The non-perturbative part of Wilson loops

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Motivation I

- 1978: Shifman, Vainshtein and Zakharov introduced the non-perturbative gluon condensate $\langle \frac{\alpha}{\pi} G\, G \rangle$
- Lattice gauge theory provides a promising tool to calculate it from Wilson loops.
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$\rightarrow$ P_{pert} **is needed to very high order in LPT!**

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- General interest in the behavior of perturbative series in QCD:

$$Q(n^\star) \sim \sum_{n}^{n^\star} a_n \lambda^n$$

- Series are asymptotic, and assumed that for large n the leading growth of the coefficients a_n can be parametrized

$$a_n \sim \; C_1 \, (C_2)^n \, \Gamma(n + C_3)$$

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- Standard diagrammatic approach in LPT is restricted essentially to two-loop
- Di Renzo et al. formulated the so-called Numerical Stochastic Perturbation Theory (NSPT)
- Based on the Langevin quantization method of Parisi/Wu
- NSPT drops the concept of Feynman diagrams - uses the action with the corresponding perturbative expansion of fields
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Computational realization

- Pure SU(3) gauge theory
- Lattice sizes L^4 with $L = 4, 6, 8, 12$
- Loop order $n = 20$
- Computer ressources: ITP, Computer center of Leipzig university, RCNP Osaka

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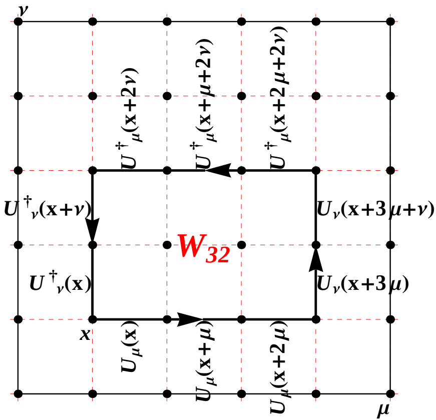
Wilson loops

Wilson loops on the lattice are ordered products of gauge link operators $U_\mu(x) = \exp\left(a\, g\, T^c\, A^c_\mu(x)\right)$



Wilson loops in NSPT

Connection to condensate ($a = 1$)

$$\begin{aligned} W_{NM} &= U_\mu(x) \ldots U_\mu(x+(N-1)\mu)\, U_\nu(x+N\mu) \ldots U_\nu(x+N\mu+(M-1)\nu) \\ &\quad U^\dagger_\mu(x+(N-1)\mu+M\nu) \ldots U^\dagger_\mu(x+M\nu)\, U^\dagger_\nu(x+(M-1)\nu) \ldots U^\dagger_\nu(x) \end{aligned}$$

(plaquette: $P = W_{11} = U_\mu(x) U_\nu(x+\mu) U^\dagger_\mu(x+\nu) U^\dagger_\nu(x)$)

Shifman (1980):

$$\langle 1 - W_{NM} \rangle \sim \frac{\pi^2}{12 N_c} \langle \frac{\alpha_s}{\pi} GG \rangle S^2_{NM} + \text{higher dim. terms}$$

S_{NM}: area of Wilson loop (here: $N \times M$)
$GG \equiv G^c_{\mu\nu} G^c_{\mu\nu}$
$G^c_{\mu\nu} = \partial_\mu A^c_\nu - \partial_\nu A^c_\mu - g f^{cab} A^a_\mu A^b_\nu$

Wilson loops in NSPT

We write the perturbative expansion for a Wilson loop W_{NM} as

$$W_{NM}(g, n^\star) = \sum_{n=0}^{n^\star} W_{NM}^{(n)} g^{2n}$$

As an example for W_{11} and a lattice of $L = 12$:

$$W_{11}(g, 20) = 1 - 0.333333g^2 - 0.033911g^4 - 0.0137061g^6 - 0.00725108g^8 - 0.00441001g^{10} - 0.00292153g^{12} - 0.0020518g^{14} - 0.00150358g^{16} - 0.00113812g^{18} - 0.00088372g^{20} - 0.000700396g^{22} - 0.000564629g^{24} - 0.000461837g^{26} - 0.000382568g^{28} - 0.000320464g^{30} - 0.000271108g^{32} - 0.000231376g^{34} - 0.000198948g^{36} - 0.000172211g^{38} - 0.000149902g^{40}$$

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Boosted series

Expansion parameter g for the perturbative computation of Wilson loops is not really small ($g^2_{min} < g^2 < g^2_{max}$):

$g^2_{max} \sim 1.04$: convergence radius for our finite lattice series
$g^2_{min} \sim 0.95$: confinement region

Expansion in bare lattice coupling g is disadvantegeous because of lattice artefacts $\rightarrow$ boosted coupling $g_b^2 = g^2 / W_{11}(g, n^\star)$

$W_{b,11}(g_b, 20) =$
$1 - 0.333333 g_b^2 + 0.0772001 g_b^4 - 0.0168321 g_b^6 + 0.00306193 g_b^8 - 0.000618686 g_b^{10} + 0.0000871841 g_b^{12} - 0.0000242642 g_b^{14} + 8.309057\, 10^{-7} g_b^{16} - 1.7372532\, 10^{-6} g_b^{18} - 2.89077030\, 10^{-7} g_b^{20} - 2.31637202\, 10^{-7} g_b^{22} - 2.05976723\, 10^{-7} g_b^{24} + 5.002386978\, 10^{-8} g_b^{26} - 1.02263213\, 10^{-7} g_b^{28} + 4.32838185\, 10^{-8} g_b^{30} - 3.40958978\, 10^{-8} g_b^{32} - 1.41692580\, 10^{-9} g_b^{34} + 8.62333\, 10^{-8} g_b^{36} - 2.9140127\, 10^{-7} g_b^{38} + 6.30326326\, 10^{-7} g_b^{40}$

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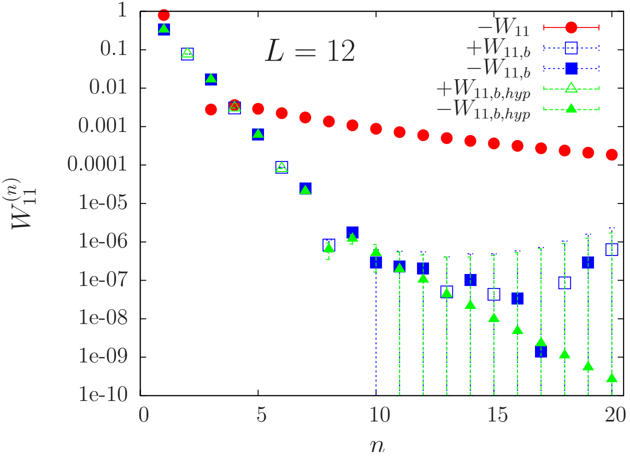
Coefficient comparison

Comparison of coefficients for naive and boosted perturbative series

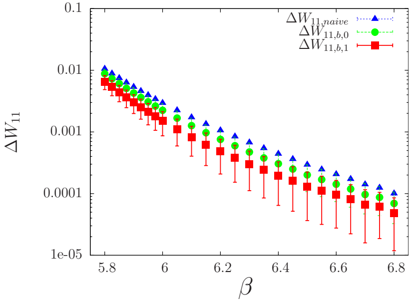


Choice of coupling

$g_b^2 > g^2$ ($g^2 = 1 \to g_b^2 = 1.683$): need the best possible perturbative determination $\to g_b^2 = g^2 / W_{b,11}(g_b, n^\star)$

Test: for large $\beta = 6/g^2$ the (MC) measured Wilson loop should be almost perturbative

Define $\Delta W_{NM}(\beta) = \frac{W_{NM,PT}(\beta) - W_{NM,MC}(\beta)}{W_{NM,MC}(\beta)}$



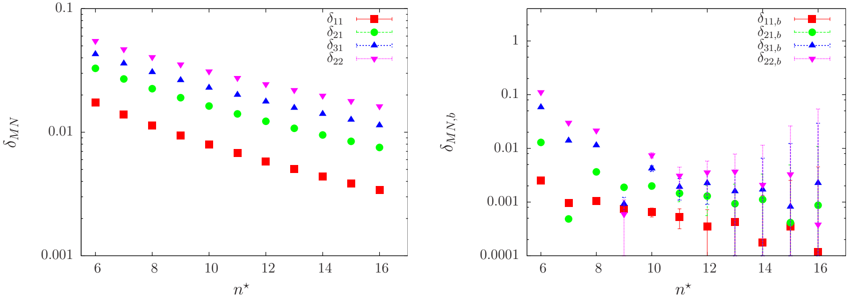
green: $g_b^2 = g^2 / W_{11}(g, 13)$; red: $g_b^2 = g^2 / W_{b,11}(g_b, 13)$

Relative convergence

Compare relative convergence behaviour from

$$\delta_{NM}(g, n^\star) = \frac{|W_{NM}(g, n^\star) - W_{NM,\infty}|}{W_{NM,\infty}}$$

for $L = 12$ and $\beta = 6$.



$W_{NM,\infty}$ - values for the summed up PT series in a model ansatz for finite L

Ratios of Wilson loops

We consider the ratios

$$R^{k,m}_{NM,N'M'} = \frac{(W_{NM,b}(g_b))^k}{(W_{N'M',b}(g_b))^m} = \sum_n [R^{k,m}_{NM,N'M'}]^{(n)} g_b^{2n}$$

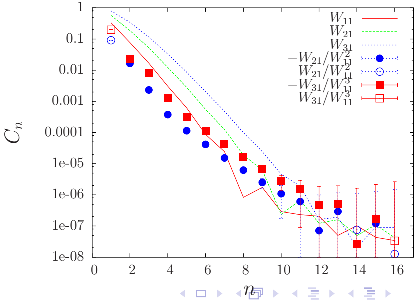
together with the "naturalness condition"

$$k \times \mathrm{Area}[W_{NM}] = m \times \mathrm{Area}[W_{N'M'}]$$

The coefficients show a similar behaviour as the boosted coefficients

$[R^{1,3}_{13,11}]^{(n)}$

$[R^{1,2}_{12,11}]^{(n)}$



Ratios of Wilson loops

We consider

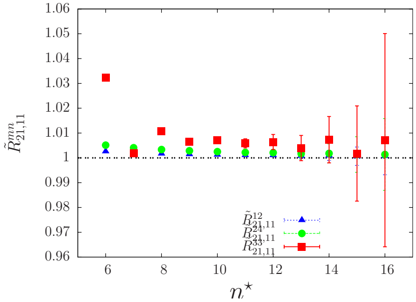
$$\tilde{\mathcal{A}} = \frac{\mathcal{A}_{PT}}{\mathcal{A}_{MC}} = \frac{\mathcal{A}_{PT}}{\mathcal{A}_{PT} + \Delta_{\mathcal{A}}} \quad \rightarrow \quad \tilde{\mathcal{A}} \simeq 1 - \frac{\Delta_{\mathcal{A}}}{\mathcal{A}_{PT}}$$

Taking for $\mathcal{A}$ the ratios introduced above

$$\tilde{R}^{k,m}_{NM,N'M'} = \left(W_{NM,b,PT}(n^\star)/W_{NM,MC}\right)^k / \left(W_{N'M',b,PT}(n^\star)/W_{N'M',MC}\right)^m$$

$\tilde{R}^{2,1}_{11,21}$, $\tilde{R}^{4,2}_{11,21} \sim 1$
(naturalness condition)

$\tilde{R}^{3,3}_{11,21}$ unsuitable

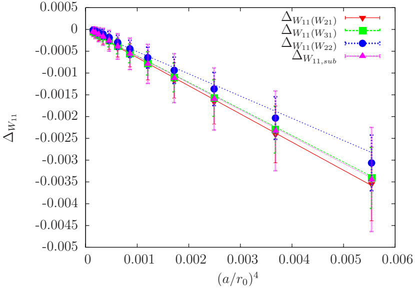


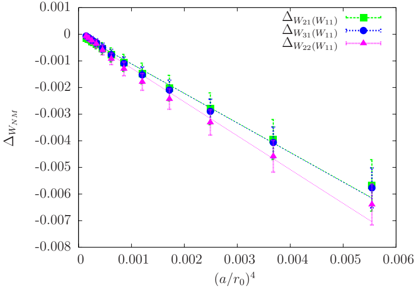
Ratios of Wilson loops

We have the exact relation

$$\Delta_{W_{NM}(W_{N'M'})} = \left(\exp\left(-\frac{d}{dk}\log\left(\tilde{R}^{k,m}_{NM,N'M'}\right)\right) - 1\right) \times W_{NM,b,PT}$$

Using the Necco-Sommer relation $\beta \leftrightarrow (a/r_0)$ we get





almost ideal $\sim a^4$ scaling behaviour

Condensate from $\Delta_{W_{11}(W_{N'M'})}$

$\Delta_{W_{11}(W_{N'M'})} \sim a^4 \quad \rightarrow$ determination of $\langle GG \rangle$ as dimension=4 quantity ($n^\star = 13$ as summation limit for boosted PT-series, $r_0 = 0.5$ fm)

$$\begin{aligned}
\Delta_{W_{11}(W_{21})} \rightarrow \langle \frac{\alpha_s}{\pi} GG \rangle &= 0.054(2)(3)\,\mathrm{GeV}^4 \\
\Delta_{W_{11}(W_{31})} \rightarrow \langle \frac{\alpha_s}{\pi} GG \rangle &= 0.051(2)(2)\,\mathrm{GeV}^4 \\
\Delta_{W_{11}(W_{22})} \rightarrow \langle \frac{\alpha_s}{\pi} GG \rangle &= 0.046(3)(1)\,\mathrm{GeV}^4 \\
\Delta_{W_{11}(sub)} \rightarrow \langle \frac{\alpha_s}{\pi} GG \rangle &= 0.059(1)(5)\,\mathrm{GeV}^4
\end{aligned}$$

Compare to $\langle \frac{\alpha_s}{\pi} GG \rangle_{SVZ} \sim 0.012\,\mathrm{GeV}^4$

Summary

- PT series coefficients do not show any factorial behaviour up to $n = 20$ (*not shown*)
- Finite lattice PT series can be represented (and parametrized) very well by a hypergeometric model (*not shown*)
- Boosted PT series are shown to be very useful
- Non perturbative parts scale like $\sim a^4$
- $\langle \frac{\alpha_s}{\pi} GG \rangle$ obtained are larger than SVZ value

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