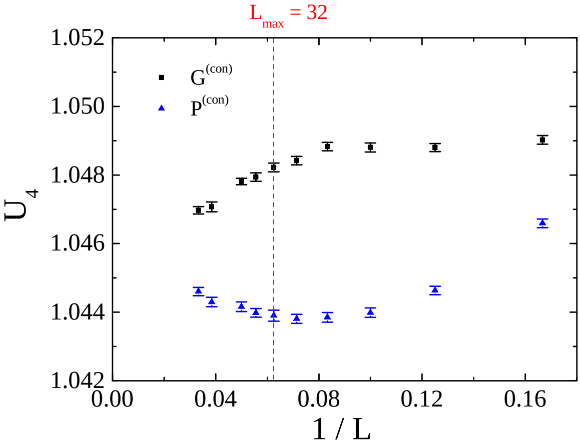
   - We denote these as: $\mathsf{G}^{(\mathrm{con})}$, $\mathsf{G}^{(\mathrm{dis})}$, $\mathsf{P}^{(\mathrm{con})}$, and $\mathsf{P}^{(\mathrm{dis})}$.

A spectacular example of non-monotonic behavior

Possible explanation of previously reported universality violations



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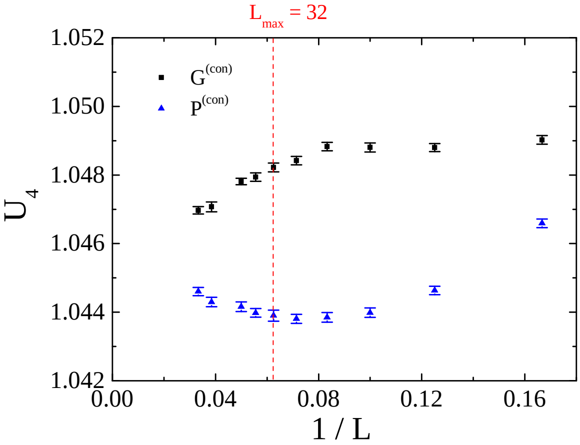
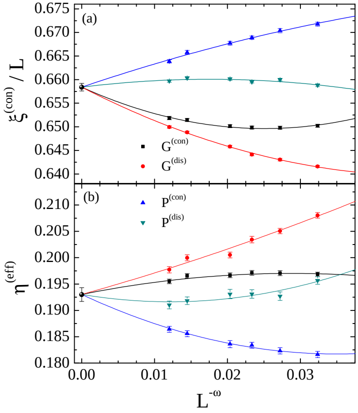
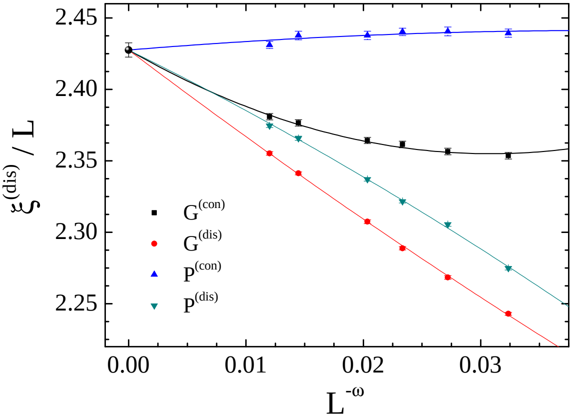


Higher-order corrections are necessary: $X_L = X^* + a_1 L^{-\omega} + a_2 L^{-2\omega}$

$$\omega = 1.30(9)\ ;\ \xi^{(\mathrm{con})}/L = 0.6584(8)\ ;\ \eta = 0.1930(13)$$
$$\chi^2/\mathrm{DOF} = 27.85/29$$

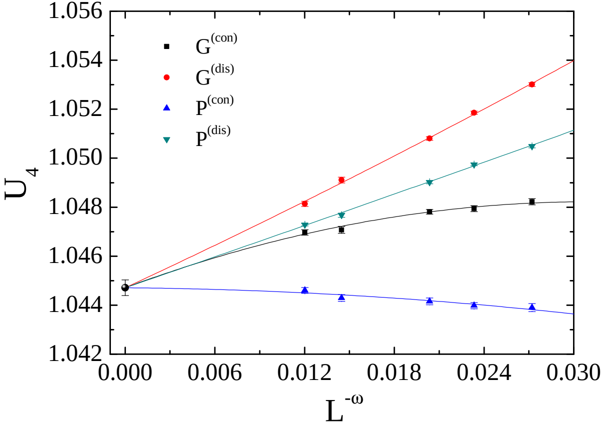


$\xi^{(\mathrm{dis})}/L = 2.4276(36)(34)$
$\chi^2/\mathrm{DOF} = 16/15$

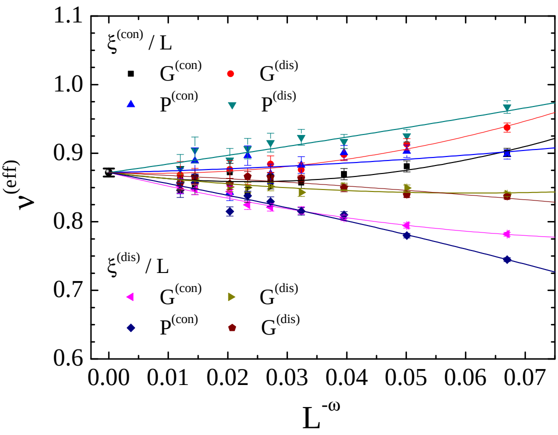


Binder cumulant U_4

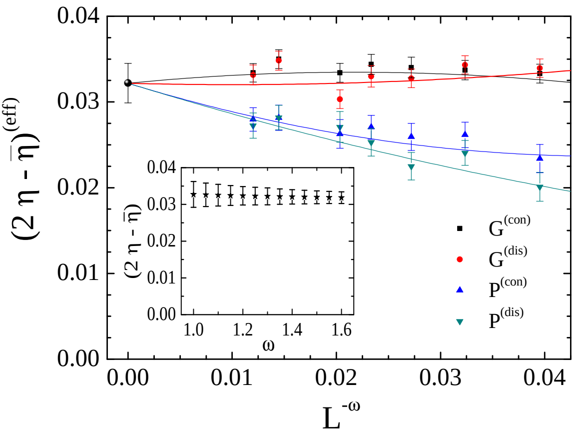
$$U_4 = 1.04471(32)(14)$$
$$\chi^2/\mathrm{DOF} = 10/11$$



$\nu = 0.8718(58)(19)$
$\chi^2/\mathrm{DOF} = 62.9/55$

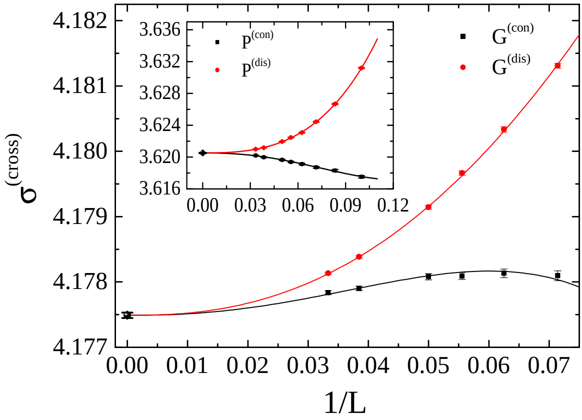


$$2\eta - \overline{\eta} = 0.0322(23)(1)$$
$$\chi^2/\mathrm{DOF} = 16.0/19$$



Critical fields

$$\sigma_{\mathrm{c},L} = \sigma_c + b_1 L^{-(\omega+\frac{1}{\nu})} + b_2 L^{-(2\omega+\frac{1}{\nu})}$$
$$\sigma_{\mathrm{c}}(G) = 4.17749(4)(2)\ ;\ \chi^2/\mathrm{DOF} = 5.6/7$$
$$\sigma_{\mathrm{c}}(P) = 3.62052(3)(8)\ ;\ \chi^2/\mathrm{DOF} = 8.85/11$$



Summary of universal ratios and exponents

	QF	χ^2/DOF
ω	1.30(9)	
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η	0.1930(13)	
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U_4	1.04471(32)(14)	10/11
$\xi^{(\mathrm{dis})}/L$	2.4276(36)(34)	16/15
ν	0.8718(58)(19)	62.9/55
$2\eta - \bar{\eta}$	0.0322 (23)(1)	16.0/19

Hartmann, PRB **65**, 174427 (2002): $\sigma_{\mathrm{c}}(G) = 4.18(1); \nu = 0.78(10)$
Middleton, arXiv:cond-mat/0208182: $\sigma_{\mathrm{c}}(G) = 4.179(2); \nu = 0.82(6)$

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- We stress the non-trivial difference $2\eta - \overline{\eta} = 0.0322(24)$ which is 10 times larger than its corresponding 3D value 0.0026(9).
- We provided decisive evidence in favor of the three-exponent scaling scenario and the spontaneous supersymmetry breaking at some $D_{\mathrm{int}} > 4$.

Acknowledgements

- Our $L = 52, 60$ lattices were simulated in the *MareNostrum* and *Picasso* supercomputers. We thankfully acknowledge the computer resources and assistance provided by the staff at the *Red Española de Supercomputación*.

- Computational time in the cluster *Memento* (BIFI Institute, Zaragoza).

- Coventry University for providing a Research Sabbatical Fellowship during which this work has been completed.

$$\nu^{(5\mathrm{D\ RFIM})} = 0.626(15) \approx 0.629971(4) = \nu^{(3\mathrm{D\ IM})} \Longrightarrow \mathbf{D_{int} \approx 5}$$