Random-field Ising magnet with correlated disorder

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CompPhys11, Leipzig, 25. November 2011

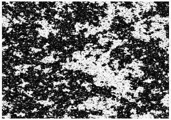
Outline

Computer Science

helps →

← **helps**

Physics



- Random-field systems
- Mapping to maximum-flow/minimum $s-t$ cut problem
- *Ferro* $\leftrightarrow$ *para* phase transition
- Correlated disordered

[AKH and H. Rieger, *Optimization Algorithms in Physics*, Wiley-VCH 2001]

[AKH and H. Rieger (eds.), *New Optimization Algorithms in Physics*, Wiley-VCH 2004]

[AKH and M. Weigt, *Phase Transitions in Optimization Problems*, Wiley-VCH 2005]

Diluted Antiferromagnets

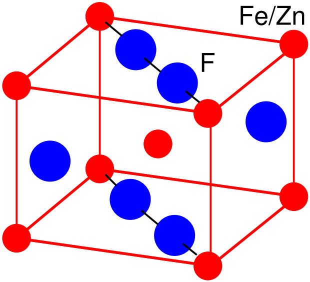
FeF_2: antiferromagnet

disorder:

Replace fraction $(1 - x)$ of iron by zinc (not magnetic)

frustration:

put into magnetic field B



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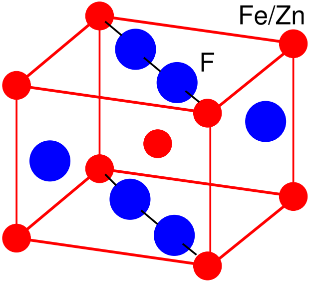
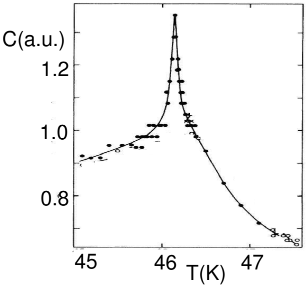


Measurement of specific heat



[D.P. Belanger et al., 1983]

phase transition with

$C(T) \sim \log |(T - T_c)/T_c|$

(like $d = 2$ ferromagnet)

$\rightarrow$ Find suitable model

Random-field Ising Model

- Ising spins $\sigma_i = \pm 1$ on a regular lattice
- Hamiltonian: $$\mathcal{H} = -\sum_{\langle i,j \rangle} J\sigma_i\sigma_j - \sum_i B_i\sigma_i$$

  quenched: $B_i = \pm h$ or Gaussian (mean 0, std.dev. h)
  Belief: DAFF $\Leftrightarrow$ RFIM [S. Fishman, A. Aharony, 1979] [J.L. Cardy, 1984]

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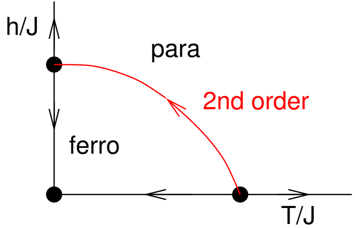
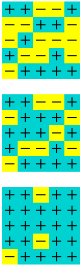
- Phase diagram ($d > 2$): (mf, RG)

  critical exponents?

[J.L. Cardy, Scaling ..., 1996]





$h \approx h_c$

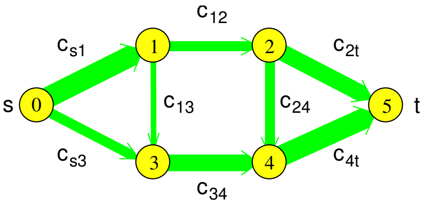
$h < h_c$

$h \approx 0$

- Monte Carlo: small sizes $L \leq 16$ [H. Rieger, 1995]
  $\rightarrow$ do GS calculations (large sizes, exact)

Networks

- Idea: mapping of RFIM $\leftrightarrow$ network



- Network = graph + edge capacities: $G = (V, E),\ E \subset V \times V$ $c_{ij} > 0,\ s, t \in V$
- Now: network $\rightarrow$ RFIM

- Much transit traffic in Austria
  $\rightarrow$ blockade 25. October 2002

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  $S \cup \overline{S} = V, S \cap \overline{S} = \emptyset,$
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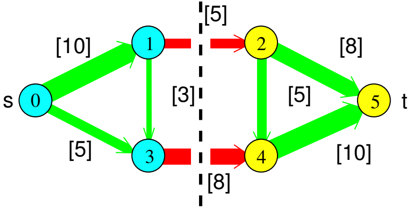

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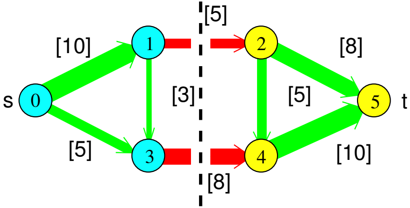
  $\sim$ capacity of cut

$$C(S, \overline{S}) = \sum_{i \in S, j \in \overline{S}} c_{ij} \, .$$



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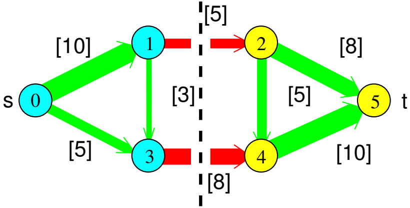

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- With $\underline{X} = (x_0, \ldots, x_{n+1})$, $x_i = 0/1$, $x_i = 1 \Leftrightarrow i \in S$
  [J.-C. Picard and H.D. Ratliff, Networks 1975]

$$C(\underline{X}) = \sum_{ij} c_{ij} x_i (1 - x_j) = -\sum_{ij} c_{ij} x_i x_j + \sum_i (\sum_j c_{ij}) x_i$$

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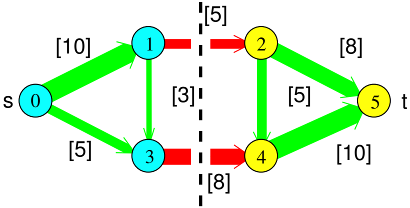

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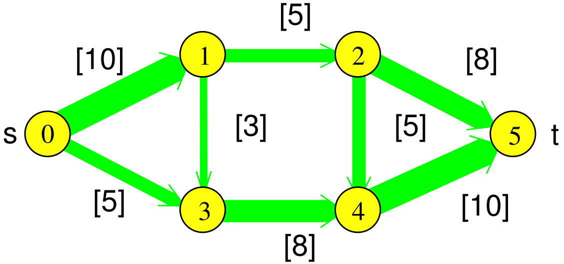
- Minimum energy = capacity of min. cut = max. flow

- Calculate maximum flow: Ford-Fulkerson algorithm (1956)



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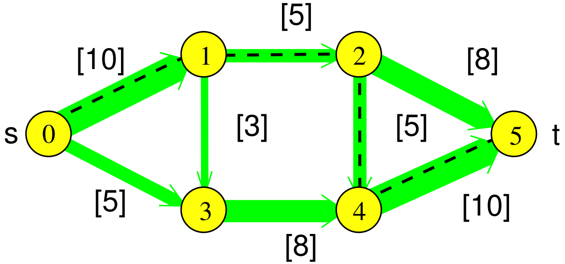

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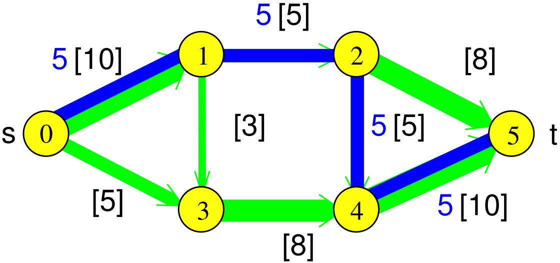

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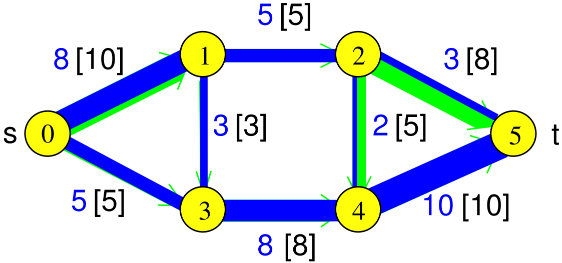

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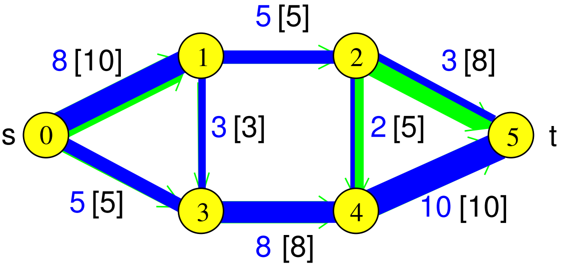


- Modern algorithms (computer science): concurrent flow increments

  [R.E. Tarjan, *Data Struc. + Netw. Algorithms* 1983]

  [A.V. Goldberg, 1988-1998]

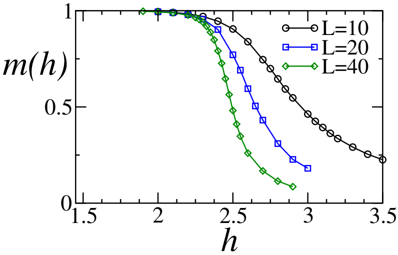
- Exact ground state of large systeme, e.g. with 100^3 spins.

Magnetization

- Magnetization

$$m = \frac{1}{N}\sum_i \sigma_i$$

[AKH, U. Nowak, Eur.Phys.J B 1998]



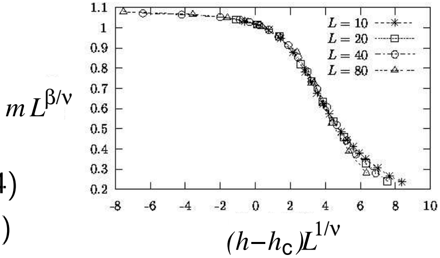
- Finite-size scaling

$$m(h,L) =$$

$$L^{-\beta/\nu}\tilde{m}\left((h-h_c)L^{1/\nu}\right)$$

$$\Rightarrow \quad h_c = 2.29(4)$$

$$\beta = 0.02(1) \quad \nu = 1.19(8)$$

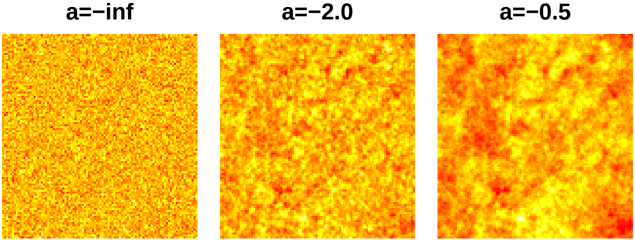


- Susceptibility via $m(H)$, $\chi = \frac{\partial m}{\partial H}\Big|_{H\to 0}$ [AKH, A.P.Young, PRB 2001]

Correlated disorder

Correlation between random fields $\{h_i\}$:

$$\mathcal{C}(|\vec{r}|) = \langle h(\vec{x})h(\vec{x}+\vec{r})\rangle \sim (1+|\vec{r}|^2)^{a/2} \quad a<0$$



$a=-\infty$ $\quad$ $a=-2$ $\quad$ $a=-0.5$

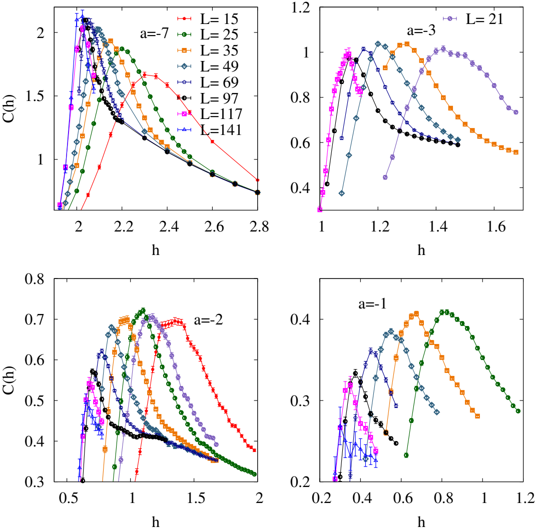
Realization: via Fourier transform of field of random numbers multiplied by correlation kernel

3-dim RFIM

$-3 < a < -2$: criticalality only weakly affected (shifted h_c, same ν, changed γ)

$a = -1, -2$: results compatible with $h_c = 0$

[B. Ahrens & AKH, Phys. Rev. B 2011]



Summary

- Disordered magnets: random-field Ising magnet
- Mapping on maximum-flow problem
  $\rightarrow$ polynomial-time exact algorithms
- Ferromagnet $\leftrightarrow$ Paramagnet transition
- Weak influence of correlation ($-3 \leq a < -2$)
  destruction of transition ($a \geq -2$)
- Thank you for your attention!