Finite-size scaling study of pseudocritical temperatures in spin glasses

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The thermodynamic limit and the 'experimental limit'







- A proper phase transition takes place only in the idealized limit of infinitely many degrees of freedom.
- Even if this limit is never realized in the laboratory, everyday experience suggests that macroscopic samples of material are infinite for all practical purposes.

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- Spin glasses are an exception, because of their sluggish dynamics.
- Even for experimental waiting times (t_w) of several hours, the spatial size of the glassy domains is of ξ(t_w) ~ O(10²) lattice spacings.
- In a sense, the non-equilibrium infinite system behaves as if composed of many equilibrium systems of size ξ(t_w)^D.
- This statement can be made quantitative through a time-length dictionary.

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Time-length dictionary (Janus Collaboration)

Equilibrium: finite size at infinite t_w Non-equilibrium: infinite system at finite t_w

Quantitative: at $T = 0.64 T_c$, choose t_w such that $L = 3.7\xi(t_w)$

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The experimental scale

- Experimental time scale: 1 hour \sim 4 \times 10¹⁵ MC steps.
- $\xi(t_w) \sim t_w^{1/z(T)}$: relevant equilibrium size L = 110

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- These fluctuations typically decrease with system size, but if we want to understand their possible experimental relevance we need to know how their distribution evolves with system size.

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- These fluctuations typically decrease with system size, but if we want to understand their possible experimental relevance we need to know how their distribution evolves with system size.
- In particular, the finite-system pseudocritical temperature is a relevant (but elusive) quantity.
- We want to characterise the statistical properties of this quantity for three-dimensional (Edwards-Anderson) and mean-field (Sherrington-Kirkpatrick) spin glasses.

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- χ_{SG} ~ N for all T < T_c (actually, a strictly decreasing function of T),
 so we need a more sophisticated approach.
- We will consider dimensionless single-sample observables *O^J*, which scale as

$$O^J(T,L) \simeq G((T-T^J_{c}(L))L^{1/\nu})$$

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Observables for the Edwards-Anderson model (I)

The EA model

$$\mathcal{H} = -\sum_{\langle x,y \rangle} S_x J_{xy} S_y, \qquad S_x = \pm 1, \ J_{xy} = \pm 1.$$

Simulations from the Janus Collaboration (up to L = 32 down to $T = 0.64 T_c$).

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Propagator

• We consider the overlap field q_x and its Fourier transform, $\phi(\mathbf{k})$.

$$\phi(\mathbf{k}) = \sum_{x} S_{x}^{(a)} S_{x}^{(b)} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} = \sum_{x} q_{x} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}$$

- The two-point propagator is $G^{J}(\mathbf{k}) = \langle \phi(\mathbf{k})\phi(-\mathbf{k}) \rangle$.
- The smallest momentum with periodic boundaries is $k = 2\pi/L$: $k_1^{(1)} = (2\pi/L, 0, 0), k_1^{(2)} = (0, 2\pi/L, 0), k_1^{(3)} = (0, 0, 2\pi/L).$
- We define $G^{J}(\mathbf{k}_{1}) = \frac{1}{3} \sum_{i} G^{J}(\mathbf{k}_{1}^{(i)}).$

We are going to use several dimensionless ratios to search for T^J_c:

$$\xi^{J}/L = \frac{1}{2L\sin(\pi/L)} \left[\frac{G^{J}(0)}{G^{J}(k_{1})} - 1\right]^{1/2}$$

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• We are going to use several dimensionless ratios to search for T_c^J :

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$$S^{J}/L = rac{1}{2L\sin(\pi/L)} \left[rac{G^{J}(0)}{G^{J}(m{k}_{1})} - 1
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angle_{J}^{2}}, B^{J}_{G} = rac{\sum_{i} \langle [\phi(m{k}_{1}^{(i)})\phi(-m{k}_{1}^{(i)})]^{2}
angle_{J}}{[G^{J}(m{k}_{1})]^{2}},$$

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$$\begin{split} \xi^{J}/L &= \frac{1}{2L\sin(\pi/L)} \left[\frac{G^{J}(0)}{G^{J}(\boldsymbol{k}_{1})} - 1 \right]^{1/2}, \\ B^{J} &= \frac{\langle q^{4} \rangle_{J}}{\langle q^{2} \rangle_{J}^{2}}, \\ B^{J}_{G} &= \frac{\sum_{i} \langle [\phi(\boldsymbol{k}_{1}^{(i)})\phi(-\boldsymbol{k}_{1}^{(i)})]^{2} \rangle_{J}}{[G^{J}(\boldsymbol{k}_{1})]^{2}}, \\ R^{J}_{12} &= \frac{G^{J}(\boldsymbol{k}_{1})}{G^{J}(\boldsymbol{k}_{2})}. \end{split}$$

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- Let us take the sample-averaged version *O* of any of the *O^J*.
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• Of course, different choices of *O* and *y* give different $T_{O,y}^J$, but we expect the scaling to be the same.

Example: Binder ratio



The sample-averaged Binder ratio. We take $y = 1.51 \approx B(T_c)$

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Example: Binder ratio



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- Several solutions (green): take the largest.
- No solution (blue): only upper bound (less than 1% of samples).

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Finite size scaling of T_{c}^{J}

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- We cannot consider the arithmetic mean of the $T_{Q_V}^J$.
- Instead, we analyse the median T^J_{O,y} of the distribution, unaffected by the few samples without a solution.
- Finally, we can also consider the pseudocritical temperature of the susceptibility, given by

$$\chi^{J}_{\mathrm{SG}}(T^{J}_{\chi,y}) = \chi_{\mathrm{SG}}(T_{\mathrm{c}})y,$$

with $y \simeq 1$.

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- In principle, for each O^J and y, a fit gives T_c and ν .
- However, we have few degrees of freedom.
- We consider all the O^J and several y at the same time in a joint fit.
- For the same *L* the different $\tilde{T}_{O,y}^J$ are correlated, so we consider the full covariance matrix:

$$\chi^{2} = \sum_{\alpha,\beta=1}^{n} \sum_{a,b=1}^{\mathcal{L}} \left[\tilde{T}_{\alpha}^{J}(L_{a}) - T_{c} - A_{\alpha}L_{a}^{-1/\nu} \right] \sigma_{(ia)(jb)}^{-1} \left[\tilde{T}_{\beta}^{J}(L_{b}) - T_{c} - A_{\beta}L_{b}^{-1/\nu} \right]$$

(here α, β label both the O^J and the y).

• The fit parameters are $\{T_c, \nu, A_{y_1}, A_{y_2}, \ldots\}$.



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- Hasenbusch et al. consider corrections to scaling ($\omega = 1$).
- We do not have enough data for small L to fit for ω and ν simultaneously.
- We fix ω , ν to the value of Hasenbusch et al. and fit for T_c , $L \ge 8$:

$$T_{\rm c} = 1.105(8), \quad \chi^2/{
m d.o.f.} = 40.9/38$$

The width of the distribution

- These results make us confident that our definition of T^J_{α} makes sense.
- We want to study the fluctuations.
- Define T_{α}^+ and T_{α}^- : $P(T_{\alpha}^J > T_{\alpha}^+) = 0.16$, $P(T_{\alpha}^J < T_{\alpha}^-) = 0.16$.
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The SK model

$$\mathcal{H} = -\frac{1}{\sqrt{N}}\sum_{x,y}S_x J_{xy}S_y, \qquad S_x = \pm 1, \ J_{xy} = \pm 1.$$

Simulations from Aspelmeier et al. ($N \le 4096$, $T \ge 0.4T_c$)

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• This scaling can be justified studying the stability of the TAP states.

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- We fit only for the amplitude for $N \ge 256$, $(\chi^2/d.o.f. = 4.44/4)$.
- We can include a scaling correction with $N^{-2/3}$. Now all sizes $N \ge 64$ fit, with $\chi^2/d.o.f. = 2.07/5$.

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Results for SK (II)



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Results for SK (II)



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- Without corrections: $N \ge 256$, $\chi^2/d.o.f. = 5.36/4$.
- With corrections: $N \ge 64$, $\chi^2/d.o.f. = 2.57/5$.

Comclusions

- We have presented a simple method to study the probability distribution of the pseudocritical temperatures in spin glasses.
- We have applied it to the Edwards-Anderson and the Sherrington-Kirkpatrick models.
- The T^J_{α} are shown to follow a straightforward finite-size scaling.
- For EA, our computed values for T_c and ν are compatible with state-of-the-art results and of similar precision.