The Gonihedric Ising Models: Order parameter(s)

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Plan of Talk

(Brief!) History of the Gonihedric model

Symmetries

Anisotropic model

Order parameter

History of the Gonihedric (String) Model

String Theory

Polyakov Action

$$S = \int d^2\sigma \sqrt{detg} g^{ab} \partial_a X_\mu \partial_b X^\mu$$

Partition function

$$Z = \int [Dg][DX] \exp(-S)$$

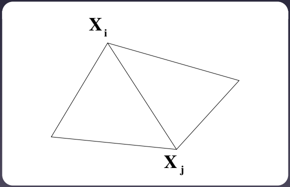
Triangulated Surfaces

Discretize worldsheet with triangles

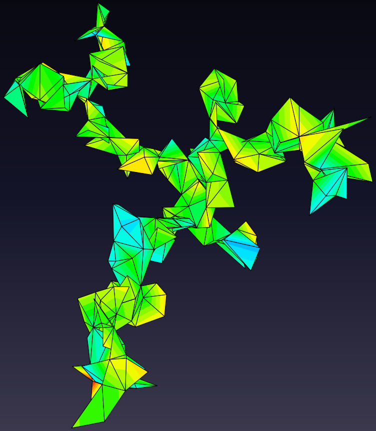
Gaussian Action

$$S \sim \sum_{ij} \left(X^{\mu}(i) - X^{\mu}(j)\right)^2$$

$$Z = \sum_{T} \int \prod_{i} dX_i \exp\left(-S\right)$$



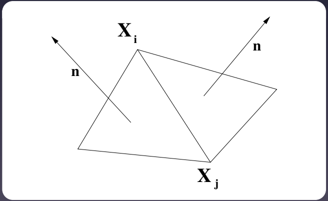
Typical Surfaces



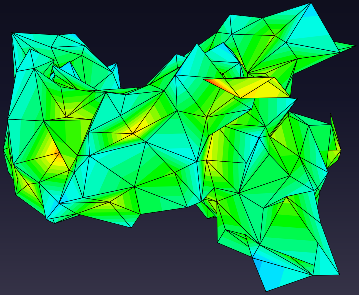
Modifying the Gaussian Action

Add extrinsic curvature term

$$
\begin{aligned}
S &= \sum_{ij} \left(X^\mu(i) - X^\mu(j)\right)^2 \\
&+ \lambda \sum_{\Delta_i, \Delta_j} (1 - \vec{n}_i \cdot \vec{n}_j)
\end{aligned}
$$



Smoothed Surfaces



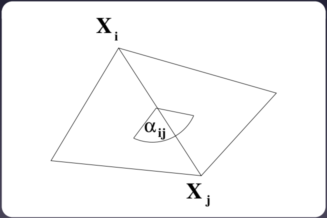
The Gonihedric action

Savvidy "Gonihedric" action

$$
\begin{aligned}
S &= \sum_{ij} |X^\mu(i) - X^\mu(j)|\theta_{ij} \\
\theta_{ij} &= |\pi - \alpha_{ij}|
\end{aligned}
$$

Gonia: angle

Hedra: face



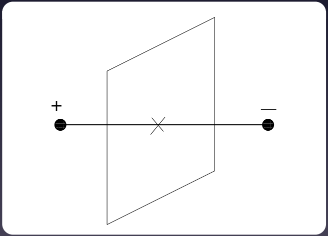
Gonihedric Surfaces on a Cubic Lattice

Spins on dual lattice $\rightarrow$ plaquettes

Spin cluster boundaries $\rightarrow$ surfaces

Edge spins: $U_{ij} = -1$

Vertex spins: $\sigma_i \sigma_j = -1$



Counting configurations with spins

Spin gadgets to count area, bends and intersections

Area

$$S(\sigma) = \sum_{\langle ij \rangle} \frac{1}{2}(1 - \sigma_i \sigma_j)$$

Self intersection

$$I = -\frac{1}{8}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_4 + \sigma_4\sigma_1)$$
$$+\frac{1}{8}(\sigma_1\sigma_3 + \sigma_2\sigma_4) + \frac{1}{8}(\sigma_1\sigma_2\sigma_3\sigma_4) + \frac{1}{8}$$

Still Counting configurations with spins

Bends (and two from a crossing)

$$
\begin{aligned}
C = & - \frac{1}{4}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_4 + \sigma_4\sigma_1) \\
& + \frac{1}{4}(\sigma_1\sigma_3 + \sigma_2\sigma_4) - \frac{1}{4}(\sigma_1\sigma_2\sigma_3\sigma_4) + \frac{3}{4}
\end{aligned}
$$

A Generalized Ising action

In general energy comes from areas, edges and intersections (*A. Cappi, P Colangelo, G. Gonella and A. Maritan*)

$$H = J_1 \sum_{<ij>} \sigma_i \sigma_j + J_2 \sum_{<<ij>>} \sigma_i \sigma_j + J_3 \sum_{[i,j,k,l]} \sigma_i \sigma_j \sigma_k \sigma_l$$

$$\beta_A = 2J_1 + 8J_2, \quad \beta_C = 2J_3 - 2J_2, \quad \beta_I = -4J_2 - 4J_3$$

$$H = \sum (\beta_A n_A + \beta_C n_C + \beta_I n_I)$$

A Particular Generalized Ising action

For the Gonihedric model, we want *no* area weight

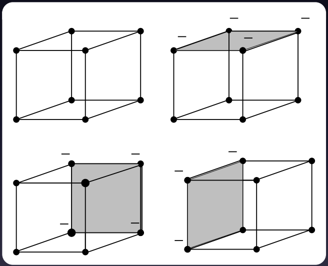
$$H = 2\kappa \sum_{<ij>} \sigma_i \sigma_j - \frac{\kappa}{2} \sum_{<<ij>>} \sigma_i \sigma_j + \frac{1-\kappa}{2} \sum_{[i,j,k,l]} \sigma_i \sigma_j \sigma_k \sigma_l$$

$\kappa = 0$ simple (and interesting)

$$H = \frac{1}{2} \sum_{[i,j,k,l]} \sigma_i \sigma_j \sigma_k \sigma_l$$

Symmetries

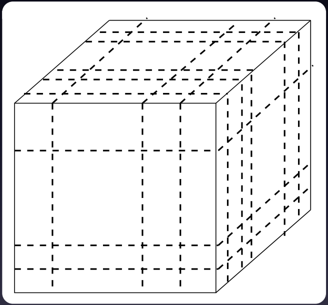
Possible to flip layers of spins at zero energy cost



These can intersect

Groundstate(s)

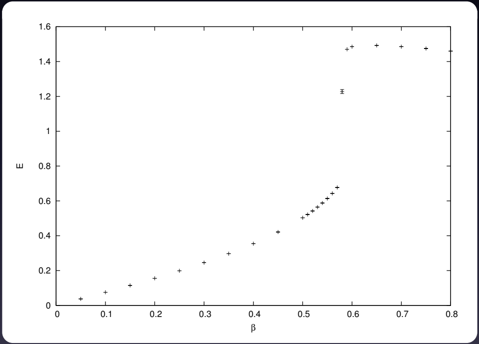
Groundstate



No simple magnetic order

Transition

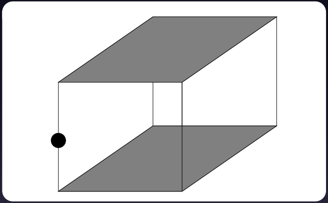
There is clearly a first order transition



Order parameter?

Anisotropic

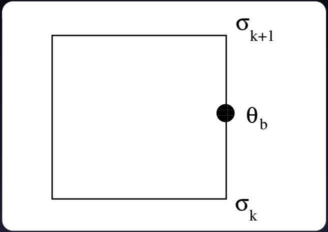
Switch off horizontal plaquette coupling



Vertical - Stack of 2D Gonihedric Ising

Horizontal (“Fuki-nuke”)- Stack of 2D Ising

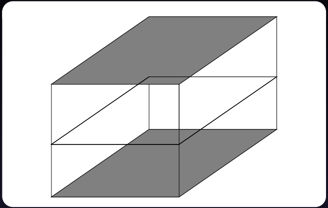
Anisotropic: Stack of 2D Ising Models



$b = (i, j, k + 1/2)$

$H = -J \sum_b (\theta_b \theta_{b+\hat{x}} + \theta_b \theta_{b+\hat{y}})$

Quasi-3D order parameter

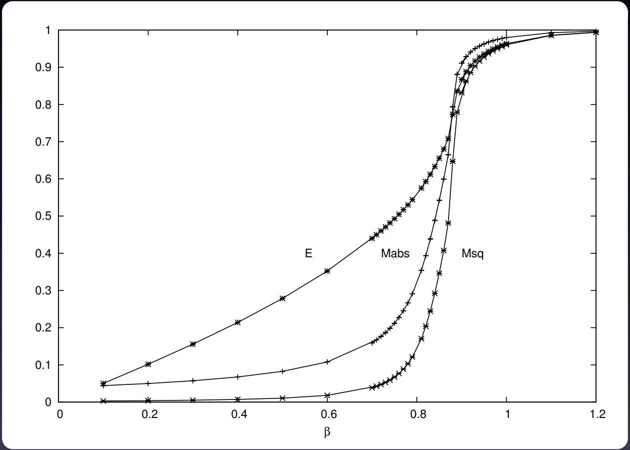


$$\left\langle \frac{1}{N^2} \sum_{single\ plane} \theta_{\mathbf{b}} \right\rangle = \left\langle \frac{1}{N^2} \sum_{single\ plane} \sigma_i \sigma_{i+\hat{e}_z} \right\rangle$$

$$M_{abs} = \frac{1}{N} \sum_{planes} \left\langle \left| \frac{1}{N^2} \sum_{single\ plane} \sigma_i \sigma_{i+\hat{e}_z} \right| \right\rangle$$

$$M_{sq} = \frac{1}{N} \sum_{planes} \left\langle \left(\frac{1}{N^2} \sum_{single\ plane} \sigma_i \sigma_{i+\hat{e}_z} \right)^2 \right\rangle$$

Quasi-3D order parameter



Isotropic order parameter

Suzuki - use this for isotropic case too

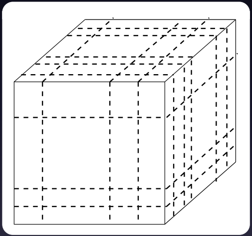
$$M^x_{abs} \;=\; \frac{1}{N} \sum_{yz\, planes} \left\langle \left| \frac{1}{N^2} \sum_{single\ plane} \sigma_i \sigma_{i+\hat{e}_x} \right| \right\rangle$$

$$M^x_{sq} \;=\; \frac{1}{N} \sum_{yz\, planes} \left\langle \left(\frac{1}{N^2} \sum_{single\ plane} \sigma_i \sigma_{i+\hat{e}_x} \right)^2 \right\rangle$$

And similarly for y, z directions

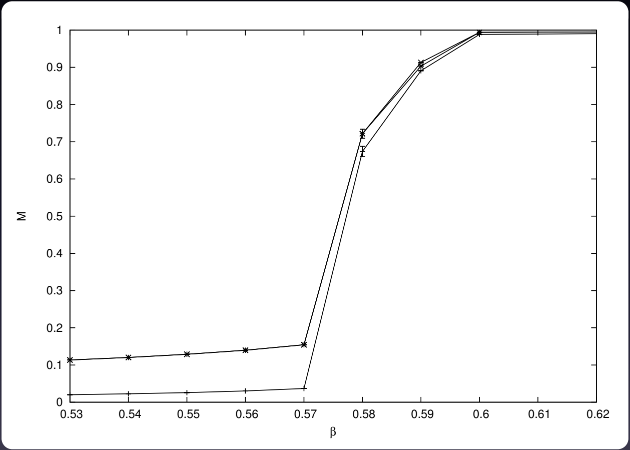
Test cases

"Ground state" - zero temperature, $M_{abs} = M_{sq} = 1$



Random - infinite temperature, $M_{abs} = M_{sq} = 0$

Isotropic order parameter(s)

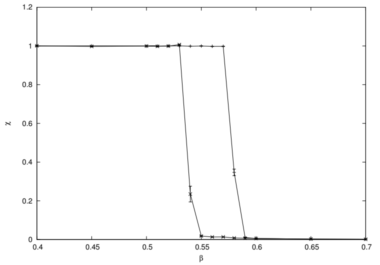


M_{abs} (upper) and M_{sq} on a 20^3 lattice

A Puzzle

Standard magnetic susceptibility *also* appears to behave as order parameter

$$\chi = \frac{1}{N^3} \sum_i \sum_j \langle \sigma_i \sigma_j \rangle = \frac{1}{N^3} \sum_i \sum_n \langle \sigma_i \sigma_{i+n} \rangle$$



... but is one on both “flipped” and random configurations

Conclusions

3D Gonihedric model has novel ground state "flip" symmetry

Persists at finite T in $\kappa = 0$ (plaquette-only) model

"Planar" order

Behaviour of χ ?