Disentanglement of two spins coupled to an Ising chain : sudden quench dynamics

Pierre Wendenbaum

work done with Dragi Karevski

Institut Jean Lamour, Groupe de Physique Statistique, Université de Lorraine-CNRS, Nancy

CompPhys13, Leipzig

November 29, 2013

Introduction

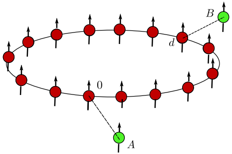
- Entanglement used as a resource for new quantum technologies (quantum computation, quantum teleportation...) Nielsen & Chuang 00, Bennett *et al.* 93
- Necessity to preserve entanglement between two distant objects
- Interaction with the surrounding environment → decoherence and loss of entanglement Zurek 02, Zurek 03
- We propose to study the disentanglement of two spins coupled locally to an Ising chain after the quench of its magnetic field

Model and theoretical description

- We consider two defect spins coupled to a quantum Ising chain
  Quan *et al.* 06, Yuan *et al.* 07, Cucchietti *et al.* 07, Mukherjee *et al.* 07, Rossini *et al.* 07, Cormick & Paz 07

Hamiltonian

$$H = -\sum_{n=0}^{N-1} \sigma_n^x \sigma_{n+1}^x - \lambda \sum_{n=0}^{N-1} \sigma_n^z - \varepsilon(|\uparrow\rangle\langle\uparrow|_A \otimes \sigma_0^z + |\uparrow\rangle\langle\uparrow|_B \otimes \sigma_d^z)$$



- Initial state $\rightarrow |\psi(0)\rangle = |\phi\rangle_{AB} \otimes |G(\lambda_i)\rangle_b$ with $|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$
- Sudden quench $\lambda_i \rightarrow \lambda_f$
- At a latter time $t \rightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \otimes |\varphi_{\uparrow\uparrow}(t)\rangle_b + |\downarrow\downarrow\rangle \otimes |\varphi_{\downarrow\downarrow}(t)\rangle_b\right)$ where

$$|\varphi_{\uparrow\uparrow}(t)\rangle_b = e^{-iH_{\uparrow\uparrow}(\lambda_f)t}|G(\lambda_i)\rangle_b$$
$$|\varphi_{\downarrow\downarrow}(t)\rangle_b = e^{-iH_{\downarrow\downarrow}(\lambda_f)t}|G(\lambda_i)\rangle_b$$

with the two effective Hamiltonians $H_{\downarrow\downarrow}(\lambda_f) = H_b(\lambda_f)$ and $H_{\uparrow\uparrow}(\lambda_f) = H_b(\lambda_f) - \varepsilon(\sigma_0^z + \sigma_d^z)$

Model and theoretical description

Reduced density matrix

$$\rho_s(t) = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & D_{\uparrow\uparrow,\downarrow\downarrow}(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ D_{\downarrow\downarrow,\uparrow\uparrow}(t) & 0 & 0 & 1 \end{pmatrix}$$

with $D_{\uparrow\uparrow,\downarrow\downarrow}(t) = \langle\varphi_{\downarrow\downarrow}(t)|\varphi_{\uparrow\uparrow}(t)\rangle = \langle G(\lambda_i)|e^{iH_{\downarrow\downarrow}(\lambda_f)t}e^{-iH_{\uparrow\uparrow}(\lambda_f)t}|G(\lambda_i)\rangle \in [0,1]$ the decoherence factor

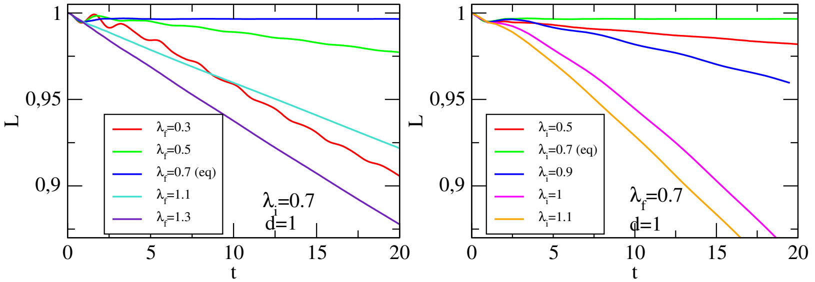
- Entanglement determined by the concurrence $\mathcal{C}(t) \in [0:1]$ Wootters 98

$$\mathcal{C}_{AB}(t) = \max\{0, \sqrt{\mathscr{L}(t)}\}$$

where $\mathscr{L}(t) = \left|D_{\uparrow\uparrow,\downarrow\downarrow}(t)\right|^2 = \left|\langle G(\lambda_i)|e^{iH_{\downarrow\downarrow}(\lambda_f)t}e^{-iH_{\uparrow\uparrow}(\lambda_f)t}|G(\lambda_i)\rangle\right|^2$ is the Loschmidt Echo

Results : Effect of the quench

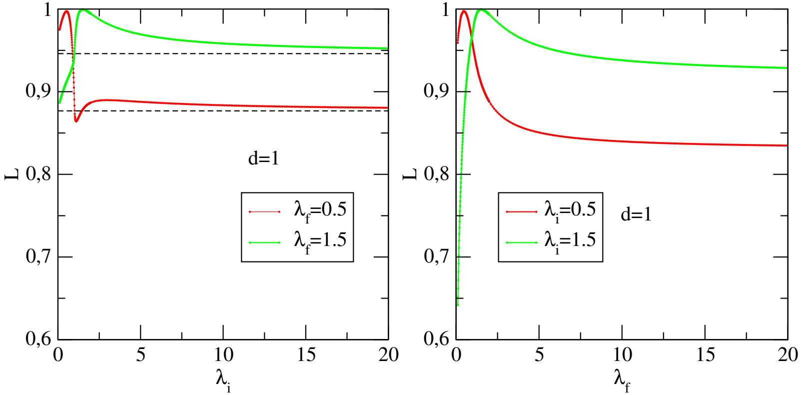
- Effect of the quench on the Loschmidt Echo



- Smaller decoherence in the equilibrium situation → Disentanglement enhanced by the quench
- Bigger disentanglement for strong quench amplitude $|\lambda_f - \lambda_i|$

Results : Effect of the quench

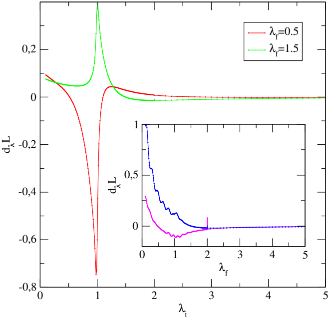
- $\mathscr{L}(t = 10)$ as a function of λ_i (left) and λ_f (right)

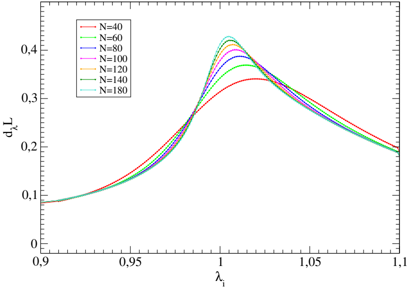


- $\mathscr{L}(t = 10)$ increasing for $\lambda_i < \lambda_f$ (left) and $\lambda_f < \lambda_i$ (right)
- For very high initial field → saturation of the echo corresponding to a completely polarized initial state (dashed lines)

Results : Effect of the quench

- Signature of the quantum phase transition experienced by the bath for the λ_i's variation





- Changes in the ground state properties for λ_i close the critical value 1
- Finite size scaling in the derivative of the Loschmidt Echo
- $|\lambda_c - \lambda_{max}| \sim c_1 N^\gamma$
- $\left.\dfrac{d\mathscr{L}}{d\lambda_i}\right|_{\lambda_{max}} \sim c_2 \ln N + \text{constant}$
- Scaling coherent with the litterature Osterloh *et al.* 02

Results : Short times behavior

- Gaussian evolution for short times

$$\mathscr{L}(t) = e^{-\alpha t^2} \approx 1 - \alpha t^2 \quad \text{with} \quad \alpha = \langle H_i^2 \rangle - \langle H_i \rangle^2, \quad \langle . \rangle = \langle G(\lambda_i)|.|G(\lambda_i)\rangle$$

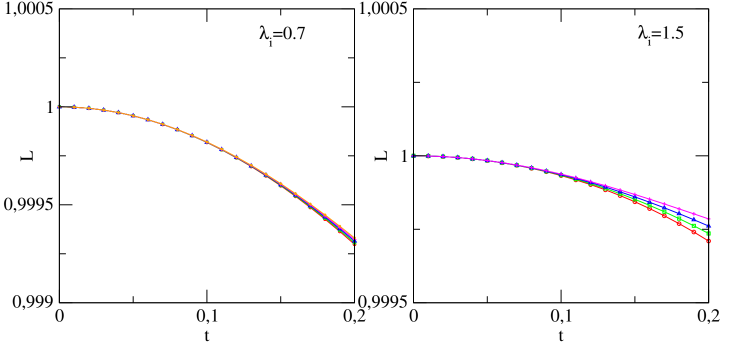
$\rightarrow$ short times evolution independant of the quench in the bath

Results : Short times behavior

- Gaussian evolution for short times

$$\mathscr{L}(t) = e^{-\alpha t^2} \approx 1 - \alpha t^2 \quad \text{with} \quad \alpha = \langle H_i^2 \rangle - \langle H_i \rangle^2, \quad \langle . \rangle = \langle G(\lambda_i) | . | G(\lambda_i) \rangle$$

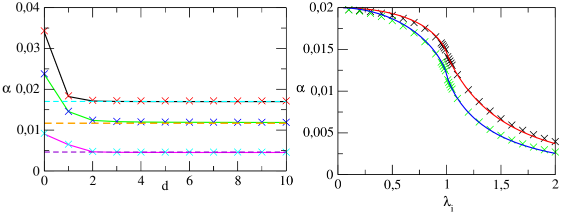
$\rightarrow$ short times evolution independant of the quench in the bath



- $\alpha = 4\varepsilon^2 \sum_{k \neq l} \Big[(g_{0k}h_{0l})^2 + (g_{dk}h_{dl})^2 + 2h_{dl}h_{0l}g_{dk}g_{0k} - 2h_{dk}h_{0l}g_{dl}g_{0k} - h_{0k}h_{0l}g_{0k}g_{0k} - h_{dk}h_{dl}g_{dk}g_{dk} \Big]$

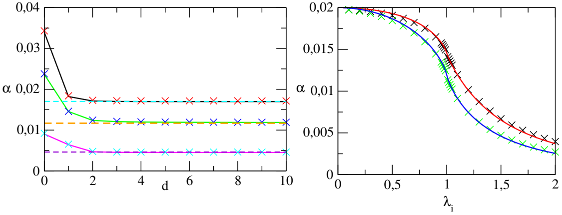
Results : Short time behavior

- Dependence of α with the parameters

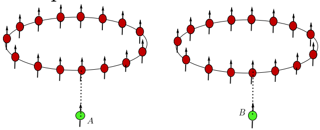


Results : Short time behavior

- Dependence of α with the parameters

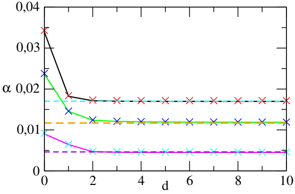


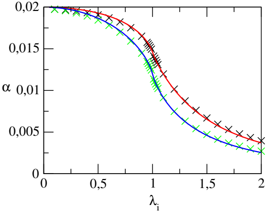
- Saturation of α with the distance :
  the spins decohere as in the situation



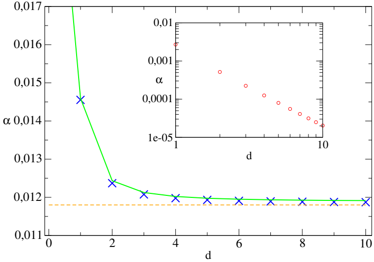
Results : Short time behavior

- Dependence of α with the parameters

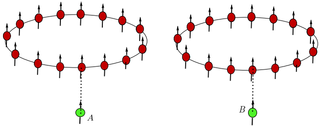




- Except when the bath is prepared at criticality

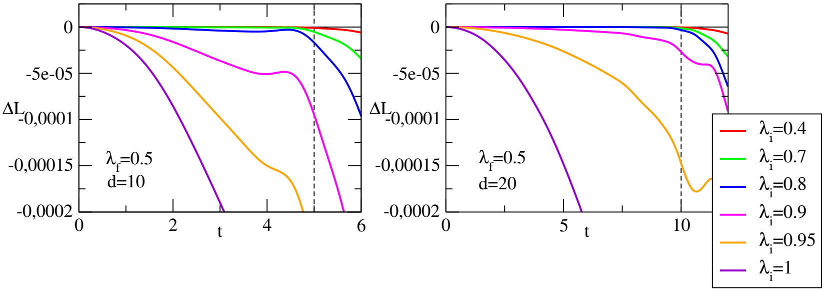


- Saturation of α with the distance : the spins decohere as in the situation



Results : Independant dynamics

- We look at $\Delta\mathscr{L}$, the difference between the Echo of two spins coupled to two different baths and coupled to a single bath → part of the decoherence directly due to the mutual interaction through the bath



- $\Delta\mathscr{L}$ starts to be different from 0 at the beginning of the evolution for intial magnetic fields close to 1
  → Long range correlations in the initial state

Conclusion

- Disentanglement always stronger in the quench situation than in the equilibrium one
- Short times dynamics independant of the quench
- Signature of the quantum phase transition in the Loschmidt Echo

Calculation of the Loschmidt Echo

- $\mathscr{L}(t) = \left|\langle\varphi_{\downarrow\downarrow}(t)|\varphi_{\uparrow\uparrow}(t)\rangle\right|^2 = \sqrt{\left|\det(\mathbb{1} - (C_{\downarrow\downarrow}(t) - C_{\uparrow\uparrow}(t)))\right|}$ with $C_{\downarrow\downarrow}(t)$ and $C_{\uparrow\uparrow}(t)$ the covariance matrices associated to the states $|\varphi_{\downarrow\downarrow}(t)\rangle$ and $|\varphi_{\uparrow\uparrow}(t)\rangle$
  Kiel & Schlingemann 10
- Finally

$$\mathscr{L}(t) = \mathcal{C}_{AB}^2 = \sqrt{\left|\det(\mathbb{1} - (e^{-it\mathcal{H}_{\downarrow\downarrow}(\lambda_f)}C(0)e^{it\mathcal{H}_{\downarrow\downarrow}(\lambda_f)} - e^{-it\mathcal{H}_{\uparrow\uparrow}(\lambda_f)}C(0)e^{it\mathcal{H}_{\uparrow\uparrow}(\lambda_f)}))\right|}$$