Precision Calculation of spin-spin correlators in the Ising model via worm updates

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- worm updates by N.V. Prokof'ev, B.V. Svistunov: PRL 87, 160601 (2001)
- sample graphs of the HTSE
- Ising model : dimers $k_i = 0, 1$
- one Giga Byte $\approx 10^{10} \approx 2^{33} = 2 \times 2^{16} \times 2^{16}$
- calculate the two point functions $\Gamma(R)$, $< \sigma_i \sigma_j >$ on a $N = 32768$ box
- in the high temperature phase of the 2D Ising model
- Avogardo 6×10^{23}
- exact result for $\Gamma(t)$ by WU, McCoy, Tracy, Barouch, PRB 13, 316 (1976) for Γ in the continuum limit

$$T \to T_c$$

$$t = \frac{R}{\xi} = \text{constant}$$

- it's a sportive project

- partition function with two spin insertions

$$Z(u,v) = \sum_{Conf.} e^{\beta \sum_{Bonds} s(i)s(j)} s(u)s(v)$$

- two point function

$$G(u-v) = \frac{Z(u,v)}{Z(u,u)}$$

- Cartesian coordinates $u = (u_x, u_y)$ and $v = (v_x, v_y)$
- zero momentum on $N_x \times N_y$ box

$$\Gamma(R) = \frac{1}{N_y} \sum_{u,v} G(u-v)\delta^1(R - u_x + v_x)$$

- simple exponential decay

$$\lim_{R\to\infty}\Gamma(R) = Ae^{-\frac{R}{\xi_0}}$$

- $\xi_0 = -[2\beta + log(tanh(\beta))]^{-1}$ along main axis and high T
- the point point correlator along the main axis is

$$< \sigma_{0,0}\sigma_{0,R} >= G[u = (0,0), v = (0,R)]$$

- dimers $k_{i,\eta} = 0, 1$, $\eta = 1, 2$ on a $2D$ $N \times N$ box
- local dimer sum

$$\Sigma(i) = \sum_{\eta=1,2} (k_{i,\eta} + k_{i-\eta,\eta})$$

- odd at u and v

$$\mathrm{mod}(\Sigma(u), 2) = \mathrm{mod}(\Sigma(v), 2) = 1$$

- even at $i \neq u, v$

$$\mathrm{mod}(\Sigma(i), 2) = 0$$

- loops of dimers and one worm

- dimer chemical potential

$$\tanh(\beta) = e^{-\mu}$$

$$\mu = -\ln[\tanh(\beta)]$$

- dimer partition function

$$Z(u,v) = \sum_{Dimers} e^{-\mu \sum_{i,\eta} k_{i,\eta}} \times \delta^1(1 - \mathrm{mod}(\Sigma(u), 2))$$
$$\times \delta^1(1 - \mathrm{mod}(\Sigma(v), 2))$$
$$\times \prod_{i \neq u,v} \delta^1(0 - \mathrm{mod}(\Sigma(i), 2))$$

- the worm update of PS performs single dimer moves at either u or v, uniform in the directions η and $-\eta$
- with a Metropolis accept reject decision
- yields a 'Random Walk' of the worm
- if the worm closes at $u = v$
  - Wolff says : kick-off to a random position $u' = v'$ and start the worm there, NPB 810, 491 (2009)
  - Evertz conjectures : even $u = v = u' = v' \mathrm{const}$ may work (no-kickoff)

- the Histogram $H(u,v)$ determines Γ
- Multicanonical sampling

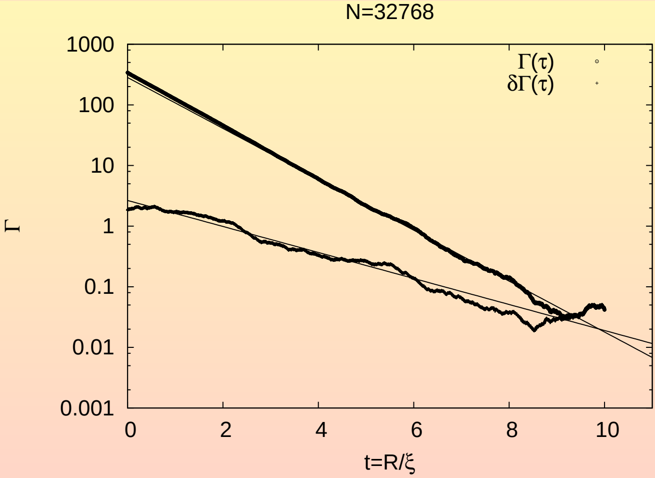
$$Z(u,v,W_{MUCA}) = \sum_{Dimers} e^{-\mu \sum_{i,\eta} k_{i,\eta}} \times \delta^1(1-\mathrm{mod}(\Sigma(u),2))$$
$$\times\, \delta^1(1-\mathrm{mod}(\Sigma(v),2))$$
$$\times \prod_{i \neq u,v} \delta^1(0-\mathrm{mod}(\Sigma(i),2))$$
$$\times\, e^{+W_{MUCA}(u_x-v_x)}$$

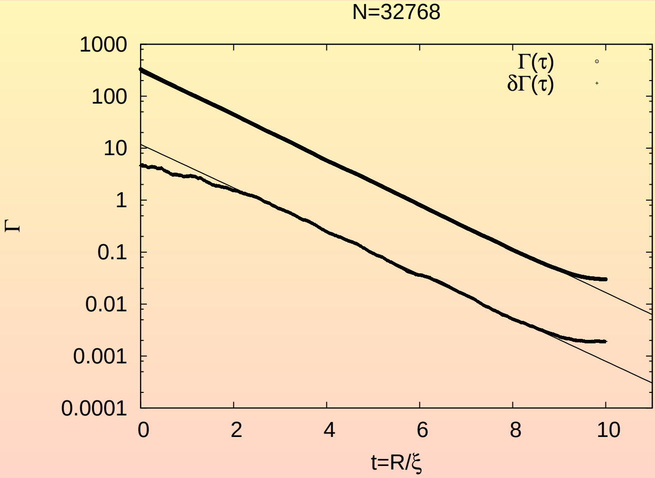
- with a guidance correlation length $\xi_G = 1.05\xi_0$
- and the weight

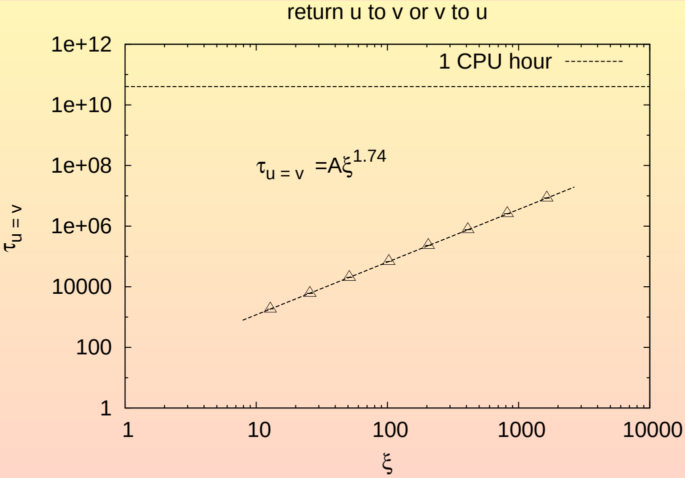
$$e^{-W_{MUCA}(u_x-v_x)} = \frac{1}{2}[e^{-\frac{u_X-v_X}{\xi_G}} + e^{-\frac{N-u_X+v_X}{\xi_G}}]$$

parameters

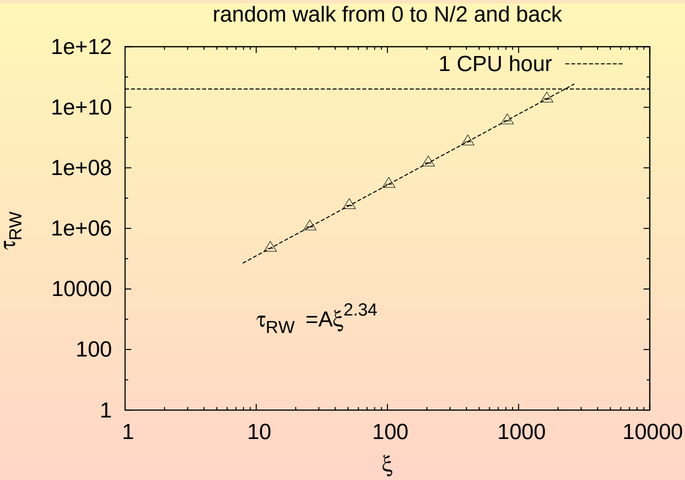
N	β	ξ_0
256	0.42142526	12.8
512	0.43098861	25.6
1024	0.43582087	51.2
2048	0.43824960	102.4
4096	0.43946715	204.8
8192	0.44007674	409.6
16384	0.44038173	819.20
32768	0.44053422	1638.4


N=32768
1000
100
10
1
0.1
0.01
0.001
Γ
Γ(τ)
δΓ(τ)
0
2
4
6
8
10
t=R/ξ

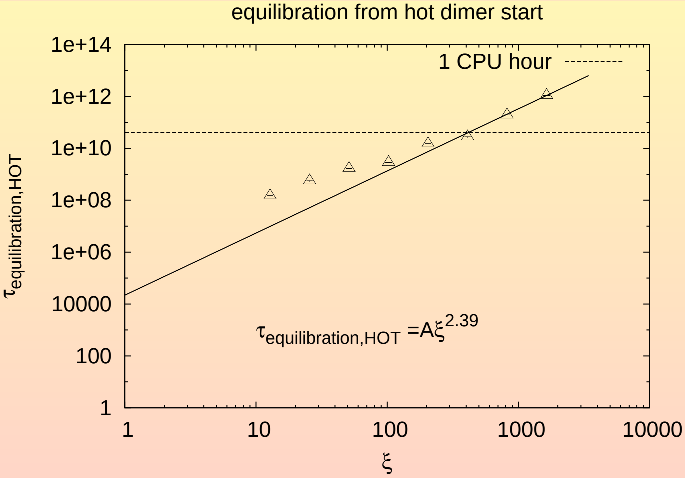

N=32768
1000
100
10
1
0.1
0.01
0.001
0.0001
Γ
0
2
4
6
8
10
t=R/ξ
Γ(τ)
δΓ(τ)



return u to v or v to u
1 CPU hour
$\tau_{u=v} = A\xi^{1.74}$
$\tau_{u=v}$
ξ
1e+12
1e+10
1e+08
1e+06
10000
100
1
1
10
100
1000
10000



random walk from 0 to N/2 and back
1 CPU hour
1e+12
1e+10
1e+08
1e+06
10000
100
1
1
10
100
1000
10000
τ_{RW}
ξ
$\tau_{RW} = A\xi^{2.34}$


equilibration from hot dimer start
1 CPU hour
$\tau_{\mathrm{equilibration,HOT}} = A\xi^{2.39}$
$\tau_{\mathrm{equilibration,HOT}}$
ξ
1e+14
1e+12
1e+10
1e+08
1e+06
10000
100
1
1
10
100
1000
10000

ONKICK

N	β	ξ_0	Steps	ξ_0	1	BIT
256	0.42142526	12.8	0.10E+14	12.7999(7)	0.99999(6)	1
512	0.43098861	25.6	0.10E+14	25.603(3)	1.0001(1)	0
1024	0.43582087	51.2	0.10E+14	51.216(9)	1.0003(2)	0
2048	0.43824960	102.4	0.10E+14	102.43(5)	1.0003(4)	1
4096	0.43946715	204.8	0.24E+14	204.9(2)	1.0007(6)	0
8192	0.44007674	409.6	0.23E+14	409.9(6)	1.000(2)	1
16384	0.44038173	819.2	0.24E+14	822(4)	1.003(5)	1
32768	0.44053422	1638.4	0.16E+14	1639(9)	1.000(6)	1

NOKICK

N	β	ξ_0	Steps	ξ_0	1	BIT
256	0.42142526	12.8	0.80E+13	12.7997(6)	0.99998(5)	1
512	0.43098861	25.6	0.80E+13	25.600(3)	1.00002(9)	1
1024	0.43582087	51.2	0.80E+13	51.23(2)	1.0007(3)	0
2048	0.43824960	102.4	0.80E+13	102.38(6)	0.9998(5)	1
4096	0.43946715	204.8	0.80E+13	204.6(3)	0.999(2)	1
8192	0.44007674	409.6	0.78E+13	411.4(9)	1.004(3)	0
16384	0.44038173	819.20	0.70E+13	820(7)	1.002(8)	1
32768	0.44053422	1638.4	0.42E+13	1628(26)	0.99(2)	1

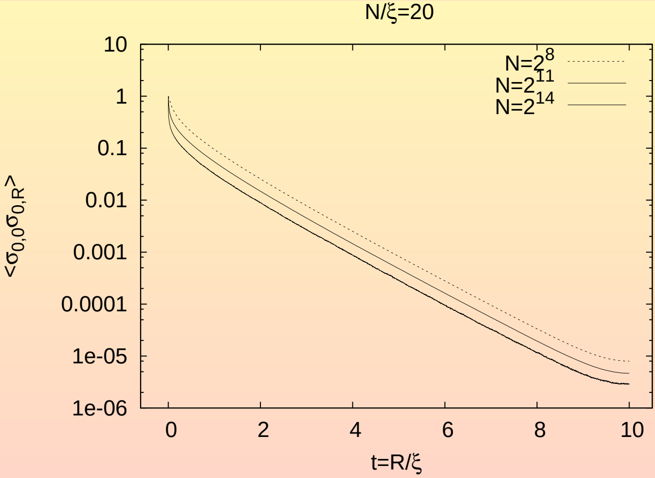
- WU, McCoy, Tracy, Barouch

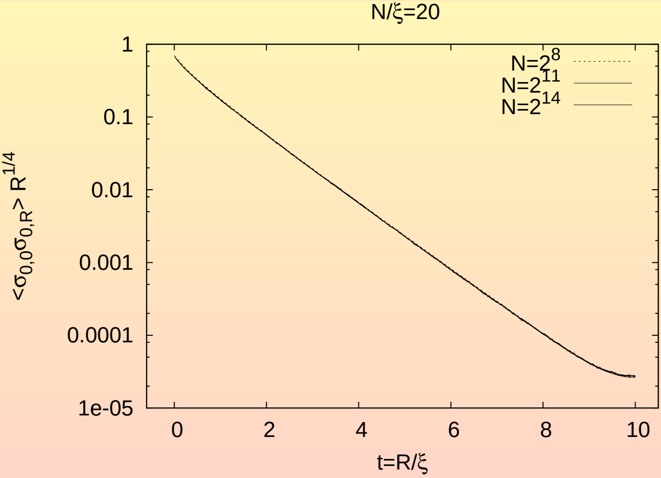
$$t = \frac{R}{\xi}$$

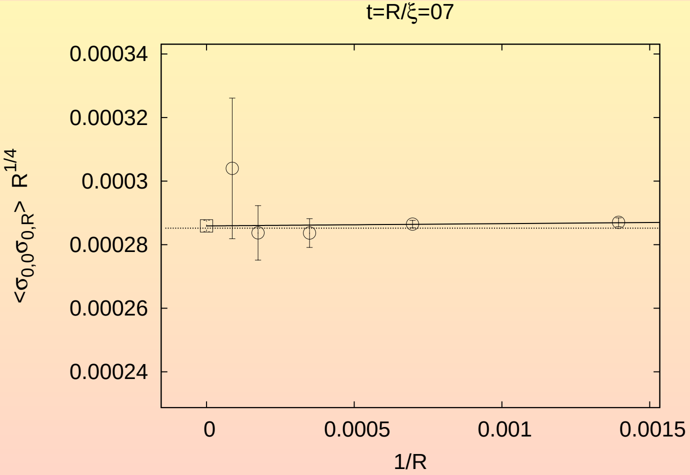
$$< \sigma_{0,0}\sigma_{0,R} >= R^{-1/4}F_{+}(t) + R^{-5/4}F_{1+}(t) + \mathcal{O}(R^{-5/4})$$

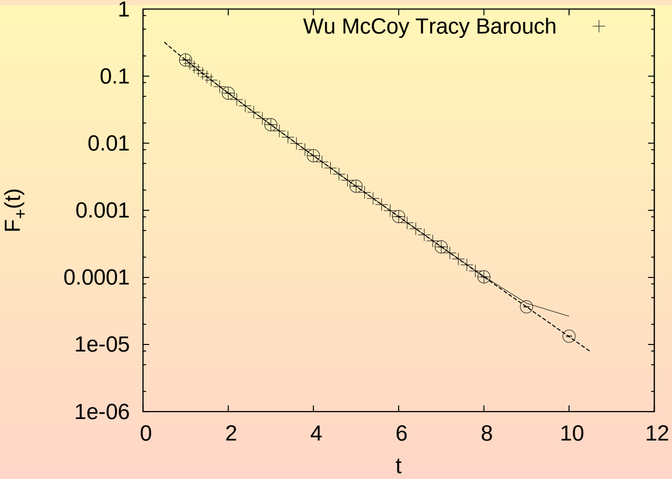
symmetric box

$$F_{1+}(t) = t\, 2^{-3/2}F_{+}(t)$$


N/ξ=20
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
0
2
4
6
8
10
t=R/ξ
<σ0,0σ0,R>
N=2^8
N=2^11
N=2^14


N/ξ=20
1
0.1
0.01
0.001
0.0001
1e-05
0
2
4
6
8
10
t=R/ξ
<σ0,0σ0,R> R1/4
N=2^8
N=2^11
N=2^14


t=R/ξ=07
0.00034
0.00032
0.0003
0.00028
0.00026
0.00024
0
0.0005
0.001
0.0015
1/R
<σ0,0σ0,R> R1/4


Wu McCoy Tracy Barouch
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
$F_+(t)$
0
2
4
6
8
10
12
t

t	F_+	F_+	$(F_+ - F_+)/\delta F_+$	BIT
1.	0.17380000	0.17440237 (0.00034030)	1.77	0
2.	0.05591000	0.05576761 (0.00017553)	-0.81	1
3.	0.01887000	0.01889210 (0.00007651)	0.29	1
4.	0.00651400	0.00651122 (0.00002704)	-0.10	1
5.	0.00227800	0.00229151 (0.00000993)	1.36	0
6.	0.00080360	0.00080661 (0.00000409)	0.74	1
7.	0.00028520	0.00028530 (0.00000176)	0.06	1
8.	0.00010160	0.00010208 (0.00000065)	0.74	1

CONCLUSION

- continuum limit of massive 2-point function in high T phase of 2D Ising
- with re-weighted worms : correlation length of $\mathcal{O}(1600)$ is accessible in $2D$
- the 65536^2 box was not thermalized