Griffiths phase in a Potts model with correlated disorder

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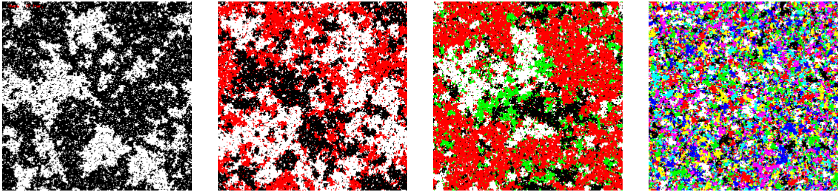
Pure Potts model

Classical “spins” lying on the nodes of a square lattice.

$$H = -J \sum_{(i,j)} \delta_{\sigma_i,\sigma_j} \qquad (\sigma_i = 0, \dots, q-1)$$

$\mathbb{Z}_q$-symmetry is spontanously broken in the low temperature phase.



Second-order phase transition for $q \leq 4$, first-order above.
q-dependent universality class.

Homogeneous uncorrelated disorder

Quenched disorder coupled to the energy density of the Potts model

$$\mathcal{H} = -\sum_{(i,j)} J_{ij}\delta_{\sigma_i,\sigma_j}$$

where $J_{ij} > 0$ (no frustration) are random variables, distributed for e.g. as

$$\wp(J_{ij}) = \frac{1}{2}\big[\delta(J_{ij} - J_1) + \delta(J_{ij} - J_2)\big]$$

Critical line in the plane $J_1 - J_2$ is given by self-duality.

Two averages (thermal fluctuations and disorder):

$$\overline{\langle X \rangle} = \int \frac{1}{\mathcal{Z}[J]} \sum_{\{\sigma\}} X(\{\sigma\}) e^{-\beta H[\sigma,J]} \prod_{(i,j)} \wp(J_{ij}) dJ_{ij}$$

Homogeneous disorder (2)

- **Regime** $q \leq 4$: New universality class for $q < 2 \leq 4$ in agreement with the Harris criterion (disorder is relevant if energy-energy correlation functions decay faster than disorder correlations, equiv. $1/\nu > d/2$ or $\alpha > 0$).

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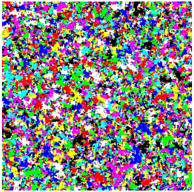
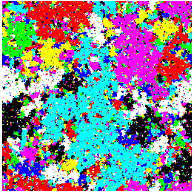
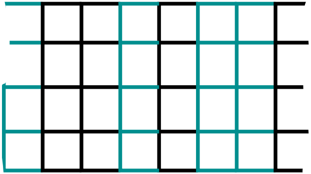




New *q*-dependent universality classes.

McCoy-Wu model

Homogeneous distribution of couplings J_{ij} in one direction and random in the second direction:



q-independent universality class (Senthil-Majumdar) !

$$\beta = (3-\sqrt{5})/2, \qquad \nu = 2$$

Griffiths phase:
Singularity of free energy in a finite range of temperatures, due to the existence of macroscopic regions with a high concentration of strong couplings and acting as super-paramagnets.

Weinrib-Halperin RG calculations

Renormalisation-Group study of the ϕ^4 model with (Gaussian) correlated disorder:

$$\overline{(J_{ij}-\bar{J})(J_{kl}-\bar{J})} \sim ||\vec{r}_{ij}-\vec{r}_{kl}||^{-a}$$

Disorder is relevant when $a < d$ and

$$\frac{1}{\nu} > \frac{a}{2}$$

New universality class:

$$\nu = \frac{2}{a} \quad \text{(exact)}, \qquad\qquad \eta = \mathcal{O}(\varepsilon^2).$$

Monte Carlo simulations for the 3D Ising model.
No results for the Potts model.

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Ashkin-Teller model (two coupled Ising models):

$$-\beta H^{\mathrm{AT}} = \sum_{(i,j)} \left[J^{\mathrm{AT}} \sigma_i \sigma_j + J^{\mathrm{AT}} \tau_i \tau_j + K^{\mathrm{AT}} \sigma_i \sigma_j \tau_i \tau_j \right]$$

Two broken $\mathbb{Z}_2$-symetries so two order parameters: $m = \sum_i \sigma_i$ and $p = \sum_i \sigma_i \tau_i$.
Self-dual critical line with varying critical exponents:

$$\beta_\sigma^{\mathrm{AT}} = \frac{2-y}{24-16y}, \quad \beta_{\sigma\tau}^{\mathrm{AT}} = \frac{1}{12-8y}, \quad \nu^{\mathrm{AT}} = \frac{2-y}{3-2y}$$

where $y \in [0; 4/3]$ and $\cos \frac{\pi y}{2} = \frac{1}{2} \left[e^{4K^{\mathrm{AT}}} - 1 \right]$

Polarisation-polarisation correlation function

$$\langle \sigma_i \tau_i \sigma_j \tau_j \rangle \sim |\vec{r}_i - \vec{r}_j|^{-2\beta_{\sigma\tau}^{\rm AT}/\nu^{\rm AT}}$$

Generate Ashkin-Teller spin configurations and associate a coupling configuration to each of them by

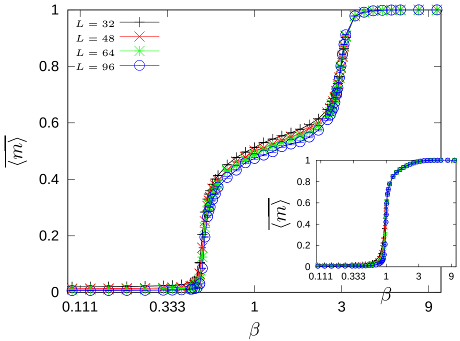
$$J_{ij} = \frac{J_1 + J_2}{2} + \frac{J_1 - J_2}{2} \sigma_i \tau_i,$$

so that

$$\overline{(J_{ij} - \bar{J})(J_{kl} - \bar{J})} \sim |\vec{r}_i - \vec{r}_k|^{-a}$$

Self-duality of the random Potts model is preserved.

Temperature behaviour of the 8-state Potts model



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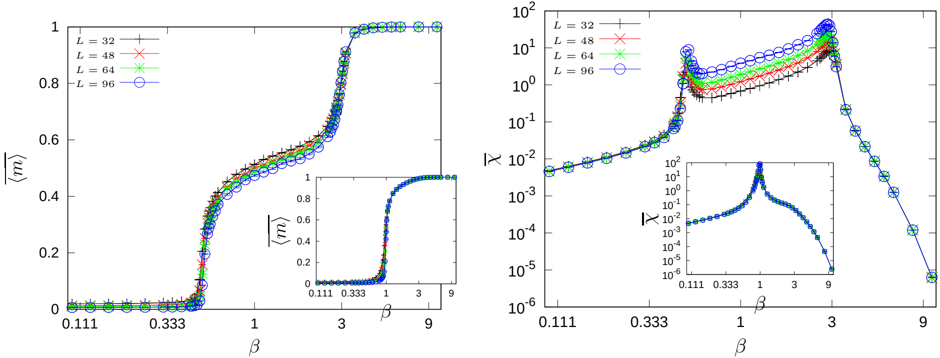
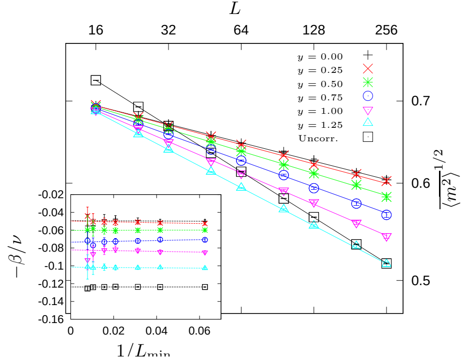


Griffiths phase!

Algebraic Finite-Size Scaling in the Griffiths region.



Critical exponent β/ν depends on disorder correlations but not on q
The hyperscaling relation is not satisfied!

$$\frac{\gamma}{\nu} = d - 2\frac{\beta}{\nu}$$

Self-averaging ratio (sample-to-sample relative fluctuations)

$$R_m = \frac{\overline{\langle m\rangle^2} - \overline{\langle m\rangle}^2}{\overline{\langle m\rangle}^2}$$

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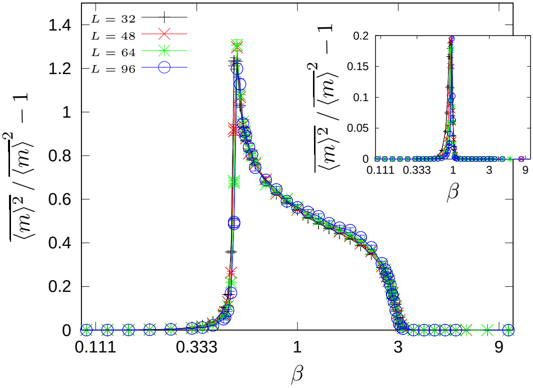

Magnetization is non-self averaging in the Griffiths region.
A constant ratio R_m implies

$$\overline{\langle m\rangle^2} - \overline{\langle m\rangle}^2 = R_m \overline{\langle m\rangle}^2 \sim L^{2\beta/\nu}$$

Decompose the susceptibility as

$$\overline{\chi} \quad = \quad \beta L^d \left[\overline{\langle m^2\rangle} - \overline{\langle m\rangle^2}\right]$$

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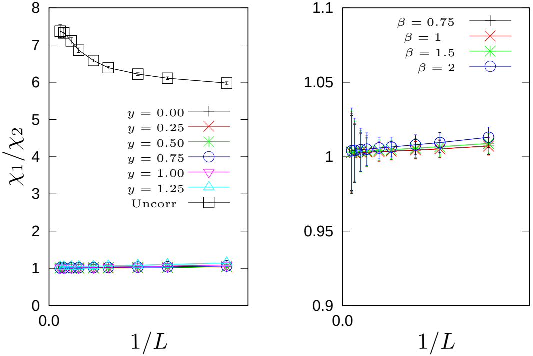
$$\begin{aligned}\overline{\chi} &= \beta L^d \left[\overline{\langle m^2\rangle} - \overline{\langle m\rangle^2}\right] \\ &= \underbrace{\beta L^d \left[\overline{\langle m^2\rangle} - \overline{\langle m\rangle}^2\right]}_{=\chi_1} - \underbrace{\beta L^d \left[\overline{\langle m\rangle^2} - \overline{\langle m\rangle}^2\right]}_{=\chi_2}\end{aligned}$$

Not only χ_1 and χ_2 have the same scaling behavior $L^{d-2\beta/\nu}$ (hyperscaling holds) but they also have the same amplitude A, i.e.

$$\chi_i = AL^{d-2\beta/\nu}\left(1 + B_i L^{-\omega_i} + \dots\right), \quad (i = 1, 2).$$

As a consequence, their difference behaves at large lattice sizes as

$$\overline{\chi} = \chi_1 - \chi_2 \sim AB_1 L^{d-2\beta/\nu-\omega_1} - AB_2 L^{d-2\beta/\nu-\omega_2}$$



Same mechanism as in the 3D Random-Field Ising model

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Violation of the hyperscaling relation is also observed in the energy sector.