3D ANISOTROPIC SPIN-GLASS MODELS

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- **Competing interactions, disorder, and frustration**
  - Uniaxial magnetic materials: $Fe_{1-xi}Mn_{xi}TiO_3$ and $Eu_{1-x}Ba_xMnO_3$
  - Neural networks

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- **Edwards - Anderson bimodal model**
  - Ferromagnetic, spin-glass, and paramagnetic phases
  - Multicritical point ; Reentrant behavior

Introduction to spin-glass models

- **Competing interactions, disorder, and frustration**
  - Uniaxial magnetic materials: $\mathrm{Fe_{1-xi}Mn_{xi}TiO_3}$ and $\mathrm{Eu_{1-x}Ba_xMnO_3}$
  - Neural networks
- **Edwards - Anderson bimodal model**
  - Ferromagnetic, spin-glass, and paramagnetic phases
  - Multicritical point ; Reentrant behavior
  - **Anisotropic cases of the Edwards - Anderson bimodal model**
    - **Transverse** and **Longitudinal** anisotropic models.
    - Phase diagrams ; Universality aspects ; Ground-state properties

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- **3D Edwards - Anderson bimodal (EAB) isotropic model**: $\{J = 1\ ; p \leq \frac{1}{2}\}$

Phase diagram of the EAB model

F - P, **SG - P**, and **F - SG** transition lines of the 3D EAB model

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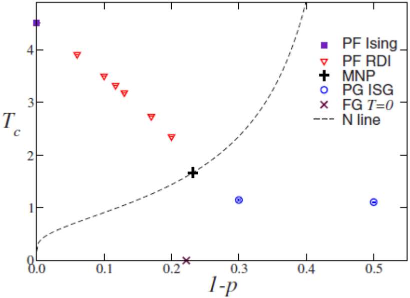

Hasenbusch *et al*, PRB (2008)

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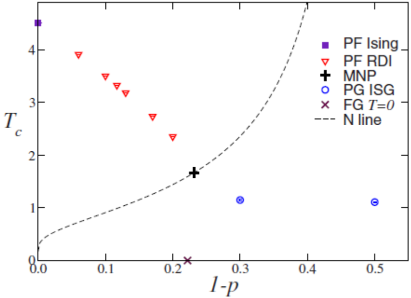

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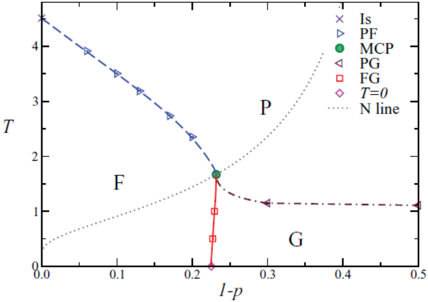
Re-entrant F - SG transition line



Ceccarelli *et al*, PRB (2011)

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- **Longitudinal** anisotropic: $J^z = J^{xy} = J(=1); p_{xy} = 0; p_z \le \frac{1}{2}$

Global phase diagrams at $K_z/K_{xy} = 0.5$ ($K = J/T$):
Re-entrant and **forward** F - SG transition lines

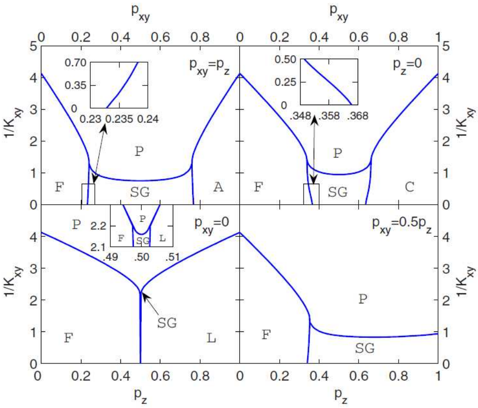
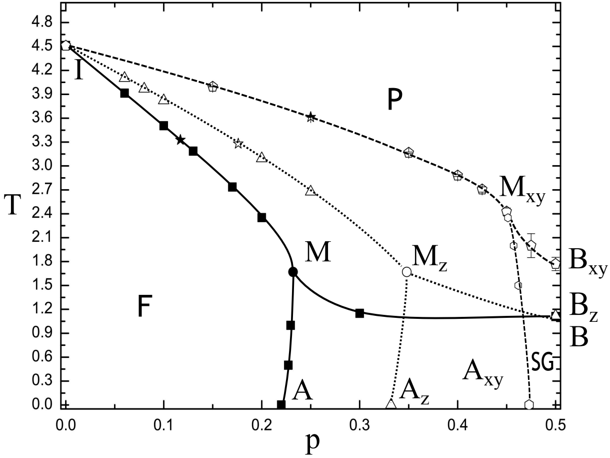


Figure: Guven *et al*, PRE (2008)

Anisotropic spin-glass models on the simple cubic lattice

Transverse and **longitudinal** models at $K_z/K_{xy} = 1$:
Re-entrant and **forward** (?) F - SG transition lines



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with

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- Selection of temperatures: constant acceptance exchange

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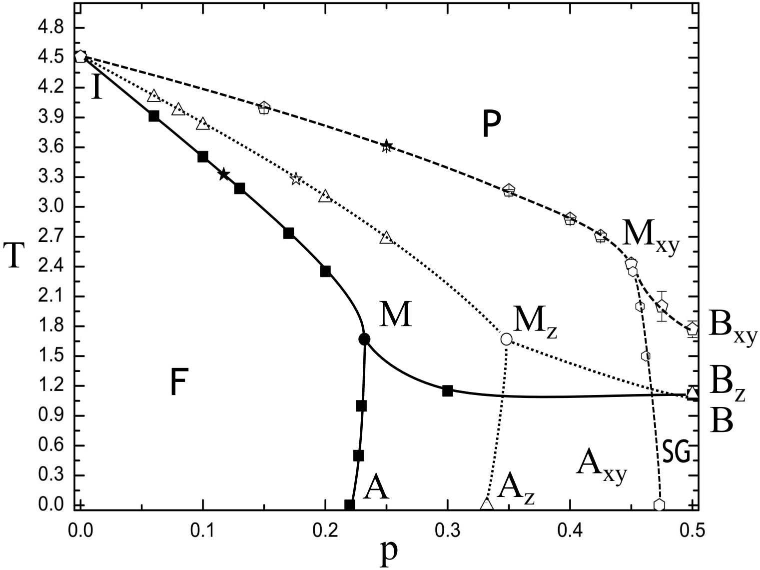
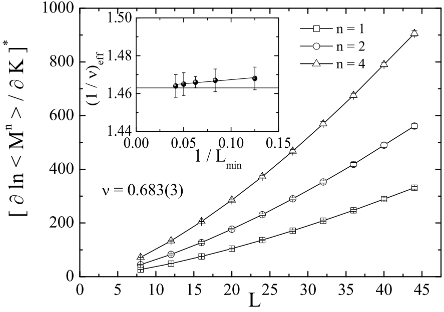
  - Data collapse: $U_Z \approx f[(T - T_c)L^{1/\nu}]$

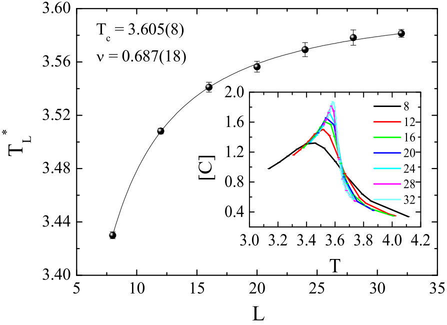
Anisotropic spin-glass models: F - P transition line



Transverse at $p_{xy} = 0.176$



Longitudinal at $p_z = 0.25$



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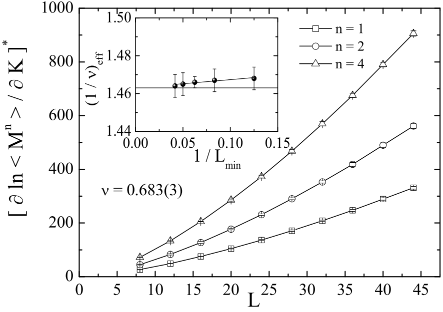

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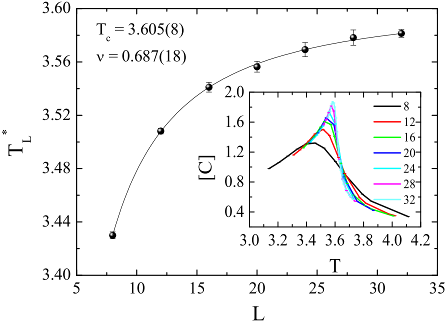
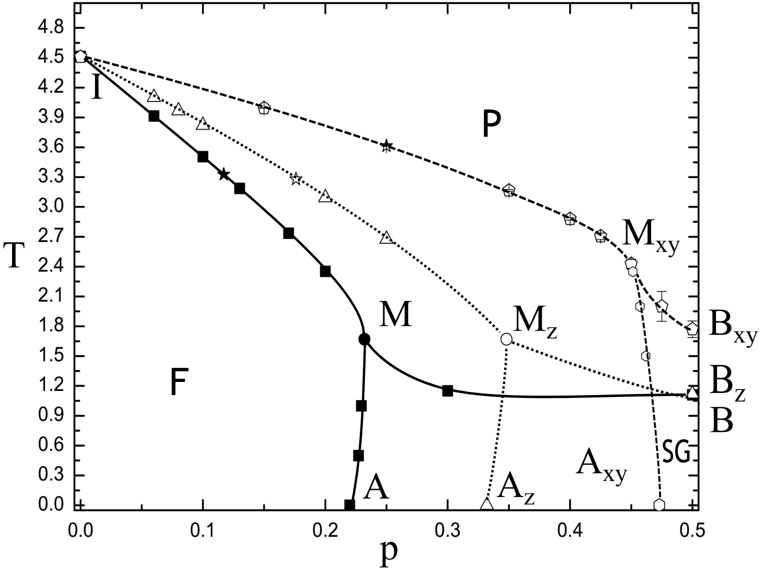
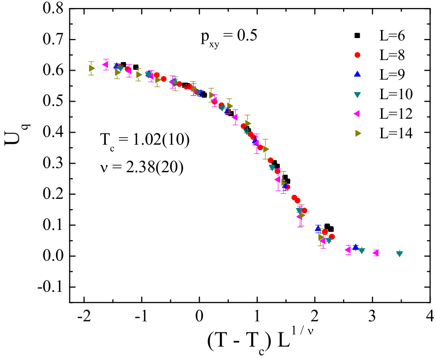


- $\nu^{\text{(transverse)}} = 0.683(3)$
- $\nu^{\text{(longitudinal)}} = 0.687(17)$
- $\nu^{\text{(RIM)}} = 0.6837(53)$ [Ballesteros *et al.*, PRB **62**, 14237 (2000)]

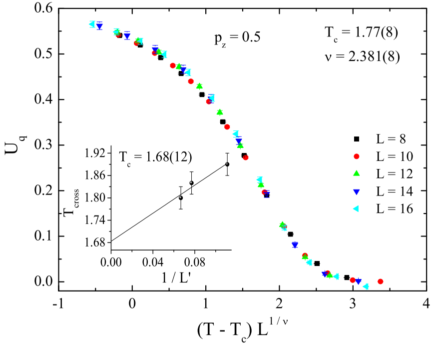
Anisotropic spin-glass models: SG - P transition line



Transverse at $p_{xy} = 0.5$



Longitudinal at $p_z = 0.5$



SG - P transition line

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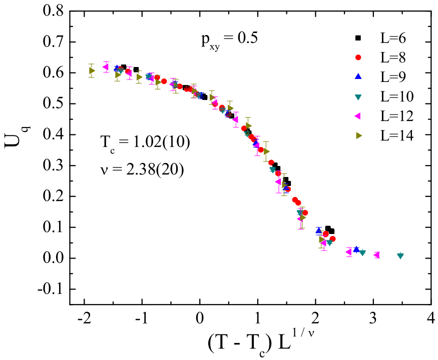
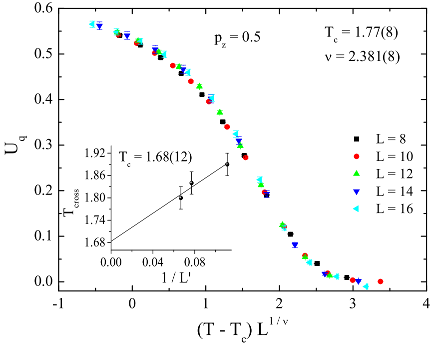
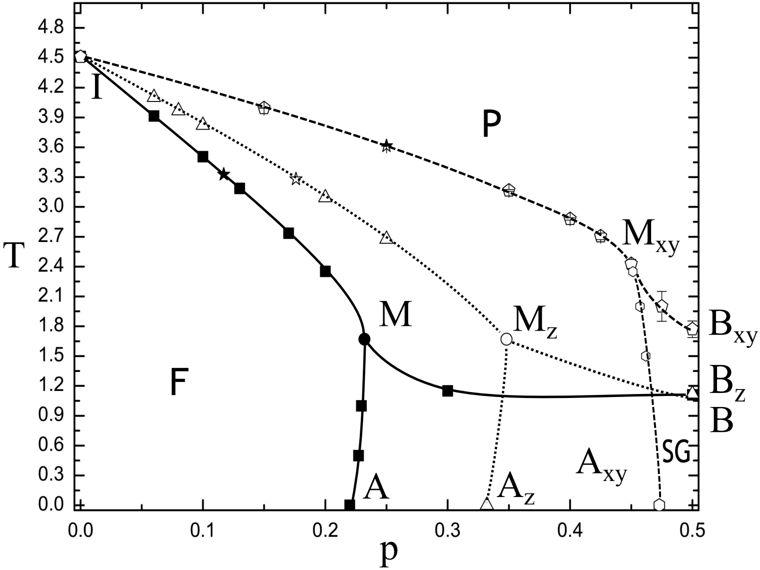


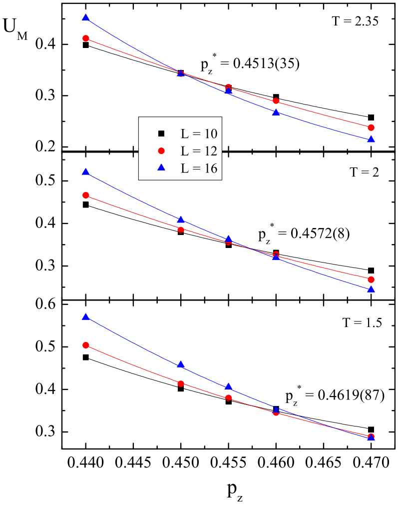
Longitudinal at $p_z = 0.5$



- **Isotropic**: $T_c = 1.109(10)$; $\nu = 2.45(15)$ [Hasenbusch *et al.*, PRB **78**, 214205 (2008)]
- **Transverse**: $T_c = 1.02(10)$; $\nu = 2.38(20)$
- **Longitudinal**: $T_c = 1.77(8)$; $\nu = 2.381(8)$


4.8
4.5
4.2
3.9
3.6
3.3
3.0
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6
0.3
0.0
T
0.0
0.1
0.2
0.3
0.4
0.5
p
I
P
F
M
M_z
M_{xy}
B_{xy}
B_z
B
SG
A
A_z
A_{xy}

Longitudinal model: F - SG transition line



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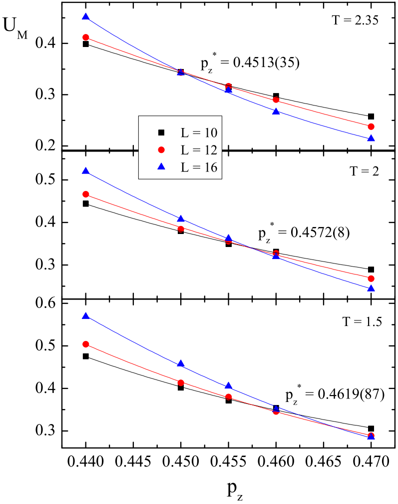


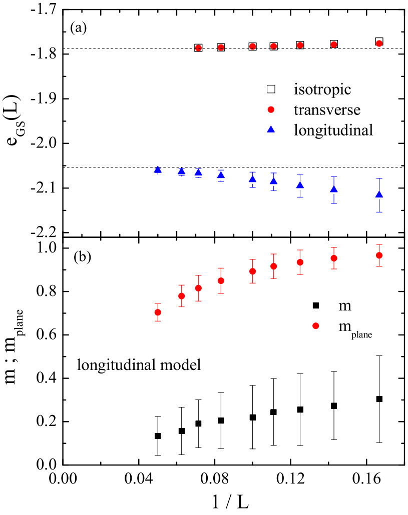
Forward behavior of the F - SG transition line

Summary of estimates for the longitudinal model

Collapse of the 4th-order Binder's cumulant

	U_M		U_q	
p_z	T_c	$1/\nu$	T_c	$1/\nu$
0.15	3.993(2)	1.465(9)	3.995(2)	1.46(2)
0.25	3.611(2)	1.464(12)	3.6128(9)	1.46(4)
0.35	3.172(2)	1.43(9)	3.166(9)	1.46(4)
0.4	2.90(2)	1.46(21)	2.880(12)	1.47(12)
0.425	2.74(3)	1.46(42)	2.705(33)	1.46(5)
0.45	2.41(12)	0.86(12)	2.43(5)	0.67(12)
0.475			2.00(15)	0.42(8)
0.5			1.77(8)	0.42(5)

Ground-state study



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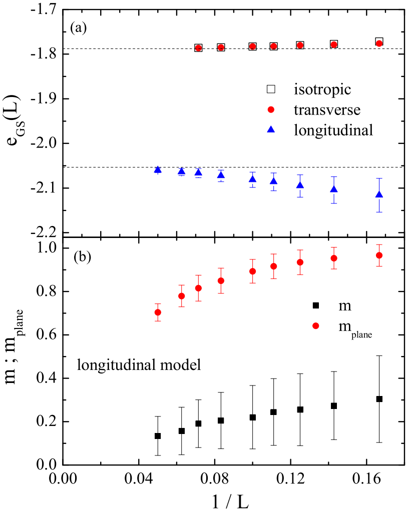
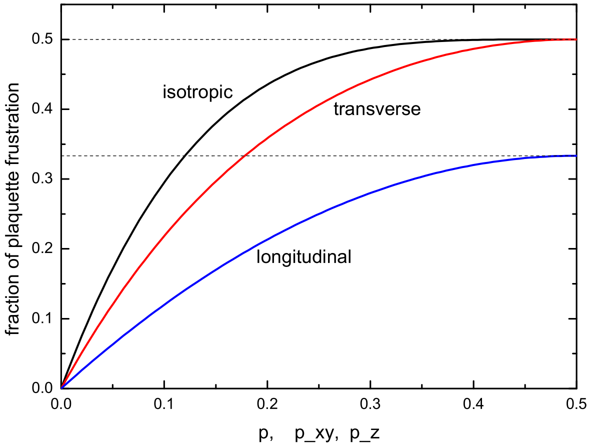


Lower e_{GS} and higher T_c for the **longitudinal** model

Disorder and Frustration



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Acknowledgements and reference papers

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- **Reference papers**:

  - T. Papakonstantinou and A.M., PRE **87**, 012132 (2013)
  - A.M. and T. Papakonstantinou, PRE **88**,013312 (2013)
  - T. Papakonstantinou, *et al.*, arXiv: 1410.8397