Universality in the $d = 3$ random-field Ising model

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Introduction

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- Some cherished concepts, i.e. the two-exponent scaling scenario ($\bar{\eta} = 2\eta$), have been recently questioned (Tissier and Tarjus, PRL **107**, 041601 (2011)).
- Universality in terms of different field distributions, or even for the same distribution but different values of the disorder strength, has been severely questioned.

Ingredients (I): Hamiltonian and the $T = 0$ scenario

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$$\mathcal{H}^{(\mathrm{RFIM})} = -J \sum_{\langle x,y \rangle} S_x S_y - \sum_x h_x S_x, \ ; \ S_x = \pm 1 \ ; \ J > 0$$

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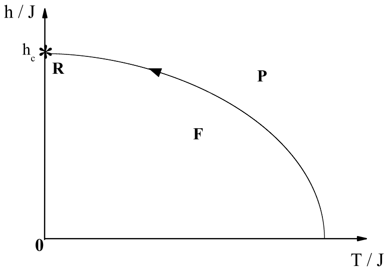
- Work at $T = 0$ using efficient optimization algorithms that calculate exact ground states. Avoid statistical errors and equilibration problems.



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- double Gaussian (dG):

$$\mathcal{W}^{(\mathrm{dG})}(h_x; h_R, \sigma) = \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{-\frac{(h_x-h_R)^2}{2\sigma^2}} + e^{-\frac{(h_x+h_R)^2}{2\sigma^2}}\right]$$

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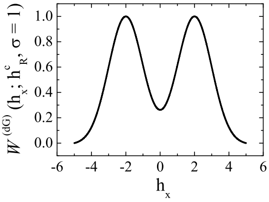

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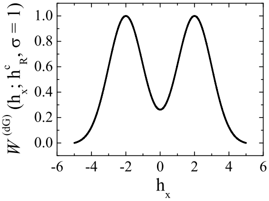

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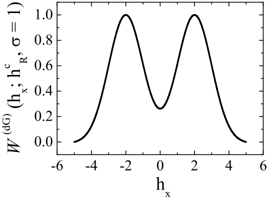

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- Poissonian (P): $\mathcal{W}^{(\mathrm{P})}(h_x; \sigma) = \frac{1}{2|\sigma|} e^{-|h_x|/\sigma}$

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- **Fluctuation-dissipation formalism**: Compute *connected* $G_{xy} = \overline{\frac{\partial \langle S_x \rangle}{\partial h_y}}$ and *disconnected* $G_{xy}^{\mathrm{dis}} = \overline{\langle S_x S_y \rangle}$ correlations functions. For *either* correlator $\rightarrow$ second-moment correlation length.

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- **Re-weighting extrapolation** on h_R and σ: From a single simulation, we extrapolate the mean value of observables to nearby parameters of the disorder distribution. Computing derivatives with respect to σ or h_R is straightforward.

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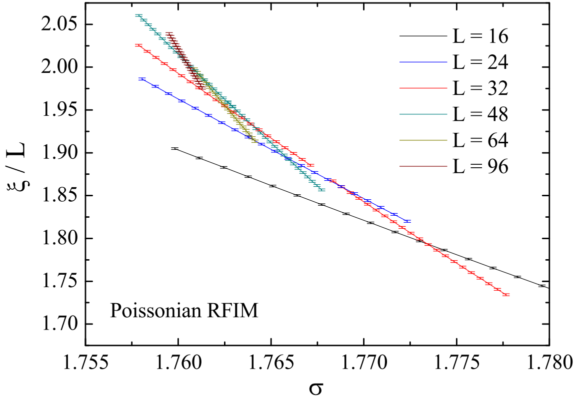

- For dimensionful quantities O, scaling in the thermodynamic limit as $\xi^{x_O/\nu}$, we consider the quotient $Q_O = O_{2L}/O_L$ at the crossing. For dimensionless magnitudes g, we focus on g_{2L}. In either case, one has:

$$Q_O^{\mathrm{cross}} = 2^{x_O/\nu} + \mathcal{O}(L^{-\omega})\,,\ g_{(2L)}^{\mathrm{cross}} = g^* + \mathcal{O}(L^{-\omega})\,,$$

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- Instances of dimensionful quantities are the derivatives of ξ, $\xi^{(\mathrm{dis})}$ ($x_\xi = 1 + \nu$), the connected and disconnected susceptibilities χ and $\chi^{(\mathrm{dis})}$ [$x_\chi = \nu(2 - \eta)$, $x_{\chi^{(\mathrm{dis})}} = \nu(4 - \bar{\eta})$], and the ratio $U_{22} = \chi^{(\mathrm{dis})}/\chi^2$ [$x_{U_{22}} = \nu(2\eta - \bar{\eta})$].

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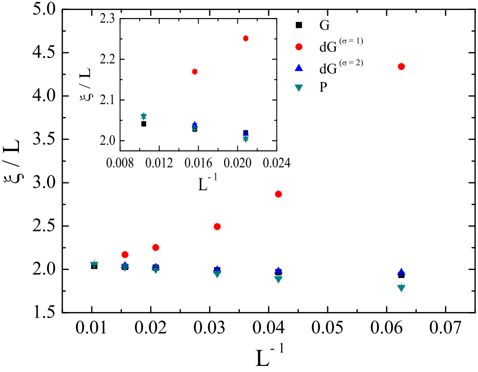

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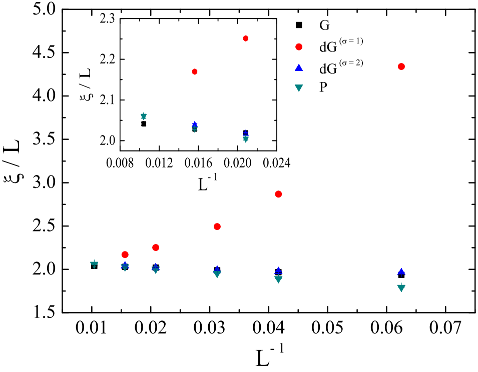
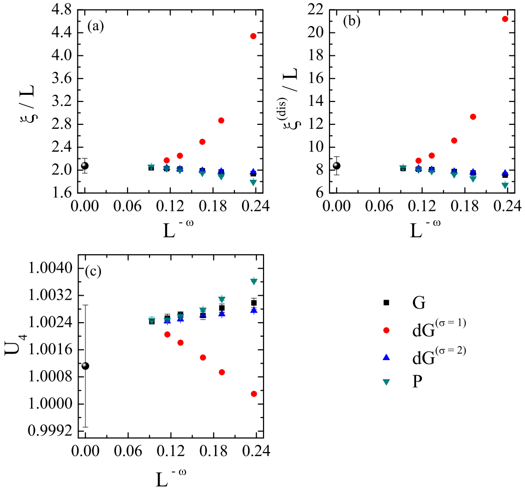


One needs extrapolation to $L \to \infty$.

Universality in the $d = 3$ RFIM

$A + BL^{-\omega} + CL^{-2\omega} + DL^{-3\omega}$; $L_{min} = 24$; $\omega = 0.52 \pm 0.11$; $\chi^2/dof = 18.83/14$, $Q = 0.17$ (full covariance-matrix!)



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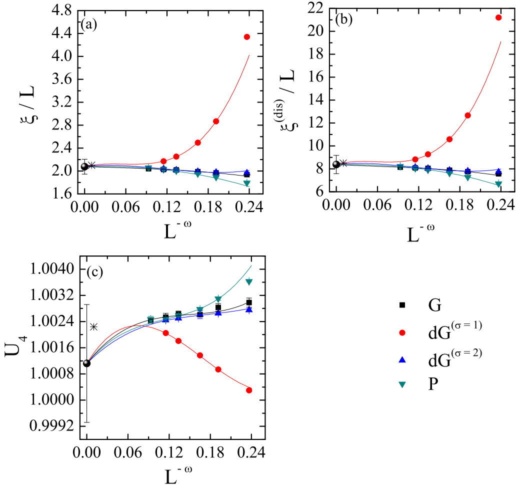
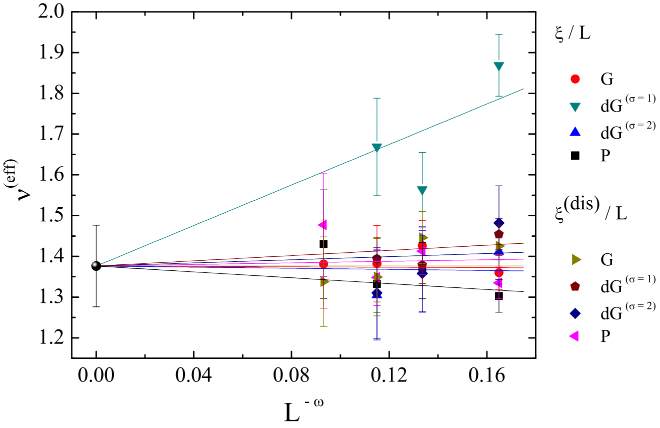


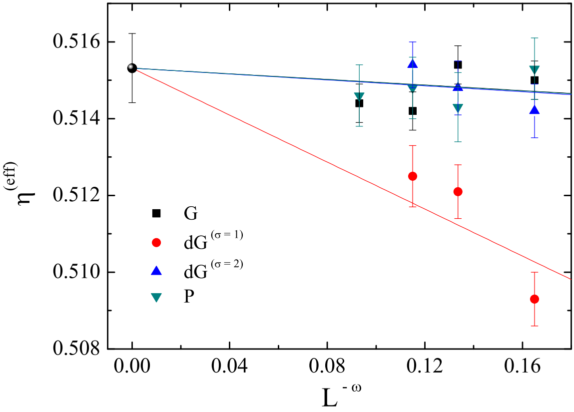
Extrapolation of ν

$\nu_L = \nu + BL^{-\omega}$; $L_{min} = 32$; $\omega = 0.52$;
$\chi^2/dof = 12.52/10, \ Q = 0.25$
Final estimate: $\nu = 1.38 \pm 0.10$



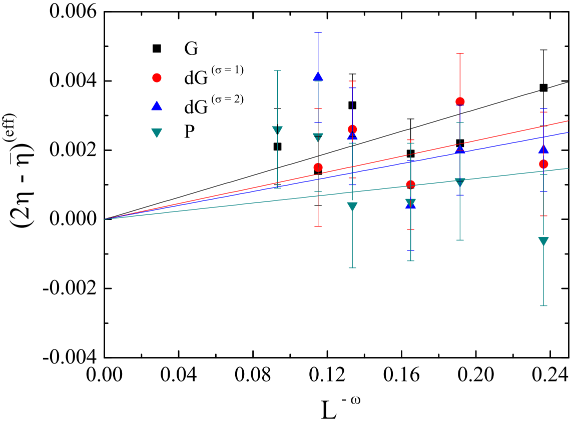
Extrapolation of η

$\eta_L = \eta + BL^{-\omega}$; $L_{min} = 32$; $\omega = 0.52$; $\chi^2/dof = 10/9$, $Q = 0.35$
Final estimate: $\eta = 0.5153 \pm 0.0009$



Extrapolation of $2\eta - \bar{\eta}$

$(2\eta - \bar{\eta})|_L = BL^{-\omega}$; $L_{min} = 16$; $\omega = 0.52$;
$\chi^2/dof = 18.26/18$, $Q = 0.44$



Summary of universal ratios and exponents

Extrapolation	χ^2/DOF	$L_{\min}$	Order in $L^{-\omega}$
$(\xi/L)\|_{L=\infty} = 2.08(13)$	18.8/14	24	third
$(\xi^{(\mathrm{dis})}/L)\|_{L=\infty} = 8.4(8)$			
$U_4\|_{L=\infty} = 1.0011(18)$			
$\omega = 0.52(11)$			
$\nu\|_{L=\infty} = 1.38(10)(0.03)$	12.5/10	32	first
$\eta\|_{L=\infty} = 0.5153(9)(2)$	10.0/9	32	first
$(2\eta - \bar{\eta})\|_{L=\infty} = 0$ (fixed)	18.3/18	16	first
$(2\eta - \bar{\eta})\|_{L=\infty} = 0.0026(9)(1)$	10.5/17	16	first
$\sigma^c[\mathrm{G}] = 2.27205(18)(4)$	3.1/3	16	second
$h_R^c[\mathrm{dG}^{(\sigma=1)}] = 1.9955(6)(24)$	2.5/1	24	second
$h_R^c[\mathrm{dG}^{(\sigma=2)}] = 1.0631(7)(10)$	0.7/2	16	second
$\sigma^c[\mathrm{P}] = 1.7583(2)(2)$	3.0/3	16	second

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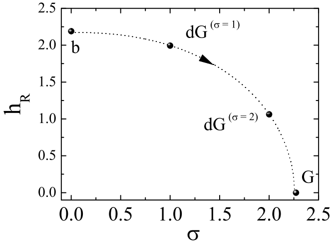

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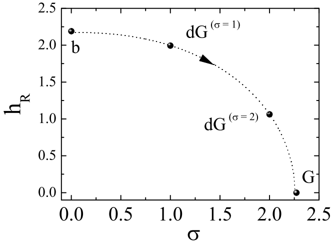

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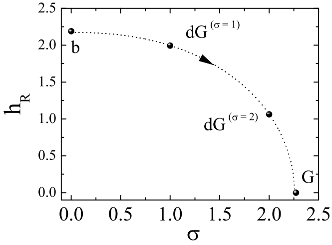

- Existence of **strong scaling corrections** that need to be carefully monitored. Very accurate computation of anomalous dimensions η, $\bar{\eta}$.
- The **two-exponent scaling scenario holds** within an accuracy of two parts in a thousand (2/1000).

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- Royal Society Research Grant in collaboration with Martin Weigel: "Simulating dirty magnets on GPUs".

Our "scientific" life in the AMRC