Open Free Fermionic Chains : The Repeated Interaction Process

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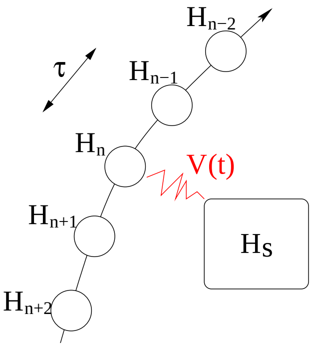
Introduction

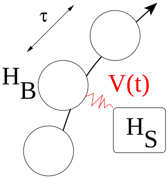
The Repeated Interaction Process



S. Attal and Y. Pautrat, Ann. Inst. Henri Poincaré 7, 59 (2006).

The Repeated Interaction Process

The set up :



✓ System : H_S

✓ Bath : $H_B = \sum_n H_n$
H_n Hamiltonian of the n^{th} particle

✓ Interaction : $V(t)$
$V(t) = V_n$ for $t \in](n-1)\tau, n\tau]$

✓ Initial State : $\rho(0) = \rho_s(0) \otimes \rho_B$

$$\rho_B(0) = \rho_1 \otimes \rho_2 \otimes \ldots \rho_n \otimes \ldots$$

Time Evolution

Key steps :

✓ Define the operator of evolution $U(n)$

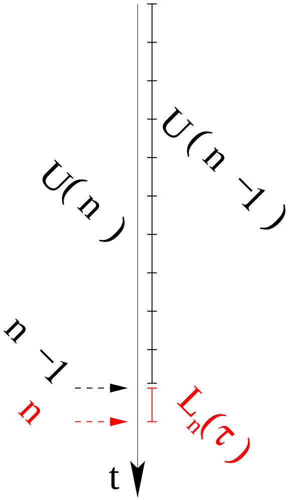
✓ Write $U(n) = L_n(\tau)U(n-1)$

with $L_n(\tau) = K_n(\tau) \otimes \exp(-i\tau \sum_{j\neq n} H_j)$

and $K_n(\tau) = \exp[-i\tau(H_s + V_n + H_n)]$

✓ Define $\rho_s(n) = Tr_B\{\rho(n)\}$

$\rho(n) = U(n)\ [\ \rho(0) \otimes \rho_B(0)\ ]\ U^\dagger(n)$



Reduced density Matrix $\rho_s(t)$

✓ Express

$$\rho_s(n+1) = Tr_n\{\ K_n(\tau)\ \ [\rho_s(n)\otimes\rho_n]\ \ K_n^\dagger(\tau)\ \} = L\,[\rho_s(n)]$$

$L\,[.]$ is a completely positive map

✓ Take the continuous limit $\tau \to 0$

$$\partial_t\rho_s(t) = \mathcal{L}[\rho_s(t)] \qquad \mathcal{L}[X] = \lim_{\tau\to 0}\frac{L[X]-X}{\tau}$$

Renormalization of the coupling system-bath $\lambda \to \lambda/\sqrt{\tau}$

S. Attal and Y. Pautrat, Ann. Inst. Henri Poincaré 7, 59 (2006).

✓ Lindblad equation

$$\partial_t\rho_s(t) = -i[H_s,\rho_s] - \sum_i\left(\left\{L_iL_i^\dagger,\rho_s\right\} - 2L_i\rho_sL_i^\dagger\right)$$

Considering Fermionic Systems

Quadratic Hamiltonian

✓ The System :

$$H_S = \sum_{i,j=1}^{L_S} (\mathbf{T}_S)_{i,j} c_i^\dagger c_j \qquad \mathbf{T}_S^\dagger = \mathbf{T}_S \qquad \{c_i^\dagger, c_j\} = \delta_{i,j}$$

✓ The Bath : $H_B = \sum_n H_n$, for each copy n of the bath :

$$H_n = \sum_{i,j=1}^{L_b} (\mathbf{T}_b)_{i,j} b_{i,n}^\dagger b_{j,n} \qquad \mathbf{T}_b^\dagger = \mathbf{T}_b \qquad \{b_{i,n}^\dagger, b_{j,n}\} = \delta_{i,j}$$

✓ The Interaction $V(t) = V_n \qquad t \in]n\tau - \tau, n\tau]$

$$V_n = \sum_i^{L_S} \sum_j^{L_b} \left[\Theta_{i,j} c_i^\dagger b_{j,n} + \Theta_{i,j}^* b_{j,n}^\dagger c_i \right]$$

Initial State

✓ The initial state is uncorrelated

✓ $\rho(0) = \rho_S(0) \otimes \rho_B(0)$

$$\rho_B(0) = \rho_1(0) \otimes \rho_2(0) \otimes \ldots \rho_n(0) \otimes \ldots$$

✓ The initial state of the entire system is Gaussian

✓ $\rho_S(0) \propto \exp[-\sum_{i,j} c_i^\dagger \mathbf{S}_{i,j} c_j]$ for any $\mathbf{S}^\dagger = \mathbf{S}$

✓ $\rho_n(0) \propto \exp[-\beta_b H_n]$ thermalized at temperature $1/\beta_b$

✓ Under the time evolution $\rho(t)$ remains Gaussian

✓ Importantly $\rightarrow$ $\rho_s(t)$ remains Gaussian

$\rightarrow$ Two-points correlators $\langle c_j c_i^\dagger \rangle$

Evolution of the Correlation Matrix : Definition

We define the matrix $\mathcal{M}^{(n)}$:

$$(\mathcal{M}_s)_{i,j} = \langle c_i c_j^\dagger \rangle \qquad (\mathcal{M}_{s,b}^{(n)})_{i,j} = \langle c_i b_{j,n}^\dagger \rangle$$

.

$$(\mathcal{M}_{b,s}^{(n)})_{i,j} = \langle b_{i,n} c_j^\dagger \rangle \quad (\mathcal{M}_b^{(n)})_{i,j} = \langle b_{i,n} b_{j,n}^\dagger \rangle$$

At $t = (n-1)\tau$ the system and the copy n of the bath enter in interaction

$$\mathcal{M}^{(n)}(n\tau - \tau) = \begin{pmatrix} \mathcal{M}_s(n\tau - \tau) & 0 \\ 0 & \mathcal{M}_b \end{pmatrix}$$

$$(\mathcal{M}_s)_{i,i} = \langle c_i c_j^\dagger \rangle = 1 - \langle n_s(i) \rangle.$$

$$(\mathcal{M}_b^{(n)})_{i,i} = 1 - \langle n_b(i) \rangle.$$

Time Evolution in one time step

Over one time step τ, we show that

$$M^{(n)}(n\tau) = \begin{pmatrix} \mathcal{M}_s(n\tau) & \mathcal{M}_{s,b} \\ \mathcal{M}_{b,s} & \mathcal{M}_b \end{pmatrix} = e^{-i\tau T} \begin{pmatrix} \mathcal{M}_s(n\tau - \tau) & 0 \\ 0 & \mathcal{M}_b \end{pmatrix} e^{+i\tau T}$$

with

$$\mathbf{T} = \begin{pmatrix} \mathbf{T}_s & \Theta \\ \Theta^\dagger & \mathbf{T}_b \end{pmatrix}$$

Focussing on $\mathcal{M}_s$ we get $\mathcal{M}_s(n\tau) = \mathcal{F}[\ \mathcal{M}_s(n\tau - \tau),\ \mathcal{M}_b\ ]$

Next step : Developing

$$e^{-i\tau\mathbf{T}} \simeq \mathbb{I} - i\tau\mathbf{T} - \frac{\tau^2}{2}\mathbf{T}^2 + \dots$$

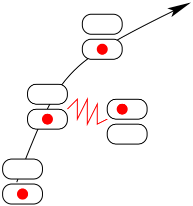
The continuous limit $\tau \to 0$

In the limit $\tau \to 0$ we finally have

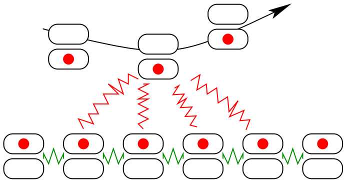
$$\partial_t \mathcal{M}_s = -i\,[\,\mathbf{T}_s, \mathcal{M}_s(t)\,] - \frac{\Lambda^2}{2}\left(\,\{\,\Theta\Theta^\dagger, \mathcal{M}_s(t)\,\} - 2\Theta\mathcal{M}_b\Theta^\dagger\,\right)$$

$$\partial_t \rho_s(t) = -i[H_s, \rho_s] - \sum_i \left(\left\{ L_i L_i^\dagger, \rho_s \right\} - 2 L_i \rho_s L_i^\dagger \right)$$

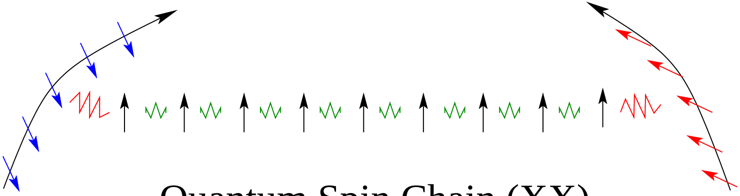
Where do we go from Here



Toy Model



Fermionic Chain



Quantum Spin Chain (XX)

The Open *XX* Chain

✓ The system is described by

$$H_{XX} = -\frac{\lambda_s}{2}\sum_j \left[\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y\right] - \frac{h}{2}\sum_j \sigma_j^z$$

✓ The bath are described by

$$H_B = \sum_n \left[H_n^{(L)} + H_n^{(R)}\right] \qquad H_n^{(L)} = -h\, S_{L,n}^z \quad H_n^{(R)} = -h\, S_{R,n}^z$$

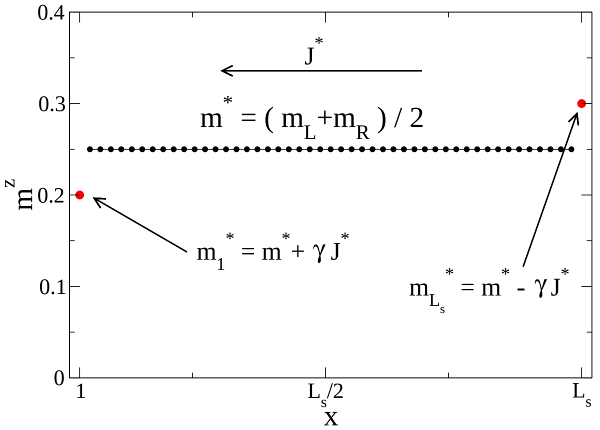
✓ The interactions are $V(t) = V_n^{(L)} + V_n^{(R)} \qquad t \in]n\tau - \tau, \tau]$

$$V_n^{(L)} = -\frac{\lambda_I}{2}\left[S_{L,n}^x \sigma_1^x + S_{L,n}^y \sigma_1^y\right], \qquad V_n^{(R)} = -\frac{\lambda_I}{2}\left[\sigma_{L_s}^x S_{R,n}^x + \sigma_{L_s}^y S_{R,n}^y\right]$$

✓ The left and right bath are thermalized

$$\rho_n^{(L)} \propto \exp\left(-\beta_L H_n^{(L)}\right), \qquad \rho_n^{(R)} \propto \exp\left(-\beta_R H_n^{(R)}\right)$$

Stationary State



✓ Flat magnetization profil $m^z(x) = m^*$

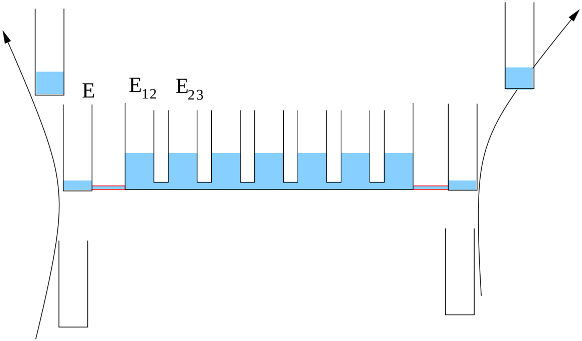
✓ Current $J = (\Delta m^z/2)\,\gamma/(1+\gamma^2)$ $\gamma = \lambda_l^2/2\lambda_s$

Correlation terms

✓ The NESS is fully characterized by

- the magnetization profile $m^z = 2\langle c_j^\dagger c_j\rangle - 1 = m^*$
- its current $J^* = \langle i c_j^\dagger c_{j+1} - i c_{j+1}^\dagger c_j\rangle$
- all other correlation terms vanish $\langle c_i^\dagger c_j + c_j^\dagger c_i\rangle = 0$

✓ Energy of interaction $E_{j,j+1} = -\lambda_s\langle c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\rangle = 0.$



The reduced density matrix

One can built ρ_S^*

$$\rho_S^* \propto e^{-\sum_l \alpha_l Q_l}, \qquad Q_l = \sum_j \left[c_{j+l}^\dagger c_j + (-1)^l c_j^\dagger c_{j+l}\right] \qquad \alpha_l = \alpha_l(m^*, J^*)$$

In the limit $J^* \ll 1$, defining

$$H^{(0)} = -(h/2)\sum_j \sigma_j^z \qquad \mathcal{J}^z = i\sum_j \left[\sigma_j^x \sigma_{j+1}^y - \sigma_{j+1}^y \sigma_j^x\right]$$

we get

$$\rho_S^* \propto \exp\left[-\beta_{eff} H^{(0)} + \frac{2J^*}{1-(m^*)^2}\mathcal{J}^z\right] \qquad \beta_{eff} = \frac{1}{h}\ln\left(\frac{1-m^*}{1+m^*}\right)$$

which at high temperature is $\beta_{eff} \simeq (\beta_L + \beta_R)/2$.

W. H. Aschbacher and C. A. Pillet, J. Stat. Phys. 112, 1153 (2003).

Summary

Using the Repeated Interaction Process,

- on quadratic fermonic systems,
- prepared in a Gaussian state :

✓ $\rho_s(t) = Tr\{\rho(t)\}$ is Gaussian $\forall\, t$

✓ Derive an equation of evolution for the 2 points correlators

$$\partial_t \mathcal{M}_s \quad = \quad -i\,[\,\mathbf{T}_s, \mathcal{M}_s(t)\,] \quad -\frac{\Lambda^2}{2}\left(\,\{\,\Theta\Theta^\dagger, \mathcal{M}_s(t)\,\} - \; 2\Theta\mathcal{M}_b\Theta^\dagger\,\right)$$

✓ Get the exact $\mathcal{M}_s^*$ for the *XX* Chains

✓ Construct the reduced density matrix ρ_s^*.

Considering disordered systems

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