Random walks in power-law correlated disordered environments

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in cooperation with

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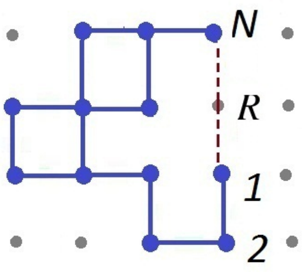
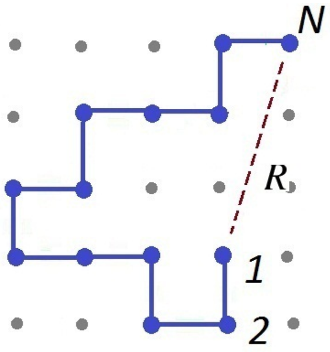




Random Walks (RWs)

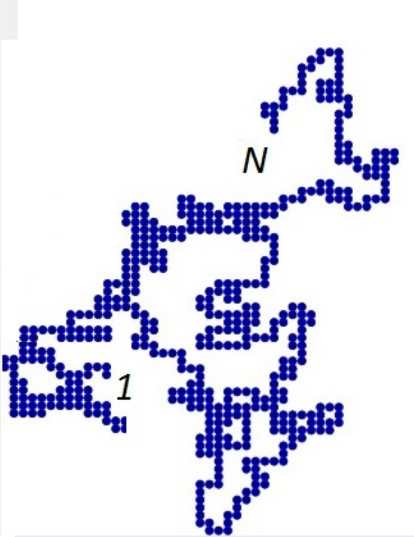
Self-Avoiding Walks (SAWs)

End-to-end distance: $\langle R^2 \rangle \sim N^{2\nu}$

Universal size exponent ν:

$$\nu_{\mathrm{RW}} = \frac{1}{2}$$

$$\nu_{\mathrm{SAW}} = \frac{3}{d+2}$$

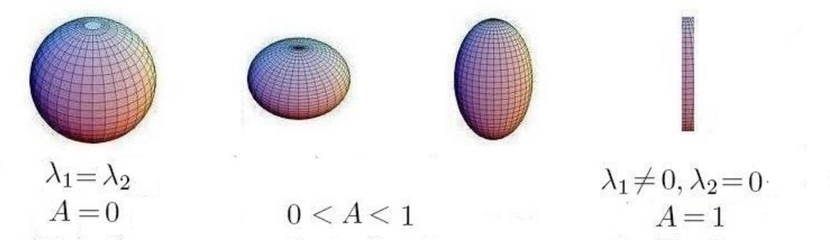


- Position vectors $\vec{R}_n = \{x_n, y_n\}$, $(n = 1, \ldots, N)$.
- Center of mass $\vec{R}_{CM}$ with $x_{CM} = \frac{\sum_{n=1}^{N} x_n}{N}$, $y_{CM} = \frac{\sum_{n=1}^{N} y_n}{N}$.
- Gyration tensor

$$Q_{xy} = \frac{1}{N} \sum_{n=1}^{N} (x_n - x_{CM})(y_n - y_{CM})$$

- Eigenvalues λ_i measures the shape

Definitions: Gyration tensor



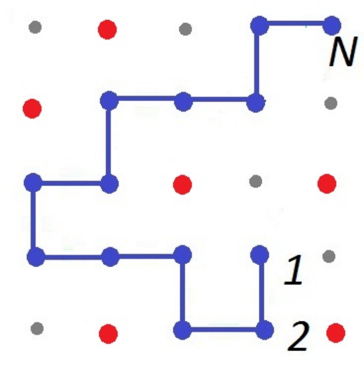
Random walk trajectories are asymmetrical (*W. Kuhn (1934); K. Solc and W.H. Stockmayer (1971)*)

Rotational invarint: Asphericity $A = \frac{1}{d(d-1)} \sum_{i=1}^{d} \frac{(\lambda_i - \overline{\lambda})^2}{\overline{\lambda}^2} = \frac{d}{d-1} \frac{\mathrm{Tr}\, \hat{\mathbf{Q}}^2}{(\mathrm{Tr}\, \mathbf{Q})^2}$

with $\overline{\lambda} \equiv \frac{1}{N} \sum_{i=1}^{d} \lambda_i = \frac{1}{N} \mathrm{Tr} \mathbf{Q}$, $\quad \hat{\mathbf{Q}} \equiv \mathbf{Q} - \mathrm{Tr} \mathbf{Q}\, \mathbf{I}$ (*J.A. Aronovitz and D.R. Nelson, J. Physique* **47** *1445 (1986).*)

- Random walk: $A_{RW}(d = 2) = 0.392 \pm 0.005$ (*J. Rudnick and G. Gaspari, J. Phys. A* **19**, *L191 (1986)*)
- Self-avoiding walk: $A_{SAW}(d = 2) = 0.501 \pm 0.003$ (*M. Bishop and C.J. Saltiel, J. Chem. Phys.* **88** *(1988)*)

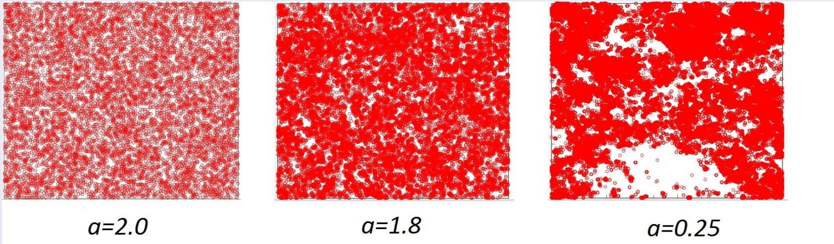
Random-walks in disordered environment



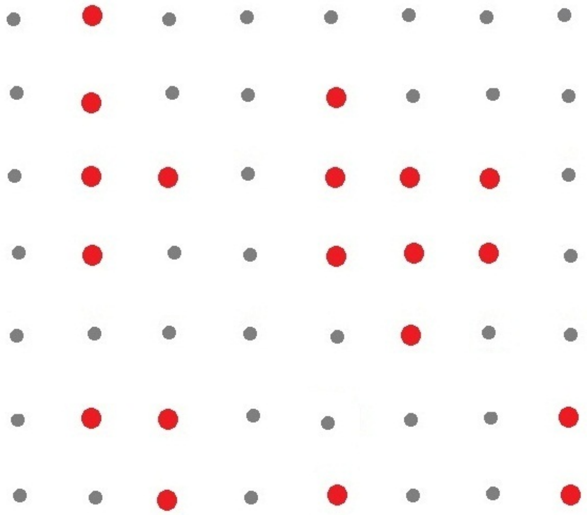
-  - site with defect
- p – concentration of disorder
- uncorrelated point-like deffects: universal size and shape characteristics are not altered unless $p > p_c$
  (*Y. Kim, J. Phys. C 16, 1345 (1983);*
  S. B. Lee and H. Nakanishi, Phys. Rev. Lett. 61, 2022 (1988).)
- p_c - percolation threshold
- $p_c(d = 2) = 0.407$
  (*R.M. Ziff, Phys. Rev. Lett. 72, 1942 (1942).*)

Long-range correlated disorder

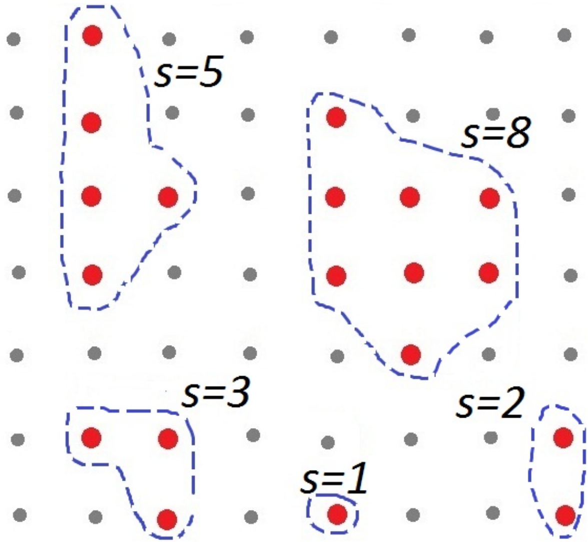
- Defects are correlated on large distances r according to a power law with a pair correlation function
$$C(r) \sim r^{-a}$$
(*A. Weinrib and B.I. Halperin, Phys. Rev. B* **27**, *413 (1983)*)
- $a = d$ – uncorrelated point-like defects
- $a < d$ – defects in the form of extended fractal clusters
- Fourier filtering method (FFM) (*S. Prakash et al., Phys. Rev. A 46 (1992); H.A. Makse et al., Phys. Rev. E 53 (1996)*)



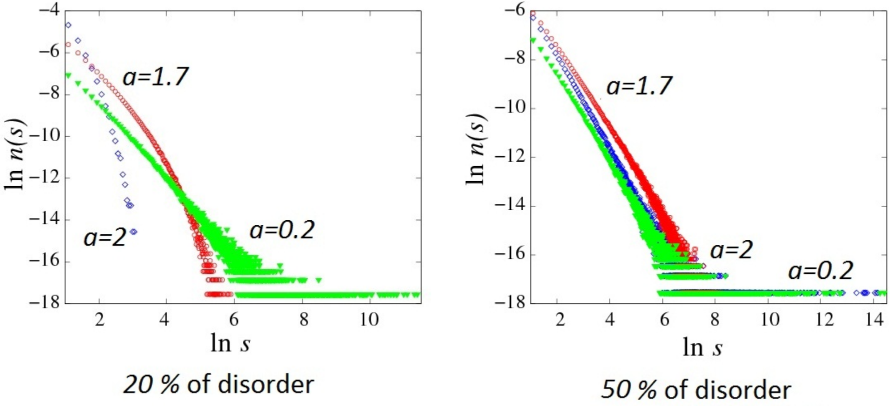
Cluster size distribution



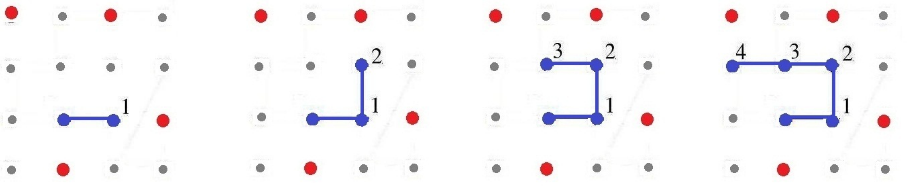
Cluster size distribution



Cluster size distribution



Pruned-enriched Rosenbluth method (PERM)



$m_1 = 3$ $m_2 = 2$ $m_3 = 2$ $m_4 = 2$

Weight of Nth step: $W_N = \prod_{l=1}^{N} m_l$ (*M.N. Rosenbluth, J. Chem. Phys.* **23**, *356 (1955)*)

Control parameters: W_n^{max} W_n^{min} (*P. Grassberger, Phys. Rev. E* **56**, *3682 (1997)*)

- $W_n < W_n^{min}$ – pruning with probability $1/2$, $W_n = 2W_n$
- $W_n > W_n^{max}$ – enrichment, $W_n = W_n/2$

Observables averaging

The configurational averaging for any observable O:

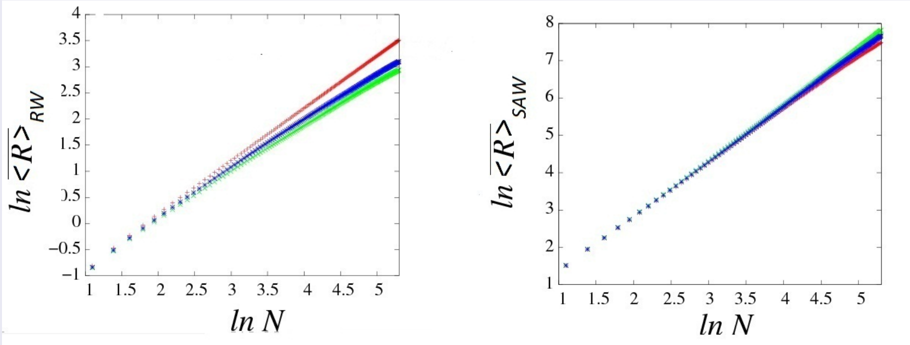
$$\langle O \rangle = \frac{\sum_{\text{conf}} W_N^{\text{conf}} O}{\sum_{\text{conf}} W_N^{\text{conf}}}, \qquad (1)$$

The averaging over different realizations of disorder, i.e., over different constructed disordered lattices

$$\overline{\langle O \rangle} = \frac{1}{M} \sum_{i=1}^{M} \langle O \rangle_i. \qquad (2)$$

We consider $M = 1000$ configurations of disorder in $d = 2$-dimensional lattices of linear size 2048×2048 at various values of correlation parameter a.

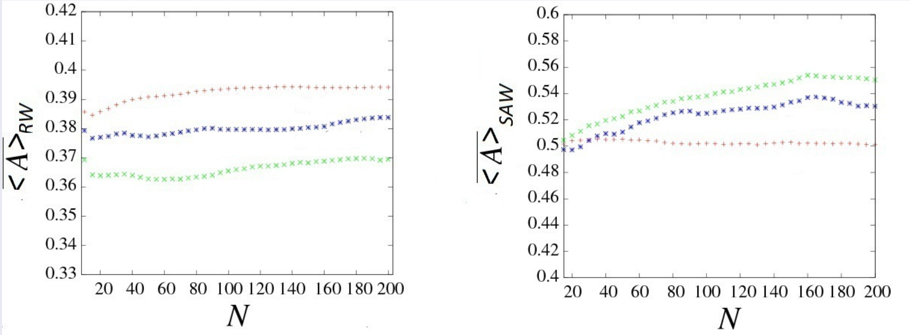
Size exponent



Averaged end-to-end distance of RW and SAW on a lattice with 20% of disorder at $a = 2.0$, $a = 1.75$, $a = 0.25$

a	ν_{RW}	ν_{SAW}
2.0	0.5 ± 0.001	0.750 ± 0.001
1.75	0.482 ± 0.001	0.757 ± 0.001
0.25	0.470 ± 0.001	0.764 ± 0.001

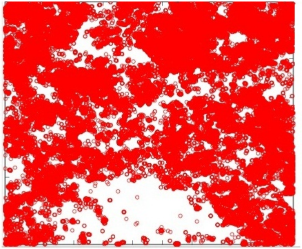
Asphericity



Averaged asphericity of RW and SAW on a lattice with 20% of disorder at $a = 2.0$, $a = 1.75$, $a = 0.25$

a	$\overline{\langle A \rangle}_{RW}$	$\overline{\langle A \rangle}_{SAW}$
2.0	0.393 ± 0.002	0.501 ± 0.003
1.75	0.380 ± 0.002	0.538 ± 0.003
0.25	0.370 ± 0.002	0.546 ± 0.003

Conclusions



- Defects are correlated on large distances r according to a power law with a pair correlation function

$$C(r) \sim r^{-a}$$

- The RWs are more compact and symmetric in correlated environments
- The SAWs are more extended and asymmetric in correlated environments