Magnetic properties of two-color QCD at finite temperature

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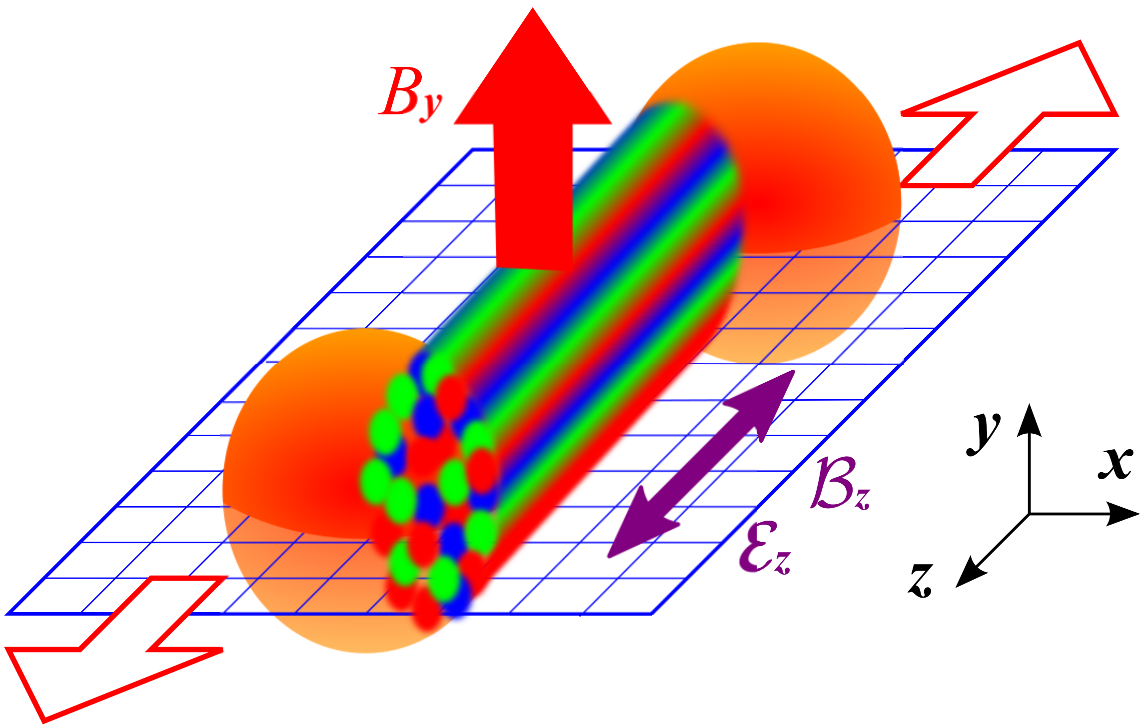
Motivations:Non-central heavy ion collisions at RHIC and LHC



$B_y \sim m_\pi^2 \sim 10^{18}$ **Gauss**

$T \sim m_\pi$

Furthermore

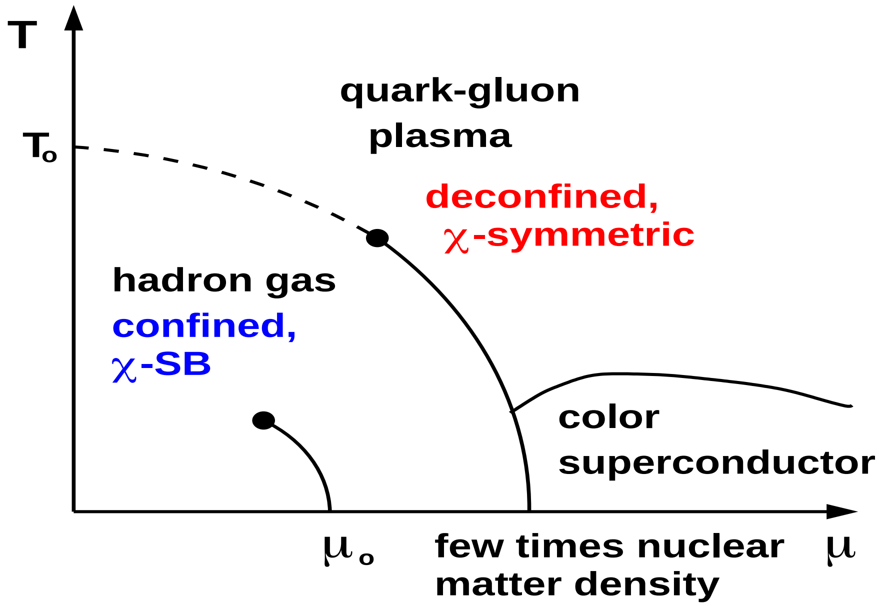
- **Very strong magnetic fields in the early universe** $\sim 10^{23}$ **Gauss**
- **Strong magnetic fields in compact stars (magnetars)** $\sim 10^{13}$ **Gauss**

(Largest man-made magnet $= 10^{6}$ **Gauss)**

Non perturbative methods necessary

- **Models**
- **Lattice simulations**

Usually: (T,μ) Phase diagram (μ = Baryon chem. pot.)



This talk: (T,B) Phase Diagram

$\bar{B} = Be_z$ B constant and homogeneous

Lattice formalism

Lattice size: $N_\sigma^3 \times N_\tau \;\equiv \mathcal{V}$

$V = (N_\sigma a)^3$

$T = \frac{1}{N_\tau a}$ $\qquad a =$ lattice spacing

$Z(T, B, \mu) = \int dP(U)$ $\qquad < \mathcal{O} >= \frac{1}{Z}\int \mathcal{O}(U) dP(U)$

U **stands for** $U_\mu(x) = e^{iA_\mu(x)} \in SU(N)$

$dP(U) = \prod dU e^{-\beta S_g(U)} Det(D(e^{iqBa^2}, e^{\mu a}) + ma)$ $\;> 0$ onlyif $\mu = 0$ for $N > 2$

$\beta =$ **bare coupling constant,** $\quad ma$= **bare quark mass,** $\quad a = a(\beta)$

Our calculations

$SU(2) \quad \mu = 0 \qquad N_f = 4$

$32^3 \times N_\tau \qquad N_\tau = 10, 8, 6, 4 \qquad \beta = 1.8$ **fixed scale approach**

Implementing a constant magnetic field on a periodic lattice $\Rightarrow$

$\phi = a^2 qB = 2\pi N_b / N_\sigma^2$, $\qquad N_b \in Z \qquad \phi$ **periodic** $\Rightarrow N_b < N_\tau^2/4$

Reasons for our choice

Simpler theory but similar chiral properties as in QCD

Action local on the lattice, some chiral symmetry left

Can be extended to $\mu \neq 0$

Simulations with CUDA Fortran on GPU:s

Chiral order parameter **(Exact for m=0)**

$$a^3 < \bar{\chi}\chi >= \frac{1}{4\mathcal{V}} < \mathrm{Tr}(D+m)^{-1} >= -\frac{1}{4\mathcal{V}} \; \frac{\partial}{\partial m} \log(Z)$$

$< \bar{\chi}\chi >$ **is the chiral condensate**

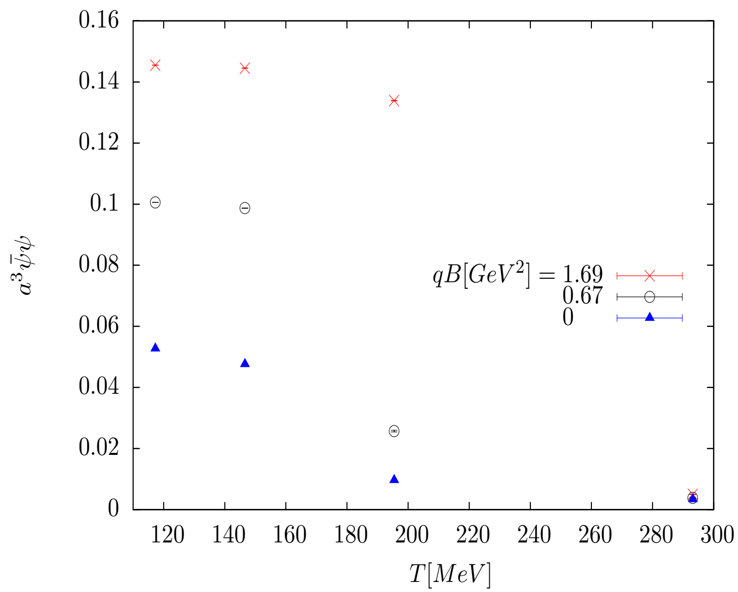
Confinement order parameter **(Exact for $m \to \infty$)**

$$< L >= \frac{1}{N_\sigma^3} \sum_{\bar{n}} < \frac{1}{2} \mathrm{Tr} \prod_{n_4=1}^{N_\tau} U(\bar{n}, n_4) > \sim \; e^{-\frac{F_q}{T}}$$

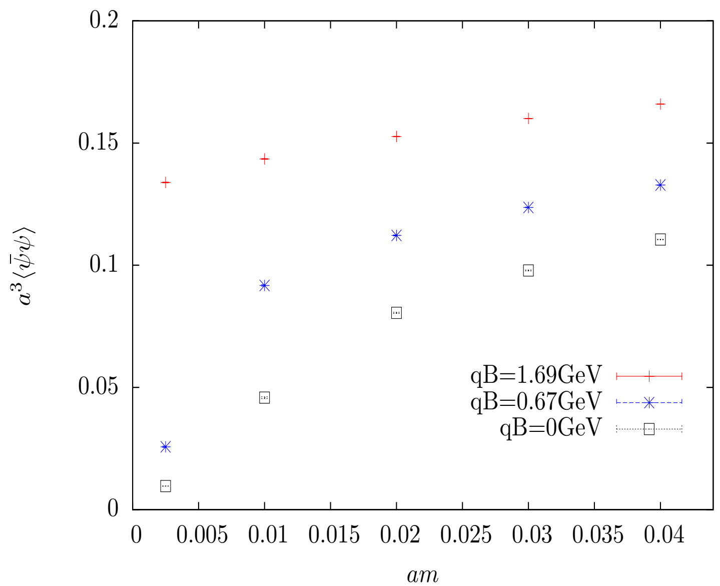
F_q **is the free energy of a static quark**

Magnetic catalysis: The chiral condensate increases with B

Scale from Static potential $ma = 0.0025 \Rightarrow m_\pi = 175\,\mathrm{MeV}$ at $B = 0$



$ma = 0.0025$



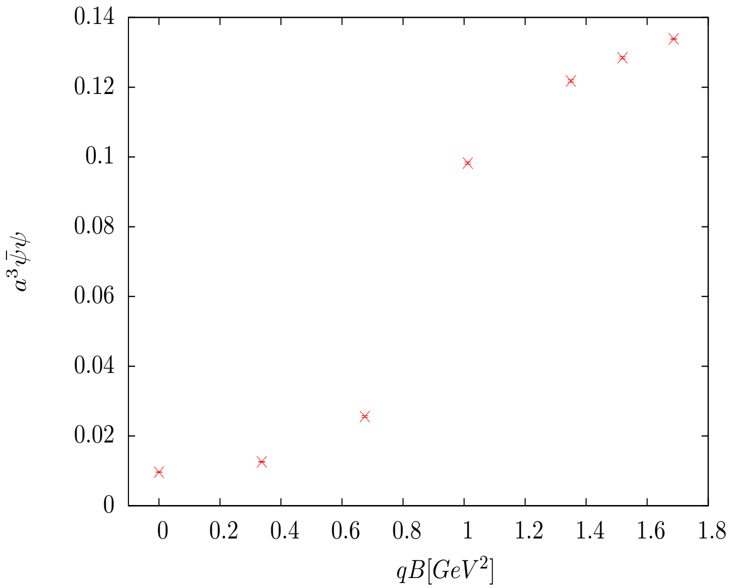
$T = 193\,\mathrm{MeV}$

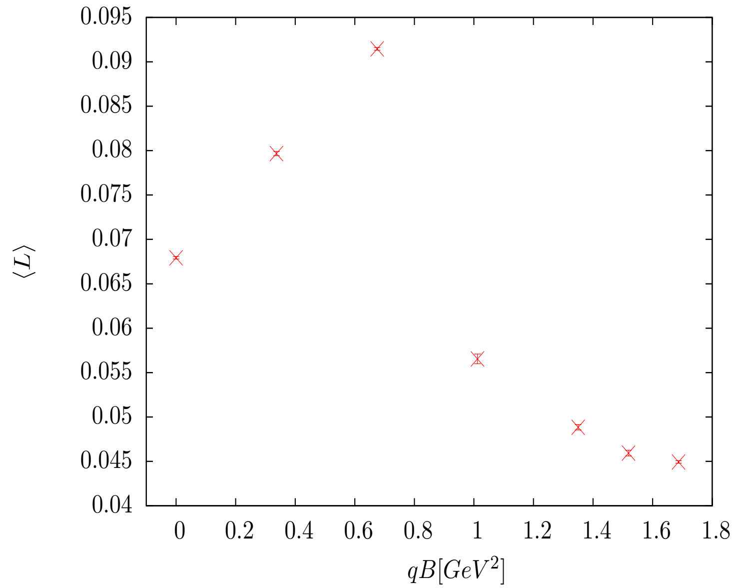
$T_c(B = 0) \approx m_\pi$

Magnetic catalysis in the transition region?

(Regensburg-Wuppertal collaboration claims the opposite!)

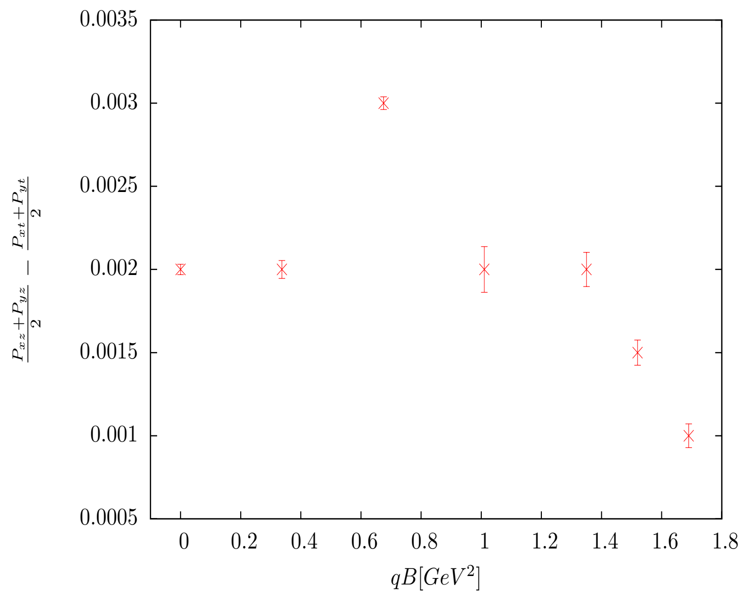
Our data

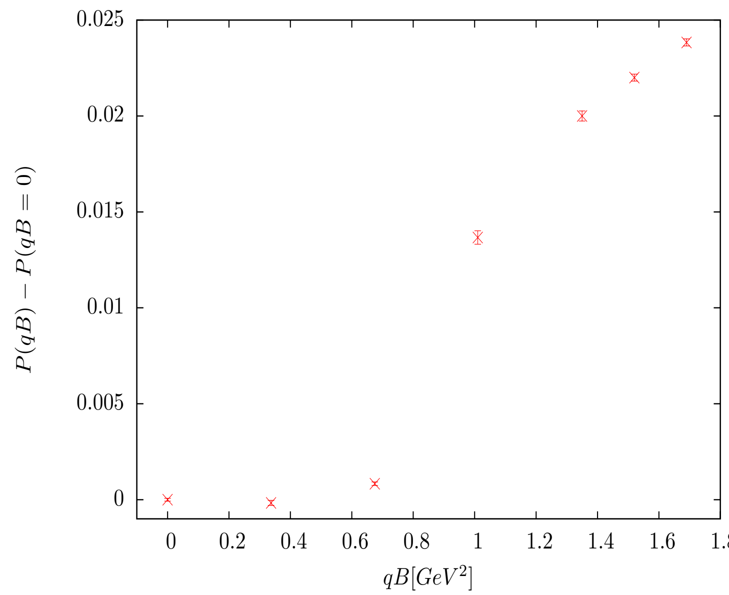




$$ma = 0.0025 \qquad T = 193\,\mathrm{MeV}$$

The indirect influence of B on the gluons





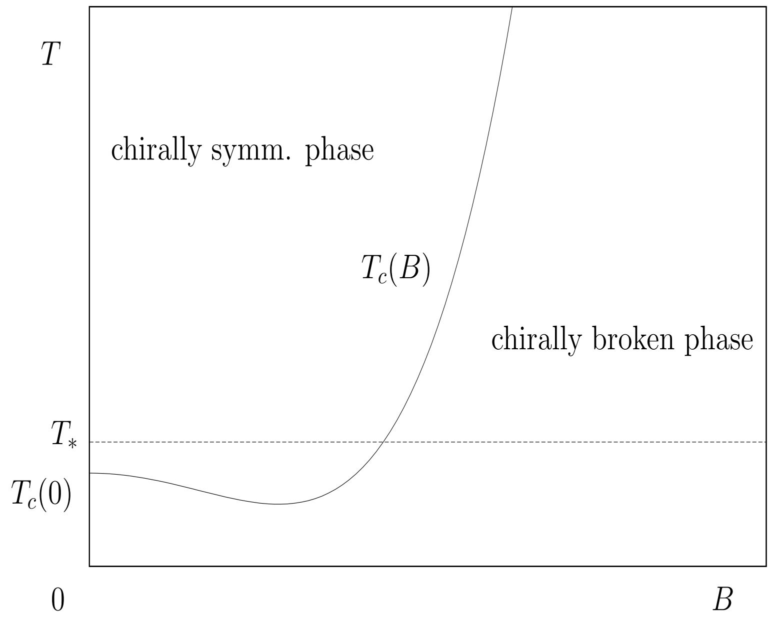
Transverse part of the gluonic energy density

Gluonic part of the conformal anomaly

$$ma = 0.0025 \qquad T = 193\,\mathrm{MeV}$$

Conclusions

Phase Diagram



Next: **Simulations nearer T_c, smaller masses ,$\mu \neq 0$**

cme-effect

$D + ma \equiv M \qquad DetM = DetM^\dagger \qquad M^\dagger M > 0$

$dP = e^{-\beta S_g}(DetM)^2 = e^{-\beta S_g}(DetM^\dagger M) = d\phi^\dagger d\phi dP e^{-\beta S_g + \phi^\dagger (M^\dagger M)^{-1}\phi}$

$(M^\dagger M)_{eo} = 0$ choose $(M^\dagger M)_{ee}$

Molecular dynamics:

$dPdpd\phi^\dagger d\phi e^{-H} \qquad H = \sum_n p_\mu^2(n) + S_{eff}(U, \phi)$

Discretized Eq. of motion, Omelyan integration

Solve $(M^\dagger M)\eta = \phi$ $M^\dagger M$ sparse $ma < \lambda < 1 \Rightarrow$

Precond. Conj. Gradient on GPU

At end of trajectory Metropolis update $\Rightarrow$ Algorithm exact