UNIVERSITÄT LEIPZIG

Computational Quantum Field Theory

Application of a steady states transport model to condensation of water droplets on a substrate

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Stochastic Mass Transport

In stochastic mass transport models the focus lies typically on the qualitative abstract understanding of transport processes. That is, some basic kind of particles moves according to a prescribed dynamics ruleset. Examples are particle are the zero-range process (ZRP) or a process extended by short range interactions where particles hop away depending on the local particle occupation plus that of direct neighbors in the latter case. Both feature particle condensation.
In this work we study a the steady state of a transport model that is based on a process generalized from the zero-range process (ZRP) [1, 2], but with a pair-factorized steady state. It most notably features extended condensates with a qualitatively tunable envelope shape.

Scaling Theory

Classical scaling [3] yields that droplet formation leads to a state where the number density of droplets has a universal form

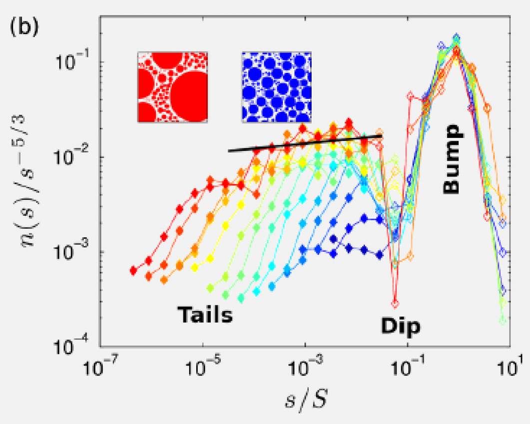
$$n(s,t) = s^{\theta} f\left(\frac{s}{S(t)}\right) \qquad (\theta_{2D} = 5/3, \theta_{1D} = 4/3)$$

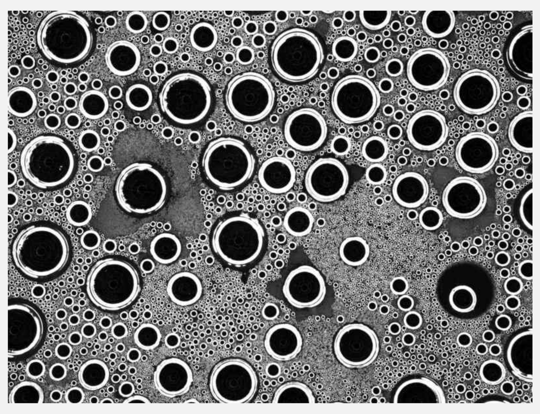
with a geometric exponent θ, the average volume of the largest droplet $S(t)$.

Blaschke et al.

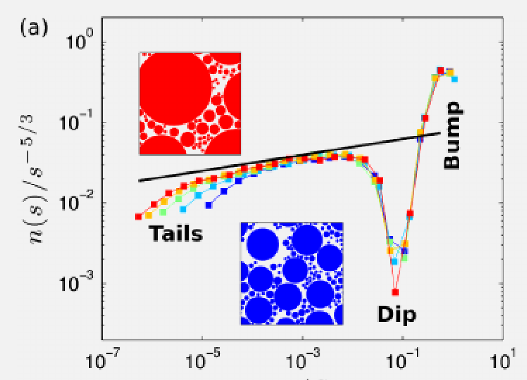
For a 2-dimensional substrate Blaschke et al. [4] measured the droplet size distributions after different times in experiment and numerical simulation (adding fixed size particles and subsequently merging overlapping droplets).

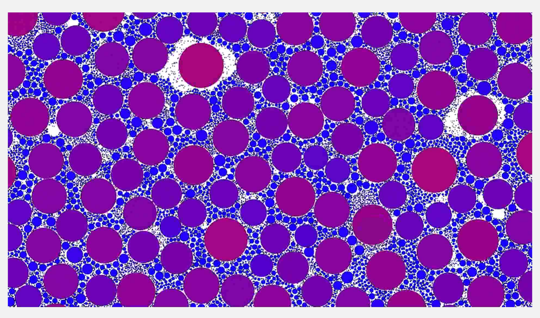
Experiment Directly observe a cooled substrate under constant influx from water condensation using a high resolution CCD camera.

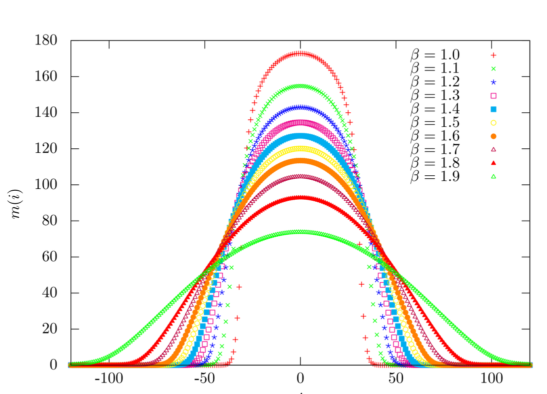


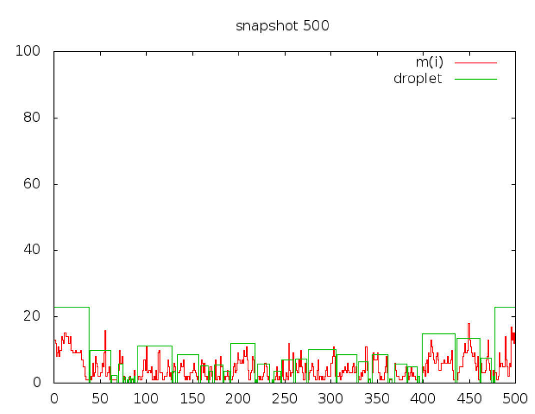


Simulation Simulate constant influx by adding droplets of size s_0 at random positions and subsequently merging overlapping droplets. Diffusion, drift, falling and physical merging of droplets are neglected.



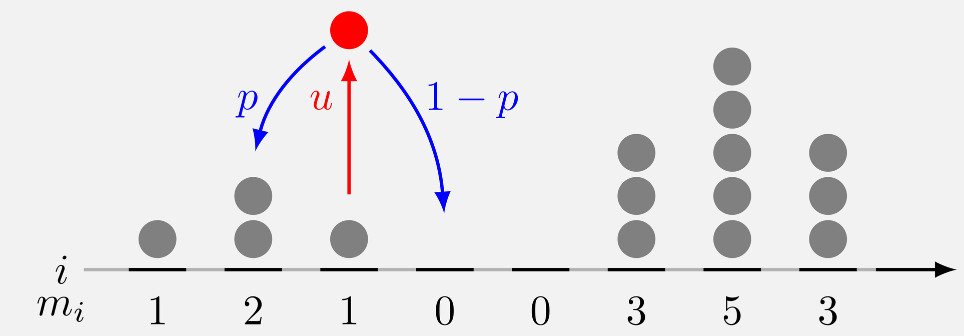






Transport Process

The model consists of a number of indistinguishable particles distributed on a peridoc ring lattice. In the discrete time stochastic process a particle may leave a randomly selected site at every time step with a hopping rate $u(m_i|m_{i-1}, m_{i+1})$ and move to either direct neighbour. This dynamics is the same as in the ZRP with an added nearest-neighbour interaction.



This also leads to a steady state that is similar to that of the ZRP, but factorizes over pairs of sites instead of single sites:

$$P(\vec{m}) = P(m_1, \ldots, m_N) = \frac{1}{Z}\prod_{i=1}^{N} g(m_i, m_{i+1})$$

The hopping rate given by the weight functions:

$$u(m_i|m_{i-1}, m_{i+1}) = \frac{g(m_i-1, m_{i-1})}{g(m_i, m_{i-1})}\frac{g(m_i-1, m_{i+1})}{g(m_i, m_{i+1})}$$

The weights are assumed to separate into zero-range and local-range interactions:

$$g(m,n) = \sqrt{p(m)p(n)}K(|m-n|)$$

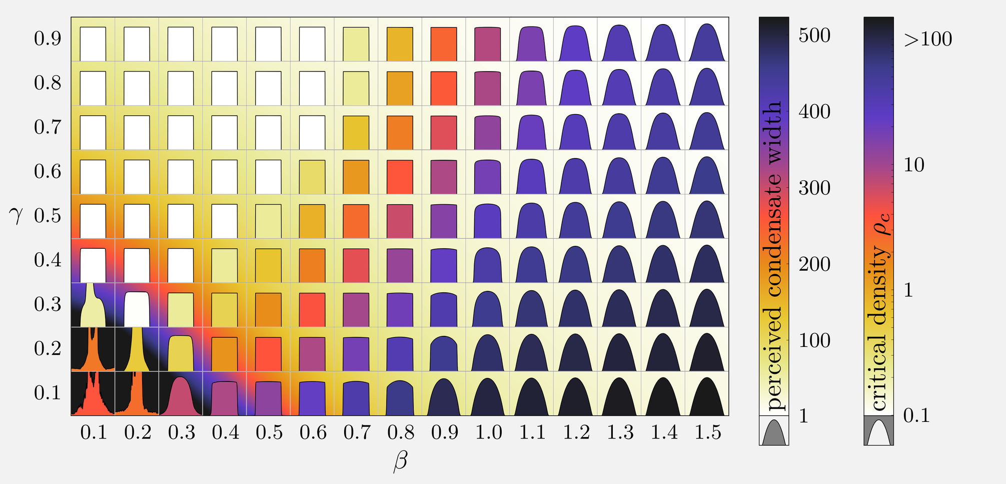
When the weight functions fall of fast enough, a critical density ρ_c exists, so that any particles added to the system, that increase the density above ρ_c add to the mass of an emerging condensate.
Well-behaving weight functions, where the zero-range interaction approaches a constant for large m and the short-range interaction falls off faster than any power law lead to an analytically known condensate shape and scaling.

To produce interesting behaviour these conditions are deliberately broken by introducing weights with tunable fall-off [5]:

$$K(x) \sim e^{-a|x|^{\beta}}, \quad p(m) \sim e^{-bm^{\gamma}}$$

Tunable Condensate Shape and Width

The shape of the condensate as well as the scaling relation of its volume to its width (contact surface) can be tuned [6]:



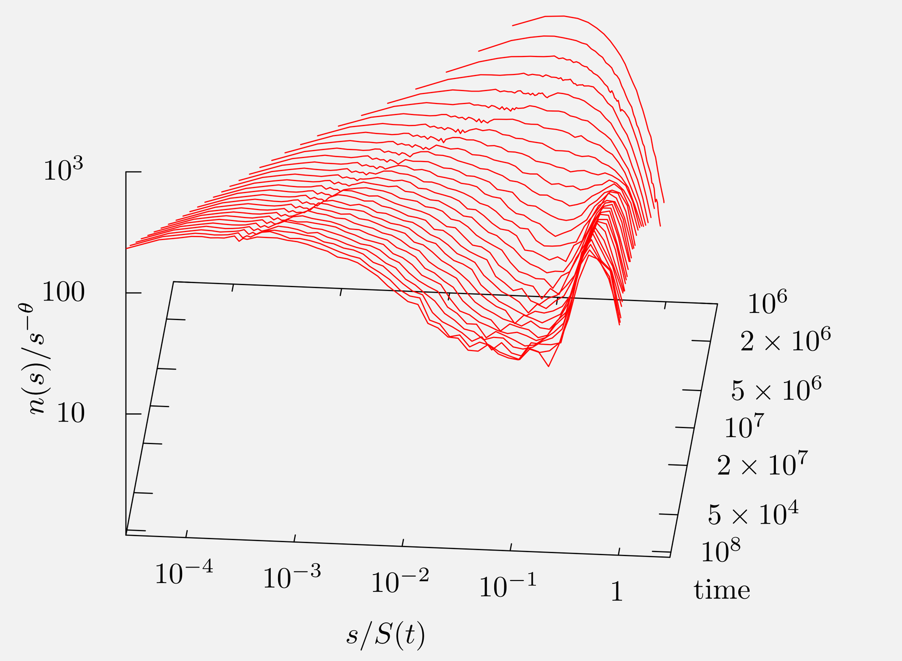
In order to simulate 3D droplets on a 1D substrate, $\theta_{1D} = 4/3$, the correct width scaling

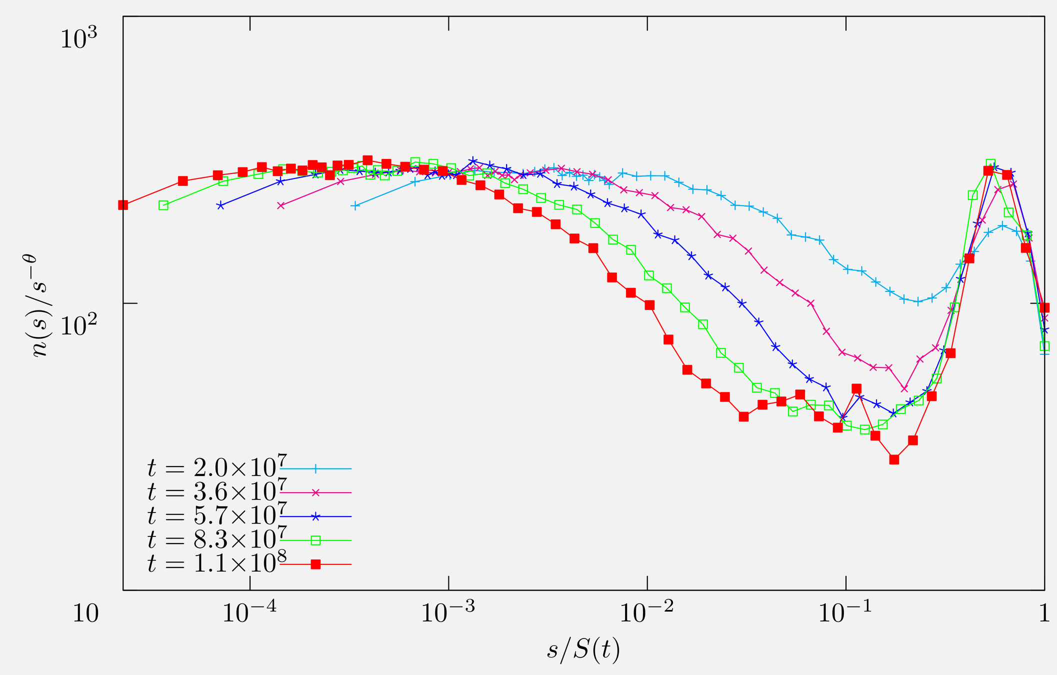
$$W \propto M'^{\alpha}, \quad \alpha = \frac{\beta-\gamma}{2\beta-\gamma} \text{ for } \beta \geq 1 \quad , \alpha = \frac{\beta-\gamma}{1+\beta-\gamma} \text{ for } \beta < 1$$

is realized for $\alpha = 1/3$, that is $\gamma = \beta/2$ for $\beta \geq 1$ and $\gamma = \beta - 1/2$ for $\beta < 1$. We select a parametrization with a low crtical density for condensation and reasonable simulation performance: $\beta = 1.2, \gamma = 0.6$.

Droplet Size Distribution

For a system with $L = 2000$ sites, symmetric hopping, a constant influx of 10^{-3} particles per sweep we estimate the droplet size distribution:





Conclusions

- Our stochastic transport model reproduces the qualitative features of the droplet size distribution of Blaschke et al., i.e. tail, dips and bump, as well as a good collapse of the distributions at least for large times
- Interestingly our results compare better to the experimental distributions than to numerical results of that group
- Effects of **diffusion**, **drift** and **merging** of droplets are implicitly included in the transport model
- The computational cost is however quite large compared to the numerical simulations [4] (even in one dimension)

References

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