$\mathbb{Z}_2$ Lattice Gerbe Theory

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Plan of talk

Gerbes?

Lattice Gauge, Gerbe Theory

$\mathbb{Z}_2$ Lattice Gerbe Theory

Gerbes?

(Abelian) Gauge theory, one forms:

$$S = \frac{1}{4g^2} \int d^4x F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

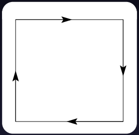
(Abelian) Gerbe theory, two forms:

$$S = \frac{1}{4g^2} \int d^dx \; H_{\mu\nu\lambda} H^{\mu\nu\lambda}$$

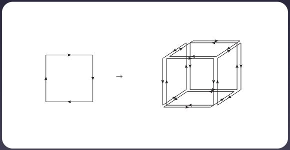
$$H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$$

Lattice Gauge/Gerbe Theory

Gauge - Hamiltonian defined on *plaquettes*



Gerbe - Hamiltonian defined on *cubets*



Higher Abelian Gauge theory

General Framework

$$H = -\sum_{C_{n+1}} \left(\prod_{C_n \in \partial C_{n+1}} U(C_n) + c.c. \right)$$

$U(C_n) = \exp(iA(C_n))$ live on the boundaries C_n of cells C_{n+1}

Hamiltonian given by the sum of products of the $U(C_n)$ around the boundary of a C_{n+1}

$\mathbb{Z}_2$ Gauge theory

Many of the properties are visible already in simplest $\mathbb{Z}_2$ case

Symmetries, observables (loops)

$$\Gamma(L) = \left\langle \prod_{C_1 \in L} U(C_1) \right\rangle$$

If a confining transition exists

$$\Gamma(L) \sim \begin{cases} \exp(-A(L)) & \beta < \beta_c \\ \exp(-P(L)) & \beta > \beta_c \end{cases}$$

$\mathbb{Z}_2$ Gerbe theory

Play same game with Gerbe theory

Symmetries, observables (surfaces)

$$\Gamma(S) = \left\langle \prod_{C_2 \in S} U(C_2) \right\rangle$$

If a confining transition exists

$$\Gamma(S) \sim \begin{cases} \exp(-V(S)) & \beta < \beta_c \\ \exp(-A(S)) & \beta > \beta_c \end{cases}$$

So Far, so general

We can use Wegner's results on duality for generalized Ising models (1971) to say more

Lattice N d-dimensional hypercubes

$M_{d,n}$ model, $N_s = \binom{d}{n-1} N$ spins sited at the centres of the $(n-1)$-dimensional hypercubes

Hamiltonians H_{dn}, product of $2n$ spins on the $(n-1)$-dimensional faces of the $N_b = \binom{d}{n} N$ n-dimensional hypercubes.

$M_{d,3}$ are lattice Gerbe theories

Schematically

Gauge $M_{d,2}$

$$H = -\sum_{\Box} U^4$$

Gerbe $M_{d,3}$

$$H = -\sum_{\cube} U^6$$

(Generalized) Ising duality

$M_{d,n}$ on d-dimensional hypercubic lattice and dual $M^*_{d,d-n}$ on dual lattice

$$Z_{d,n}(\beta) \sim Z^*_{d,d-n}(\beta^*)$$

$$\tanh(\beta) = \exp(-2\beta^*)$$

Duality also in field

$$Z_{d,n}(\beta, h) \sim Z^*_{d,d-n+1}(\beta^*, h^*)$$

$$\begin{aligned} \tanh(\beta) &= \exp(-2h^*) \\ \tanh(h) &= \exp(-2\beta^*) \end{aligned}$$

Consequences of (Generalized) Ising duality $d = 3$

$d = 3$, use in field duality

$$Z_{3,3}(\beta, 0) \sim Z^*_{3,1}(\infty, h^*)$$

Dualize back

$$Z_{3,3}(\beta, 0) \sim 2^{N_s} \left[\cosh(\beta)^{N_b} + \sinh(\beta)^{N_b}\right]$$

Analytic, no transition

Consequences of (Generalized) Ising duality $d = 4, 6..$

$d = 4$, $\mathbb{Z}_2$ Gerbe theory dual to standard Ising model

$$Z_{4,3}(\beta) \sim Z^*_{4,1}(\beta^*)$$

Continuous transition

$d = 6$, self-duality

$$Z_{2n,n}(\beta) \sim Z^*_{2n,n}(\beta^*)$$

$6d$ $\mathbb{Z}_2$ Gerbe theory is self dual, familiar critical temperature $\beta_c = \frac{1}{2}\ln(1+\sqrt{2})$

Making life more complicated

Gauge Higgs

$$H = -\sum_{\square} U^4 - \lambda \sum_{\langle ij \rangle} U \sigma^2$$

Gerbe Higgs

$$H = -\sum_{\cube} U^6 - \lambda \sum_{\square} U \sigma^4$$

Conclusions

Interest in formulating lattice Gerbe theories

Wegner has very thoughtfully calculated much of what is needed to discuss $\mathbb{Z}_2$ lattice Gerbe theory

These shed some light on Abelian and non-Abelian Gerbe theories

These models are curiously unsimulated.....

References

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