Simulated Quantum Annealing For General Ising Models

Thomas Neuhaus

Jülich Supercomputing Centre, JSC
Forschungszentrum Jülich
Jülich, Germany

e-mail : t.neuhaus@fz-juelich.de

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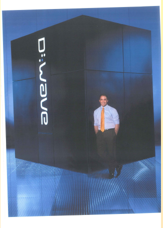
On the Talk

- Quantum Annealing
- DWAVE
- Simulated Quantum Properties

Quantum Annealing Board in a Box



supposed to do Quantum Annealing:

$$H_{\text{Ising}} = -\sum_{<ij>}^{M} J_{ij}\sigma_i^z\sigma_j^z + \sum_{i}^{N} h_i\sigma_i^z - \Gamma\sum_{i}^{N}\sigma_i^x$$

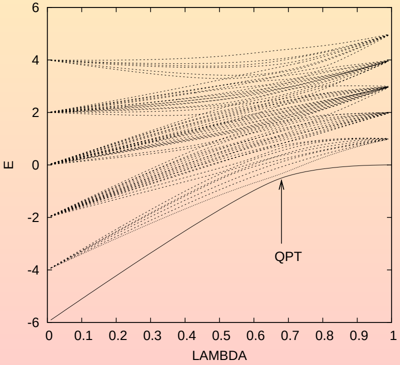
- Ising model
- $-1 \leq J_{ij} \leq +1$
- transverse field Γ
- number of clauses $\boxed{M}$

- non-planar graph
- $-1 \leq h_i \leq +1$
- number of spins $\boxed{N}$

Quantum Annealing

switch to adiabatic control: $\boxed{0 \leq \lambda \leq 1}$

$$H_{\text{Ising}} = -\lambda \sum_{<ij>}^{M} J_{ij}\sigma_i^z\sigma_j^z + \lambda \sum_{i}^{N} h_i\sigma_i^z - (1-\lambda)\sum_{i}^{N} \sigma_i^x$$



- Landau Zener crossing
- gap correlation length

$$\xi_{\text{GAP}} = \frac{1}{(E_1 - E_0)}\Big|_{\text{min}}$$

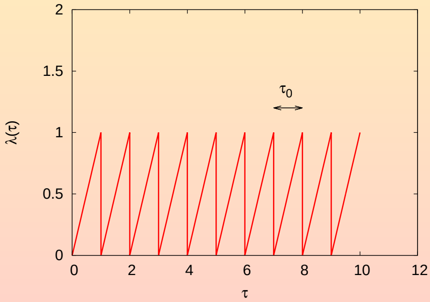
- diabatic probability

$$P_{\text{DIABATIC}} = e^{h\,\frac{+\pi^2}{\xi^2_{\text{GAP}}\,|\partial_\tau \lambda|}}$$

- $P_{\text{ADIABITC}} = 1 - P_{\text{DIABATIC}}$

Chain Tooth Sampling Schedules

- short trajectories of length $\boxed{\tau_0}$
- $\mathcal{O}(1000)$ repetitions
- dynamics: on the chip
- : Schrödinger equation
- : simul. quantum annealing
- : Newtons equation of motion
- : simulated annealing
- named QA,SQA,CA,SA

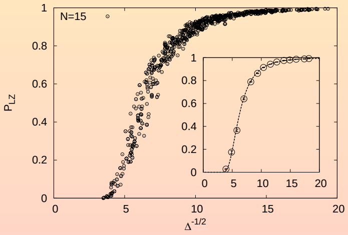


- Histogram: $\boxed{N[H_{\text{Ising}}(\lambda = 1)]}$
- success probability: $\boxed{P}$ to find a ground state
- run time at median success:

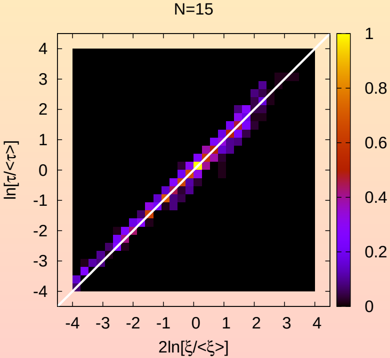
$$\boxed{\tau} = \frac{\ln(0.5)}{\ln(1 - \boxed{P})} \boxed{\tau_0}$$

Landau Zener and the Quantum Monte Carlo

- 1000 problems in HARD 2SAT
- Metropolis updates
- diabatic probabilty P_{LZ} vs. $\sqrt{\xi_{\mathrm{GAP}}}$:



- Trotter regularization
- Multi Spin Coding
- run time in MCS $\ln(\tau / < \tau >)$ vs. $2\ln(\xi_{\mathrm{GAP}} / < \xi_{\mathrm{GAP}} >)$:



Residual Energy in Annealing

- SA:
- Huse, D. Fisher PRL 1986:

Residual Energies after Slow Cooling of Disordered Systems

David A. Huse and Daniel S. Fisher
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 12 June 1986)

The residual energy, $\epsilon(\tau)$, left after cooling to zero temperature in a finite time τ is analyzed for various disordered systems, including spin-glasses and random-field magnets. We argue that the generic behavior for such frustrated systems is $\epsilon(\tau) \sim (\ln\tau)^{-\zeta}$ for large τ, with the exponent ζ depending on the system. This result is dominated in some cases by a distribution of classical two-level systems with low excitation energies, and in other cases by large-scale nonequilibrium effects.

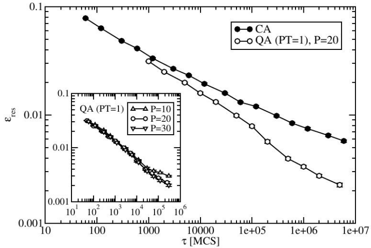
PACS numbers: 75.10.Hk, 75.50.Kj

$$\boxed{\epsilon(\tau) \propto \ln(\tau)^{-\xi}}$$

$$\xi = \min[2, 2(d-\theta)/(d_s/2+2\Psi-\theta)]$$

droplet model

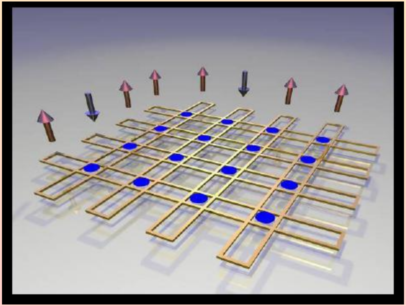
- SQA:
- Santoro, Martonak, Tosatti, Car , Sience 2002
- 2D Ising glass on 80^2
- $-2 \leq J_{ij} \leq +2$ flat distribution
- finding: $\boxed{\xi_{\mathrm{SA}} < \xi_{\mathrm{SQA}} < 6}$



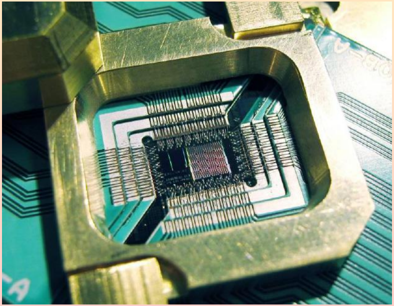
DWAVE

- 2012: USC-Lockheed Martin Quantum Computation Center
  N=128+N=512
- 2013: Nasa, QUAIL, Quantum Artificial Intelligence Laboratory
  N=512
- in the near future apply for computertime at:
  http://www.usra.edu/quantum/

- superconducting fluxes

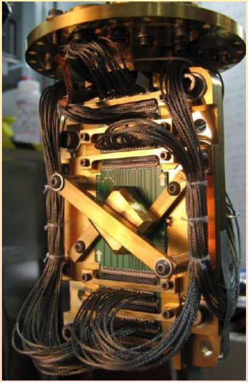


- chip

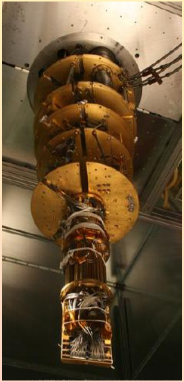


DWAVE

- chip mount



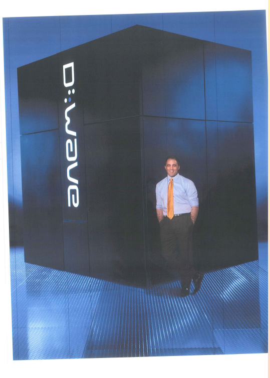
- chip mount



DWAVE

- fridge

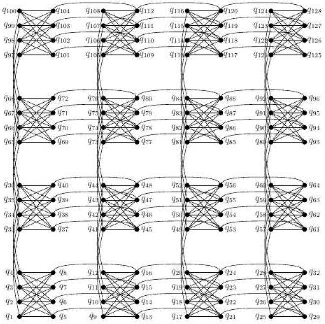
- box



Connectivity Graph

- 2D arrangement of $K_{4,4}$
- $K_{4,4}$: completely connected bipartite graph of 4+4 spins

- N=128 spins(similar for N=512)

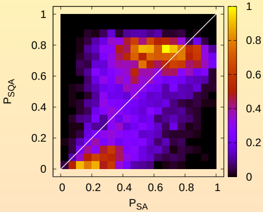


(a)

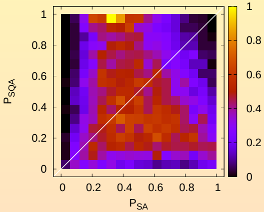
- the DWAVE graph is bipartite
- the DWAVE graph is non-planar
- regular lattice manifolds are somewhat orthogonal

Ensemble Correlations P_{SQA} vs. P_{SA}

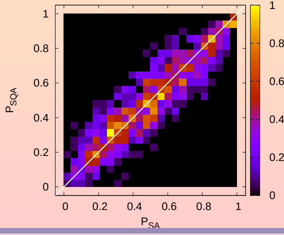
- EA spin glass non-planar:



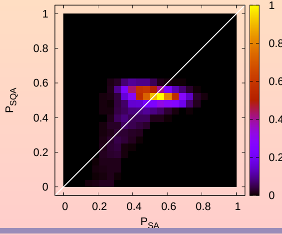
- EA spin glass planar:



- 1*d* random magnet:

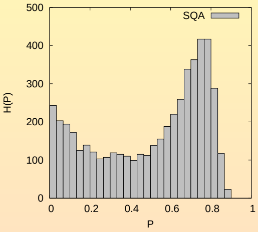


- random magnet on full graph:

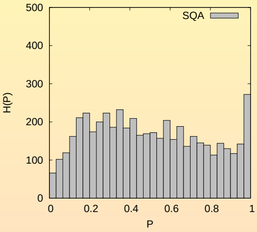


Ensemble Histograms $H[P_{\mathrm{SQA}}]$:[EA-BIMODAL]

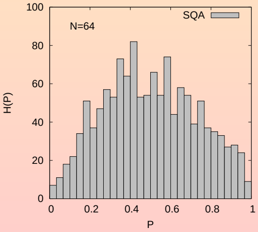
- EA spin glass non-planar:



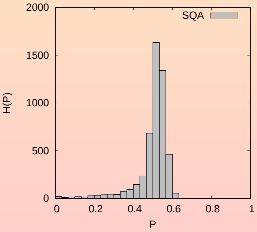
- EA spin glass planar:



- 1d random magnet:

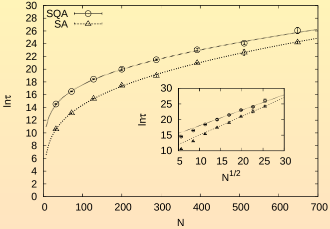


- random magnet on full graph:

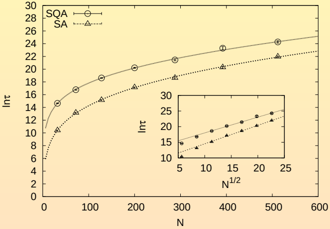


SQA + SA Run-Times :[NO-SQA-SPEEDUP]

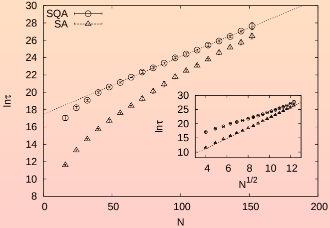
- EA spin glass non-planar:



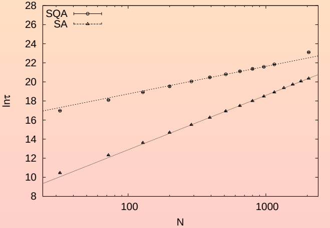
- EA spin glass planar:



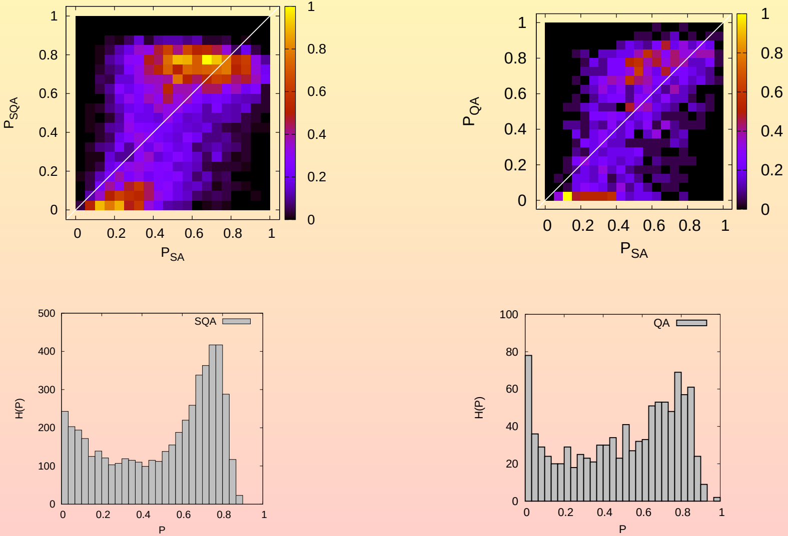
- 1*d* random magnet:



- random magnet on full graph:

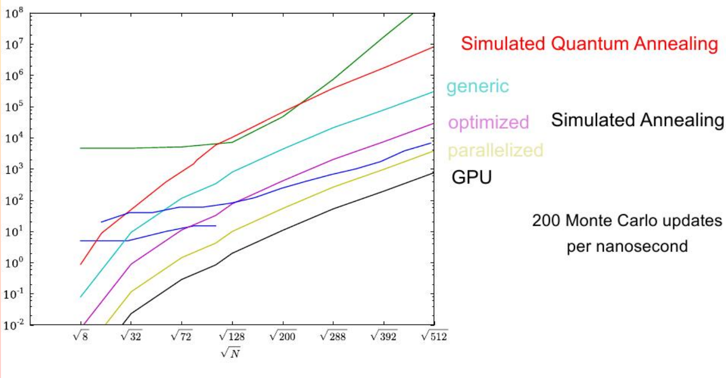


SQA vs. QA at N=108 in EA:[chip is quantum]

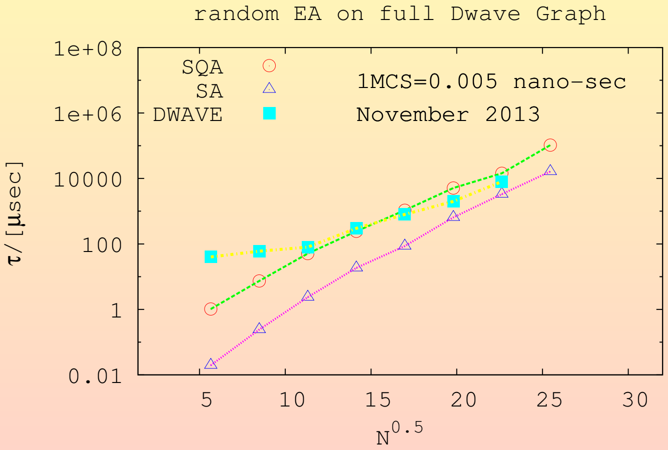


Troyer et. al. 2013

- arXiv:1304.4595v2 [quant-ph]
- EA $J = \pm 1$ on full graph
- N=108 machine finds groundstates
- N=108 machine operates quantum
- no QA speedup over SA
- N=108 quantum annealing at .5 success is $15\mu sec$
- N=108 8-core Xeon SA at .5 success is $4\mu sec$
- Troyer, Talk, N=512:



2013 DWAVE Benchmark



Conlusion

- some understanding on annealing boards
- no known non-trivial problem class where SQA 'effectivly' wins over SA
- the simulated data are not asymptotic
- no one knows what happens at $N = 2048$

appendix I

arXiv:1304.4595v2 [quant-ph]; ”Quantum annealing with more than one hundred qubits”; Sergio Boixo, Troels F. Rannow, Sergei V. Isakov, Zhihui Wang, David Wecker, Daniel A. Lidar, John M. Martinis, Matthias Troyer: Quantum technology is maturing to the point where quantum devices, such as quantum communication systems, quantum random number generators and quantum simulators, may be built with capabilities exceeding classical computers. A quantum annealer, in particular, solves hard optimisation problems by evolving a known initial configuration at non-zero temperature towards the ground state of a Hamiltonian encoding a given problem. Here, we present results from experiments on a 108 qubit D-Wave One device based on superconducting flux qubits. The strong correlations between the device and a simulated quantum annealer, in contrast with weak correlations between the device and classical annealing or classical spin dynamics, demonstrate that the device performs quantum annealing. We find additional evidence for quantum annealing in the form of small-gap avoided level crossings characterizing the hard problems. To assess the computational power of the device we compare it to optimised classical algorithms.

appendix II

arXiv:1305.4904v1 [quant-ph]; ”Classical signature of quantum annealing”; John A. Smolin, Graeme Smith: A pair of recent articles concluded that the D-Wave One machine actually operates in the quantum regime, rather than performing some classical evolution. Here we give a classical model that leads to the same behaviors used in those works to infer quantum effects. Thus, the evidence presented does not demonstrate the presence of quantum effects.

appendix III

arXiv:1305.5837v1 [quant-ph]; "Comment on: Classical signature of quantum annealing"; Lei Wang, Troels F. Rannow, Sergio Boixo, Sergei V. Isakov, Zhihui Wang, David Wecker, Daniel A. Lidar, John M. Martinis, Matthias Troyer: In a recent preprint (arXiv:1305.4904) entitled "Classical signature of quantum annealing" Smolin and Smith point out that a bimodal distribution presented in (arXiv:1304.4595) for the success probability in the D-Wave device does not in itself provide sufficient evidence for quantum annealing, by presenting a classical model that also exhibits bimodality. Here we analyze their model and in addition present a similar model derived from the semi-classical limit of quantum spin dynamics, which also exhibits a bimodal distribution. We find that in both cases the correlations between the success probabilities of these classical models and the D-Wave device are weak compared to the correlations between a simulated quantum annealer and the D-Wave device. Indeed, the evidence for quantum annealing presented in arXiv:1304.4595 is not limited to the bimodality, but relies in addition on the success probability correlations between the D-Wave device and the simulated quantum annealer. The Smolin-Smith model and our semi-classical spin model both fail this correlation test.