Estimate of the energy of vacuum fluctuations of non-Abelian gauge fields from the uncertainty relations

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Quantization of Yang-Mills fields via Heisenberg's equation of motion (time and space)

$$\frac{dE_a^m}{dt} = \frac{i}{\hbar}\left[T^{00}, E_a^m\right], \cdots \quad \longleftrightarrow \quad \text{Field equations}$$

- **Nonlinear commutation relations for field strengths**

$$\left[B_a^n(r'), E_b^l(r)\right] = i\hbar\delta_{ab}\varepsilon^{nlm}\partial'^m\delta(r'-r) - i\hbar g\varepsilon_{abc}\varepsilon^{nlm}W_c^m\delta(r'-r), \cdots$$

- **Accordance of the equation of motion with respect to space derivatives with field equations demands constraint equations**

$$\varepsilon_{abc}(G_b^{i0}W_c^m - G_b^{im}W_c^0) = 0, \quad \varepsilon_{abc}(B_b^i W_c^m - \varepsilon^{imn}E_b^n W_c^0) = 0$$

- **Derivation from the commutation relations the uncertainty relations for field strengths and estimate of the vacuum energy**

- **Finite vacuum energy of the k_{th} mode for $k \to 0$ and finite volume V**

$$\mathcal{E}_{0,k} \simeq \frac{9}{2}\frac{\hbar g}{\left(\sum_k 1\right)^{2/3}}\left(\frac{\hbar}{2gV}\right)^{1/3}$$