http://arxiv.org/abs/1309.1634

Boundary effects in interface growth processes

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work done with Malte Henkel and Jean-Yves Fortin.

November 29, 2013

Outline

$1d$ Interfaces in nature

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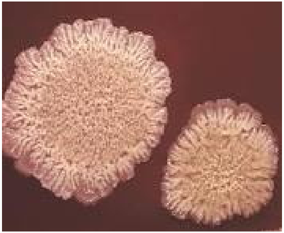

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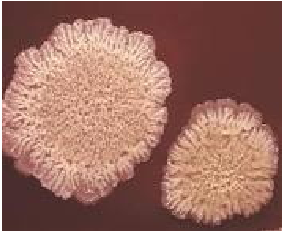

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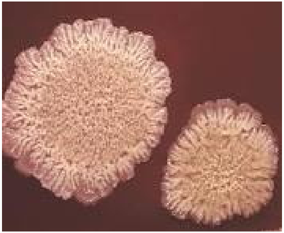

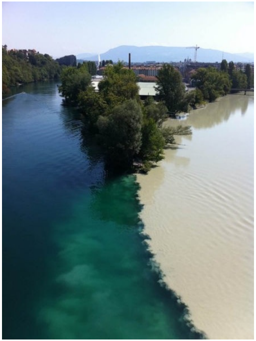

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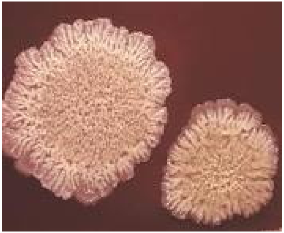

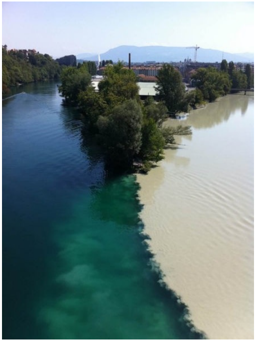

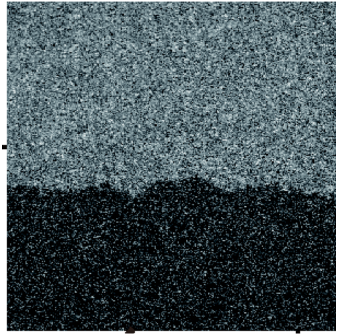

Random surface

Random deposition

- Deposition of particles randomly on the substrate (no diffusion)
- Creation of a random uncorrelated interface

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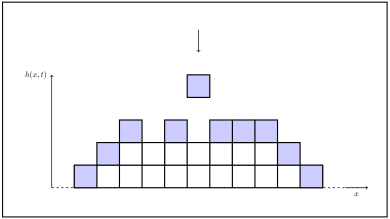

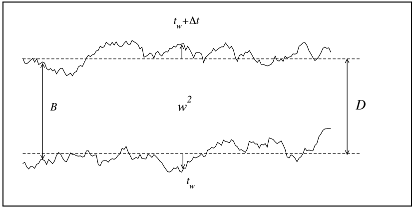

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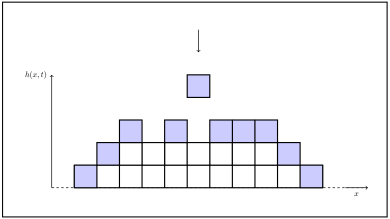
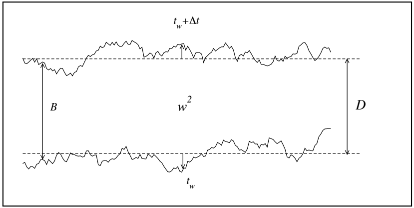


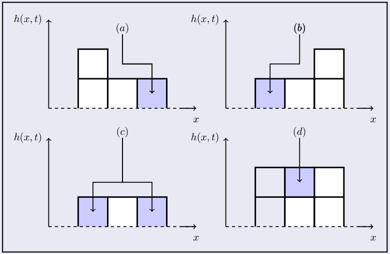


Brownian interface

- $\frac{\partial h(x,t)}{\partial t} = F + \eta(x,t)$
- Gaussian white noise $\langle \eta(x,t)\eta(x',t')\rangle = 2T\delta(x-x')\delta(t-t')$
- Solution $\rightarrow h(\mathrm{x},\mathrm{t}) = \mathrm{Ft} + \int \mathrm{dt}_1 \eta(\mathrm{x},\mathrm{t}_1)$

Family model and EW equation

Deposition relaxation process (Family Model) and discrete Langevin equation



$$\frac{\mathrm{d}h_i}{\mathrm{d}t} = \Gamma a\left(\omega_i^{(0)} + \omega_{i+1}^{(g)} + \omega_{i-1}^{(d)}\right) + \eta_i(t)$$

- $\omega_i^0 = \theta_{i+1,i}\theta_{i-1,i}$
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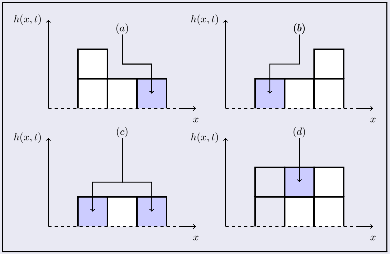

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Continuum limit and the Edwards-Wilkinson equation

$$\frac{\mathrm{d}h_i}{\mathrm{d}t} = \Gamma a M_i^0 + \eta_i(t) \rightarrow \frac{\partial h(x,t)}{\partial t} = \nu\frac{\partial^2 h(x,t)}{\partial^2 x} + \eta(x,t) \tag{1}$$

- Solution $\rightarrow h(\mathrm{x},\mathrm{t}) = \int \mathrm{dt}_1 \int \mathrm{dx}_1\ \eta(\mathrm{x}_1,\mathrm{t}_1)\mathrm{K}(\mathrm{x}-\mathrm{x}_1,\mathrm{t}-\mathrm{t}_1)$
- Heat Kernet: $\mathrm{K}(x,t) := \exp\left(-x^2/4t\right)/\sqrt{4\pi t}$

Interface growth process

Fluctuations and the Family-Vicsek scaling 991)

$$\text{width of the interface} \rightarrow \mathrm{w(t,x)} := \left\langle [\mathrm{h(t,x)} - \langle \mathrm{h(t,x)} \rangle]^2 \right\rangle^{1/2} \tag{2}$$

$$w_L(t) = \overline{w(t,x)} = t^{\beta} f\left(L t^{-1/z}\right), \ f(u) \sim \begin{cases} u^{\alpha} & \text{if } u \ll 1 \\ \text{cste} & \text{if } u \gg 1 \end{cases} \text{ with } z = \alpha/\beta. \tag{3}$$

$$\text{Critical exponents} \rightarrow \ z = 2 \text{ and } \alpha = \frac{z-d}{2} \rightarrow \textit{EW universality class} \tag{4}$$

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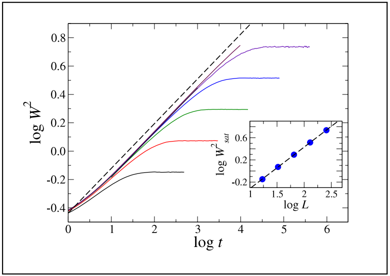

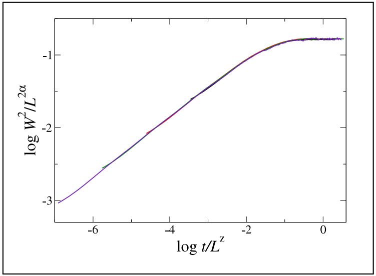

Interface growth and Family-Viczek scaling

Boundary interface growth process

- What happens to the mean profile of the interface ?
- Fluctuation close to boundary vs far from the boundary ?

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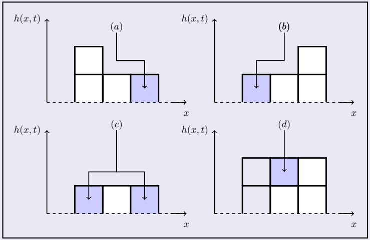
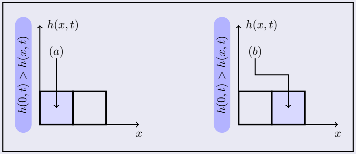
Modification of the process close to a hard wall

- semi-infinite lattice $i = 1, 2...$
- Hard wall: $h_0 > h_1$

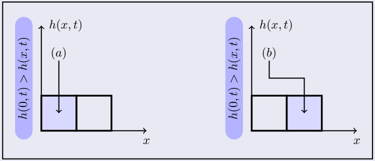




- Wall acts like an infinite energy barrier
- new exponents ? $\rightarrow$ new universality classes ?
- new phase transitions ?

From the microscopic process to a continuum equation

Discrete Langevin equation



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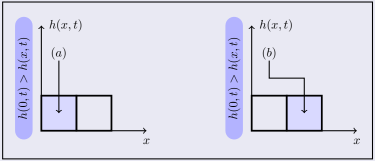

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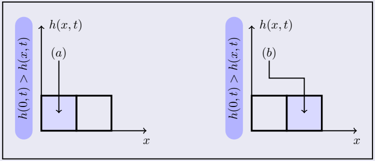

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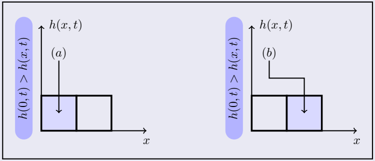

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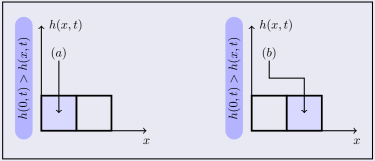

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Continuum formulation

Boundary Edwards Wilkinson equation

$$\frac{\partial h}{\partial t} - \nu \frac{\partial^2 h}{\partial^2 x} = \nu \left(\mu_1 + \mu_2 \frac{\partial h}{\partial x} \Big|_{x=0} \right) \delta(x) + \eta \qquad x \in \mathbb{R}^+ \tag{9}$$

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- Compare to MC simulations

Scaling law of the mean profile of the interface

Hypothesis: scaling ansatz

$$\langle h(x,t)\rangle = t^{1/\gamma}\Phi(\lambda) \quad \text{with} \quad \lambda = xt^{-1/z} \tag{11}$$

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$$\frac{1}{z} + \frac{1}{\gamma} = 1 \quad \text{and} \quad \frac{2}{z} = 1 \rightarrow \boxed{z = \gamma = 2} \tag{12}$$

$$\Phi''(\lambda) + \frac{\lambda^2}{2}\Phi'(\lambda) - \frac{1}{2}\Phi(\lambda) = 0 \tag{13}$$

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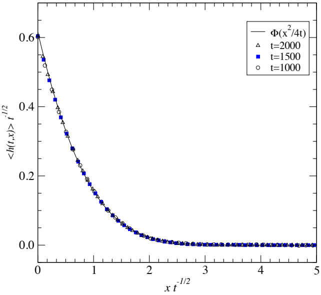

Ansatz results

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- $\Phi(\lambda) = \Phi_0\left(\mathrm{e}^{-\lambda} - \sqrt{\pi\lambda}\,\mathrm{erfc}\,\sqrt{\lambda}\right)$
- In agreement with simulations

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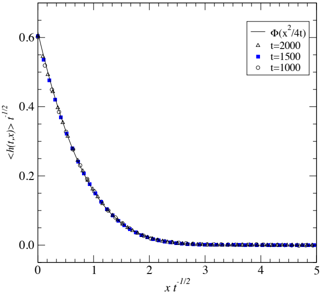

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→ *Comparison to the Exact solution*

Laplace transform solution

Boundary EW equation is linear

$$\frac{\partial h}{\partial t} - \nu \frac{\partial^2 h}{\partial^2 x} = \nu \left(\mu_1 + \frac{\partial h}{\partial x}\Big|_{x=0} \right) \delta(x) + \eta \qquad (14)$$

Linear → Laplace transform $\mathcal{L}_x.h(x,t) = h_p^*(t) = \int_{\mathbb{R}+} \mathrm{d}x \; \mathrm{e}^{-px} h(x,t), \quad p \geqslant 0$

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Solution

$$\boxed{h(t,x) = \frac{1}{4\sqrt{\pi}} \int_\lambda^\infty \frac{\mathrm{d}v}{v^{3/2} e^v} \left[x\mu_1 + 2vh_0 \left(t - \frac{x^2}{4v} \right) \right] + \zeta(t,x)} \qquad (15)$$

where

$h_0 := h(0,t) = 2_1\sqrt{\frac{t}{\pi}} + \zeta(t,0)$ *and* $\langle h(x,t) \rangle = t^{1/2} \frac{2\mu_1}{\pi} \left(\mathrm{e}^{-\lambda} - \sqrt{\pi\lambda} \; \mathrm{erfc} \sqrt{\lambda} \right)$

$$\zeta(x,t) = \int_0^t \int_0^\infty \mathrm{d}\tau \mathrm{d}u \; \mathrm{K}(x-u, t-\tau) \eta(u,\tau) \quad \to \langle \zeta \rangle = 0 \qquad (16)$$

$$\langle \zeta(t,x) \zeta(t',x') \rangle = 2T \int_0^{\min(t,t')} \mathrm{d}\tau \int_0^\infty \mathrm{d}u \; \mathrm{K}(x-u, t-\tau) \mathrm{K}(x'-u, t'-\tau) \qquad (17)$$

Exact solution: Fluctuations of the interface $w(x,t)$

$w^2(x,t) = w_1^2(t,x) + w_2^2(t,x) + w_3^2(t,x) = T\sqrt{t}\Phi_w(\lambda)$

$$w_1^2(t,x) = \frac{1}{\pi}\iint_\lambda^\infty \frac{\mathrm{d}v\,\mathrm{d}v'}{(vv')^{1/2}e^{v+v'}}\left\langle \zeta\left(t-\frac{\lambda t}{v},0\right)\zeta\left(t-\frac{\lambda t}{v'},0\right)\right\rangle \qquad (18)$$

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Asymptotic expansion $\lambda \to 0$

$$w_1^2(t,x) \simeq T\sqrt{t}\left(\frac{1}{\sqrt{2\pi}} + +\log(2) - 2 + \log(\lambda))\frac{\sqrt{\lambda}}{\pi} + O(\lambda^{3/2})\right) \tag{21}$$

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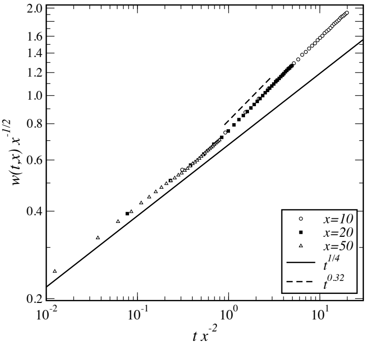
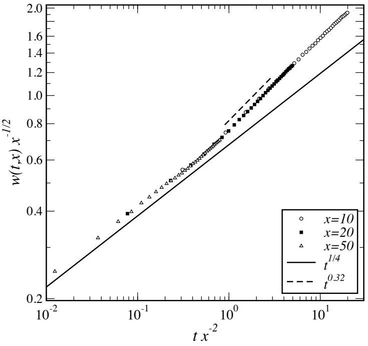


Figure : Width $w(x, t)$ simulated by the deposition-relaxation algorithm



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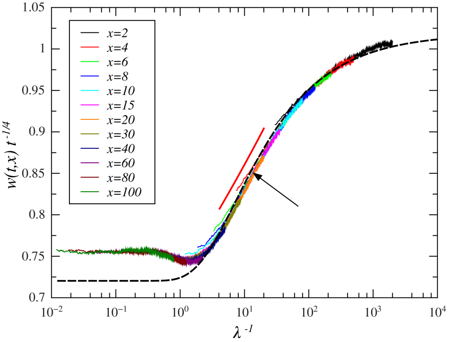
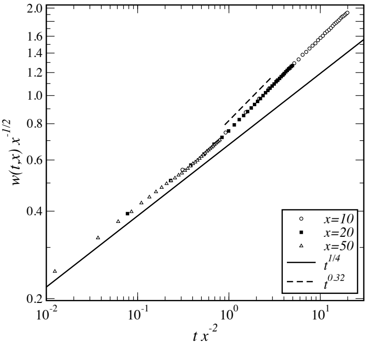


Figure : Effective correction $\beta_{\text{eff}} \approx 0.32$ to the exponent $\beta = 1/4$.



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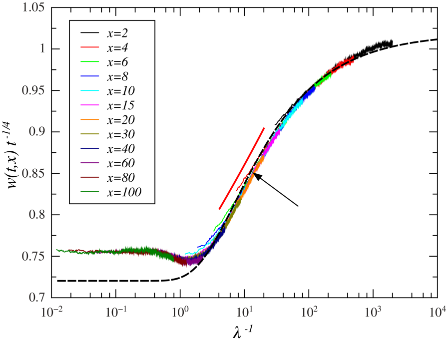

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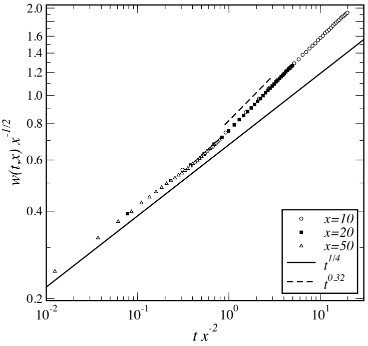
Width profile of the interface

- More complex behavior than the standard EW width $\rightarrow w(t) \sim t^{1/4}$



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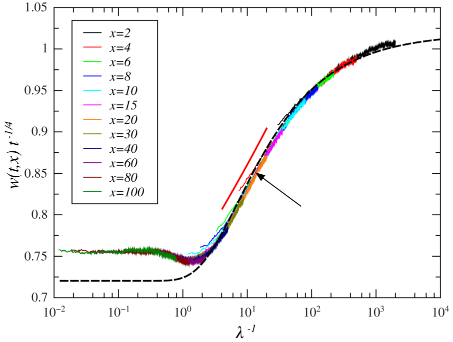

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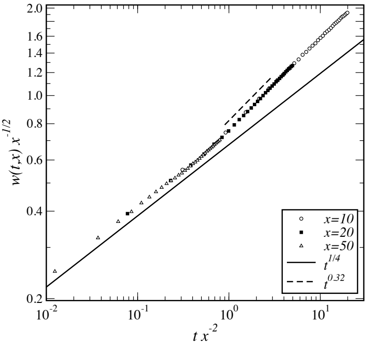

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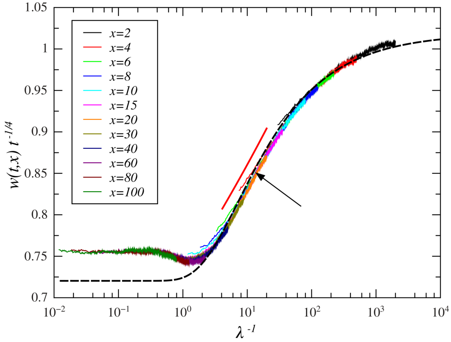

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KPZ equation

KPZ equation and generalities

$$\boxed{\frac{\partial h}{\partial t} - \nu \frac{\partial^2 h}{\partial^2 x} = \lambda \left(\frac{\partial h}{\partial x} \right)^2 + \eta} \tag{24}$$

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- Exact solutions in $1d$
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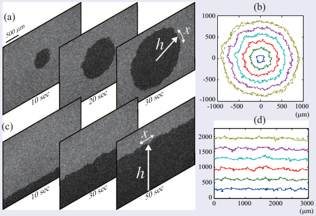

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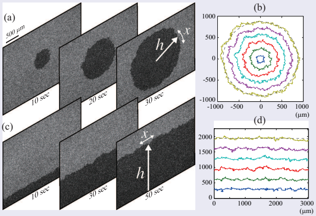
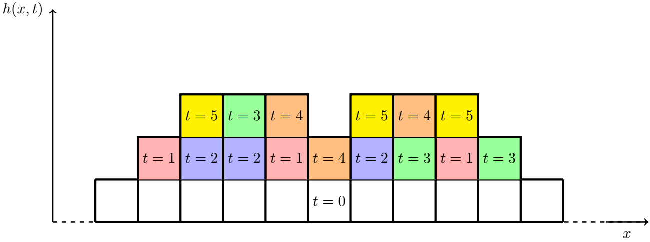


Excellent agreement with the theory

- Family Viczek scaling checked
- $\alpha = 0.50(0)$, $\beta = 0.336(11)$ same for the 2 geometries (Takeuchi et Sano 2010)
- Geometry dependent height distribution in perfect agreement with exact solutions. (Sasamoto, Spohn, Prahofer, le Doussal...)

RSOS process and KPZ equation



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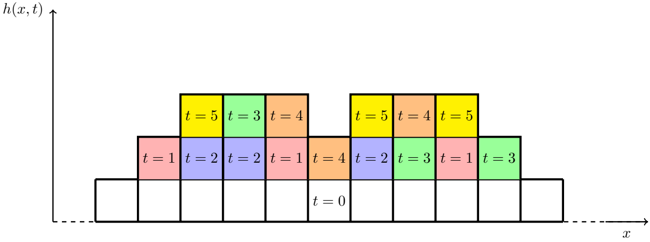

Continuum limit of the RSOS process

- Discrete Langevin equation $\frac{\mathrm{d}h_i}{\mathrm{d}t} = \Gamma a\, \omega_i^{(0)} + \eta_i(t) = \Gamma a\, \theta_{i+1,i}\theta_{i-1,i} + \eta_i(t)$

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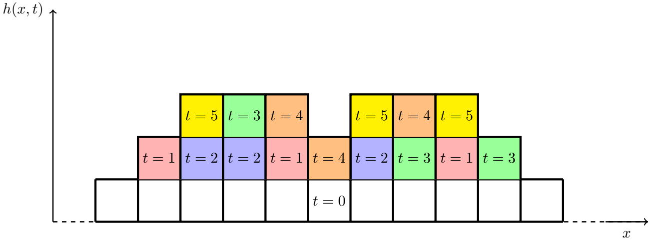

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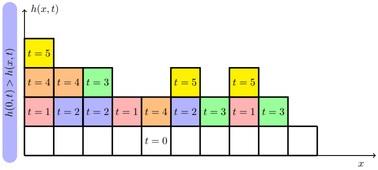
- Valid in any dimension
- Next: Boundary RSOS process ?

RSOS process and KPZ equation



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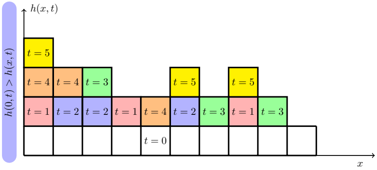

Modification of transition rates.

- $h(0,t) > h(x,t)$
- $\omega_1^{(0)} = \theta_{21}(1-\epsilon)$
- $\omega_i^{(0)} = \theta_{i+1,i}\theta_{i-1,i}$ pour $i = 2, 3..$

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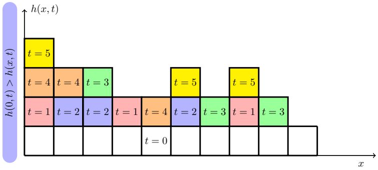

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Continuum limit of the RSOS process with a boundary in $i = 0$

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$$\frac{\mathrm{d}h_i}{\mathrm{d}t} = \Gamma a\left(\omega_i^{(0)} + \Delta\tilde{\omega}_i^{(0)}\delta_{i,1}\right) + \eta_i(t) \quad \forall i \in \mathbb{N} \tag{26}$$

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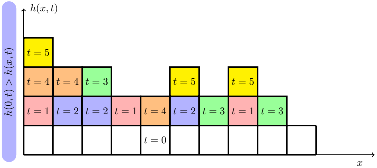

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- Is this equation right ?

RSOS Process and KPZ equation

Profile scaling ansatz

- Continuous Langevin Eq ($\mu_2 = 1$)

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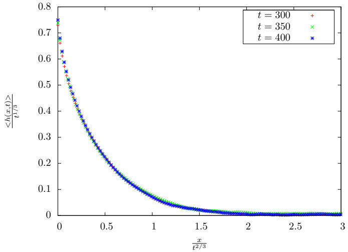

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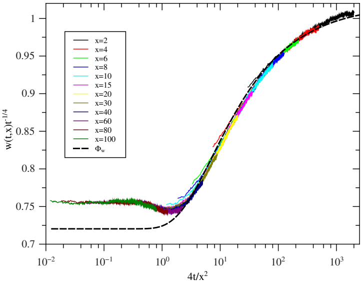

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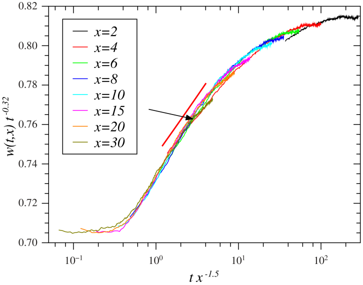

Figure : Width $w(x,t)$ for RSOS: $\beta_{\text{eff}} \sim 0.35$

Conclusion and perspectives

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Conclusion and perspectives

Thank you for your attention !!